JSUNIL TUTORIAL

CH-12 AREAS RELATED TO CIRCLES

VERY SHORT ANSWER TYPE QUESTIONS

1. Tick the correct answer in the following and justify your choice :
1. Mark the correct answer in the following. If the perimeter and area of a circle are numerically equal, then the radius of the circle is :

(a) 2 unit **(b) p units** **(c) 4 units** **(d) 7 units.**

Sol. Perimeter of a circle = area of a circle.
Suppose that 'r' is the radius of a circle.

$2\pi r = p\pi 2 \Rightarrow 2 = r$

$r = 2$ units.

∴ (a) 2 units is the correct answer.

2. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. Length of the blade of each wiper = 25cm (Given)

Therefore, $r = 25$ cm

$\theta = 115°$

Ar. cleaned at each sweep of the blades = 2 (Ar. cleaned at each sweep of one blade)

$$= 2\left(\frac{\pi r^2\theta}{360°}\right) = 2\left(\frac{22}{7} \times \frac{25 \times 25 \times 115°}{360°}\right)$$

$$= \frac{158125}{12}\ cm^2$$

3. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travellng at a speed of 66 km per hour ?

Sol. Radius of a wheel, r = 80 cm

speed of car = 66 km/hrs

$$= \frac{66 \times 1000 \times 100}{60}\ cm/minutes$$

= 110000 cm/minutes

Now Circumference of wheel = $2\pi r = 2 \times \frac{22}{7} \times 80$ cm = 502.86 cm .

4. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. Area of the radius r = 6 cm, θ = 60° (Given)

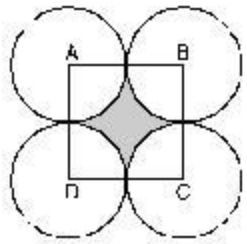
$$\text{Area of sector} = \frac{\pi r^2\theta}{360°} = \frac{22}{7} \times \frac{6 \times 6 \times 60}{360°}$$

$$= \frac{132}{7}\ cm^2$$

CH-12 AREAS RELATED TO CIRCLES

SHORT ANSWER TYPE QUESTION

1. In fig., ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Sol. In the figure, ABCD is a square fo side = 14 cm

Radius of each circle, r = $\frac{14}{2}$ cm = 7 cm

$\theta = 90°$

Ar. of four sectors = 4(Ar. of one sector)

$= 4\left(\frac{\pi r^2\theta}{360°}\right) = 4 \times \frac{22}{7} \times \frac{7 \times 7 \times 90°}{360°} = 154 \text{ cm}^2$

Ar. of four selctors = 154 cm^2

Ar. of square ABCD = $(14)^2 = (14 \times 14) = 196 \text{ cm}^2$

Therefore, Ar. of shaded region = Ar. of square ABCD – Area of four sectors

$= (196 - 154) \text{ cm}^2 = 42 \text{ cm}^2$

2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Sol. The radii of two circles are 8 cm and 6 cm. (Given)

Let $r_1 = 8$ cm

$r_2 = 6$ cm

Therefore $A_1 = \pi r_1^2 = \pi(8)^2 = 64\pi \text{ cm}^2$

$A_2 = \pi r_2^2 = \pi(6)^2 = 36\pi \text{ cm}^2$

Let r be the radius of circle.

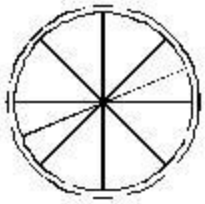
Area of required circle = $A_1 + A_2$

$\pi r^2 = 64\pi + 36\pi = 100\pi$

$r^2 = (10)^2 = 10$ cm

$r = 10$ cm

3. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as hsown in fig. Find:
(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.



Sol. Given a circle with diameter = 35 mm

and radius of the brooch, r = $\frac{35}{2}$ mm

Suppose q be the angle made by each sector at centre.

$$\theta = \frac{360°}{\text{no of sectors}} = \frac{360°}{10°} = 36°$$

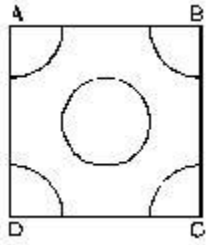
(i) Total length of the silver wire required = 2πr + 5 × (diameter of brooch)

$$= 2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 35 = 110 + 175 = 285 \text{ mm}$$

(ii) Area of each sector of the brooch = $\frac{\pi r^2\theta}{360°}$

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times \frac{36°}{360°} = \frac{385}{4} \text{ mm}^2$$

4. From each corner of a square of side 4 cm a quadrant of a circle of a radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in fig. Find the area of the remaining portion of the square.



Sol. A square ABCD, of side a = 4 cm r_1 = 1 cm

and radius of circle, $r_2 = \frac{2}{2}$ 1 cm

θ = 90°

Ar. of shaded region = Ar. of square – (Ar. of circle at centre of square) –4 (Ar. of sector at corner of square)

$$= (4)^2 - \pi r^2 - 4 \left(\frac{\pi r^2\theta}{360°} \right)$$

$$= (4)^2 - \frac{22}{7} \times 1^2 - 4 \times \frac{22}{7} \times (1)^2 \times \frac{90°}{360°}$$

$$= 16 - \frac{22}{7} - \frac{22}{7} = 16 - \frac{44}{7} = \frac{112 - 44}{7} = \frac{68}{7} \text{ cm}$$

5. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Sol. The radii of two cicles are 19 and 9 cm (Given)

Suppose $r_1 = 19$ cm

$r_2 = 9$ cm

Circumference of first circle, $c_1 = 2\pi r_1$

$= 2\pi (19) = 38\pi$ cm

Circumference of 2nd circle, $c_2 = 2\pi r_2$

$= 2\pi(9) = 18\pi$ cm

Now radius of required circle be r cm.

Circumference of required circle = $c_1 + c_2$

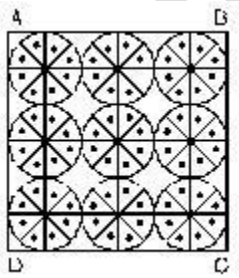
$2\pi r = 38\pi + 18\pi = 56\pi$

$2\pi = 28$ cm

$$\pi = \frac{56}{2}$$

$r = 28$ cm

6. On a square handkerchief nine circular designs each of radius 7 cm are made (see fig.) Find the area of the remaining portion of the handkerchief.

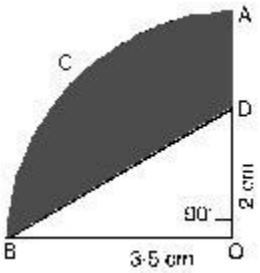


Sol. A square handkerchief nine circular disigns whose each of radius r = 7 cm

Side of square ABCD, a = 14 × 3 = 42 cm (_ *side = sum of diameter of three circular designs*)

Ar. of remaining portion = Ar. of square – 9 (Ar. of circle)

7. In fig., OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB (ii) shaded region.



Sol. In the given figure, OACB is a quadrant of a circle with centre O

and $r = 3.5$ cm

$OD = 2$ cm

$\theta = 90°$

(i) Ar. of quadrant OACB = Ar. of sector OACB

$$= \frac{\pi r^2 \theta}{360°} = \frac{22}{7} \times \frac{3.5 \times 3.5 \times 90°}{360°} = \frac{77}{8} \text{ cm}^2$$

(ii) Ar. of shaded region = Area of sector OACB – Ar. of ΔBOD

$$= \frac{\pi r^2\theta}{360^\circ} - \frac{1}{2} \times OB \times OD$$

$$= \frac{22}{7} \times \frac{3.5}{360^\circ} \times 3.5 \times 90^\circ - \frac{1}{2} \times 3.5 \times 2$$

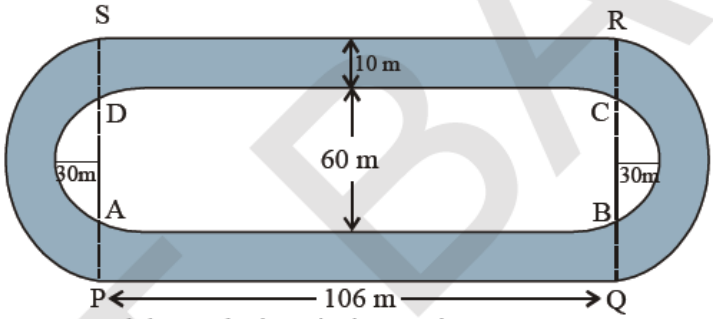
$$= \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \times \frac{1}{4} - \frac{35}{10}$$

$$= \frac{77}{8} - \frac{7}{2} = \frac{77 - 28}{8} = \frac{49}{8}\ cm^2$$

Example 8. *The given figure depicts a racing track whose left and right ends are semicircular. The distance between the inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :*

(i) the distance around the track along its inner edge.

(ii) the area of the track. **[NCERT]**



Solution. (*i*) Distance around the track along its inner edge

$$= 2 \times 106\ m + \text{Perimeter of two semi-circle of radius } \frac{60}{2} m$$

$$= 212\ m + 2 \times \frac{22}{7} \times 30\ m = 212\ m + \frac{1320}{7} m = \frac{2804}{7} m = \mathbf{400.57\ m}$$

(*ii*) Area of the track

$$= 2\ (\text{area of the rectangle SRCD}) + 2\ (\text{area of the semi-circular track})$$

$$= 2\left[106 \times 10 + \frac{1}{2} \times \frac{22}{7} \times (40^2 - 30^2)\right] m^2$$

$$= 2\left[1060 + \frac{1}{2} \times \frac{22}{7} \times (40 + 30)(40 - 30)\right] m^2$$

$$= 2\left[1060 + \frac{1}{2} \times \frac{22}{7} \times 70 \times 10\right] m^2$$

$$= 2\ (1060 + 1100)\ m^2$$

$$= 2\ (2160)\ m^2$$

$$= \mathbf{4320\ m^2}\ \textbf{Ans.}$$