Q.1 Show that the points (−4, −9), (2, 0) and (4, 3) are collinear.

Solution: Let A, B and C be the given points respectively. Then

A → (−4,−9)

B → (2, 0)

$$AB = \sqrt{(2+4)^2 + (0+9)^2} = \sqrt{6^2 + 9^2} = \sqrt{36+81} = \sqrt{117} = \sqrt{9\times 13} = 3\sqrt{13}.$$

Rough Sketch

Figure 7.24

B → (2, 0)

C → (4, 3)

$$BC = \sqrt{(4-2)^2 + (3-0)^2} = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$AC = \sqrt{(4+4)^2 + (3+9)^2} = \sqrt{8^2 + 12^2} = \sqrt{64+144} = \sqrt{208} = \sqrt{16\times 13} = 4\sqrt{13}$$

We observe that $AB + BC = AC$. $3\sqrt{13} + \sqrt{13} = 4\sqrt{13}$.

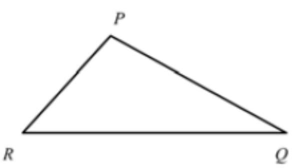
$\therefore$ A, B and C are collinear.

Q.2 Show that the points (3, −2), (2, 5) and (8, −7) form an isosceles triangle

Solution:

Let the given points be P, Q and R respectively. One way of proving that ΔPQR is an isosceles triangle is to show that two of its sides are of equal length. Here we have

Rough Sketch



$$d(P,Q) = \sqrt{(2-3)^2 + (5+2)^2} = \sqrt{1^2 + 7^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}.$$

$$d(Q,R) = \sqrt{(8-2)^2 + (-7-5)^2} = \sqrt{6^2 + 12^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}.$$

$$d(R,P) = \sqrt{(8-3)^2 + (-7+2)^2} = \sqrt{5^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$$

$\therefore d(P,Q) = d(R,P) \neq d(Q,R)$.

$\therefore$ ΔPQR is an isosceles triangle but not an equilateral triangle.

Q.3. Show that the points (0, 3), (0,1) and ()2,3 are the vertices of an equilateral triangle.

Solution: Let the points be A, B and C respectively. One way of showing that ΔABC is an equilateral triangle is to show that all its sides are of equal length. Here we find that

$$d(A,B) = \sqrt{(0-0)^2 + (1-3)^2} = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2.$$

$$d(B,C) = \sqrt{(\sqrt{3}-0)^2 + (2-1)^2} = \sqrt{3+1} = \sqrt{4} = 2.$$

$$d(C,A) = \sqrt{(0-\sqrt{3})^2 + (3-2)^2} = \sqrt{3+1} = \sqrt{4} = 2.$$

$\therefore d(A,B) = d(B,C) = d(C,A)$.

$\therefore \Delta ABC$ is an equilateral triangle.

Q.4. Examine whether the points P (7, 1), Q (−4,−1) and R (4,5) are the vertices of a right triangle.

Solution: The points P ,Q, R form a triangle. To show that ΔPQR is a right triangle, we have to show that one vertex angle is 90°. This is done by showing that the lengths of the sides of the triangle satisfy Pythagoras theorem.

$$PQ = \sqrt{(-4-7)^2 + (-1-1)^2} = \sqrt{121+4} = \sqrt{125} = 5\sqrt{5}.$$
$$QR = \sqrt{(4+4)^2 + (5+1)^2} = \sqrt{64+36} = \sqrt{100} = 10.$$
$$PR = \sqrt{(4-7)^2 + (5-1)^2} = \sqrt{9+16} = \sqrt{25} = 5.$$
$$\therefore PQ^2 = 125,\ QR^2 = 100 \text{ and } PR^2 = 25.$$
We observe that $QR^2 + PR^2 = PQ^2$.
$\therefore$ The Pythagoras formula is satisfied.
$\therefore$ ΔPQR is a right triangle with right angle at R.

Q.5. Show that the points (1, 2), (2, −1), (5, 3) and (4, 6) taken in order form a parallelogram. Is it a rectangle

Solution: Let the points be P1, P2 ,P3 and P4 respectively. One way of showing that P1 P2 P3 P4 is a parallelogram is to show that the opposite sides are of equal length.

$$P_1P_2 = \sqrt{(2-1)^2 + (-1-2)^2} = \sqrt{1+9} = \sqrt{10}.$$
$$P_2P_3 = \sqrt{(5-2)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25}.$$
$$P_3P_4 = \sqrt{(4-5)^2 + (6-3)^2} = \sqrt{1+9} = \sqrt{10}.$$
$$P_4P_1 = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25}.$$
$$\therefore P_1P_2 = P_3P_4 = \sqrt{10} \text{ and } P_2P_3 = P_4P_1 = \sqrt{25}.$$
$\therefore$ $P_1P_2\,P_3P_4$ is a parallelogram. Since
$$P_1P_3 = \sqrt{(5-1)^2 + (3-2)^2} = \sqrt{16+1} = \sqrt{17} \text{ and}$$
$$(P_1P_2)^2 + (P_2P_3)^2 = 10 + 25 = 35,\ (P_1P_3)^2 = 17,\quad (P_1P_2)^2 + (P_2P_3)^2 \neq (P_1P_3)^2.$$
$\therefore$ $\Delta P_1P_2\,P_3$ is not a right triangle.
$\therefore$ $\angle P_1P_2\,P_3$ is not a right angle.
$\therefore$ $P_1P_2\,P_3P_4$ is not a rectangle.

Q.6. Show that the points (0, −1), (−2, 3), (6, 7) and (8, 3), taken in order form the vertices of a rectangle.

Solution: Let the points be *A*, *B*, *C* and *D* respectively. One way of showing that *ABCD* is rectangle is to show that the opposite sides are of equal length and one corner angle is 90°. One way of showing that one corner angle is 90° is to show that the lengths of the sides of Δ*ABC* satisfy the Pythagoras theorem.

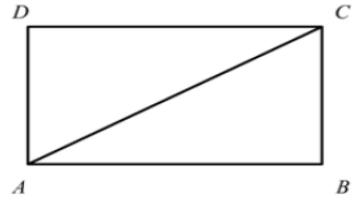
$AB = \sqrt{(-2-0)^2 + (3+1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}.$

$BC = \sqrt{(6+2)^2 + (7-3)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}.$

$CD = \sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}.$

$AD = \sqrt{(8-0)^2 + (3+1)^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}.$

$AC = \sqrt{(6-0)^2 + (7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$



We observe that $AB = CD = 2\sqrt{5}$, $BC = AD = 4\sqrt{5}$

and $AB^2 + BC^2 = 20 + 80 = 100 = AC^2$

$\therefore$ $ABCD$ is a rectangle but not a square.

Q.7 Show that the points (0, −1), (2, 1) (0, 3) and (−2, 1) taken in order form the vertices of a square.

Solution: Let *A*, *B*, *C*, *D* be the given points respectively.

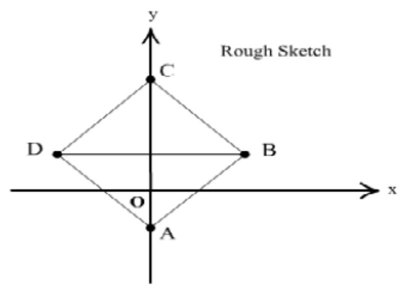
One way of showing that *ABCD* is a square is to show that all its sides are of equal length and the diagonals are of equal length.

$AB = \sqrt{(2-0)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2},$

$BC = \sqrt{(0-2)^2 + (3-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2},$

$CD = \sqrt{(-2-0)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2},$

$AD = \sqrt{(-2-0)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2},$



$BD = \sqrt{(-2-2)^2 + (1-1)^2} = \sqrt{16+0} = \sqrt{16} = 4,$

$AC = \sqrt{(0-0)^2 + (3+1)^2} = \sqrt{0+16} = \sqrt{16} = 4.$

We observe here that

$AB = BC = CD = AD = 2\sqrt{2}$ and $BD = AC = 4.$

$\therefore$ $ABCD$ is a square.

Prove that the points *A*(2, −3), *B*(6, 5), *C*(−2, 1) and *D*(−6, −7), taken in order form a rhombus but not a square.

Solution: One way of showing that *ABCD* is a rhombus is to show that all its sides are of equal length. One way is showing that a rhombus is not a square is to show that the diagonals are of unequal length

$AB = \sqrt{(6-2)^2 + (5+3)^2}$
$= \sqrt{16+64}$
$= \sqrt{80}$

$BC = \sqrt{(-2-6)^2 + (1-5)^2}$
$= \sqrt{64+16} = \sqrt{80}$

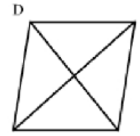
$AC = \sqrt{(-2-2)^2 + (1+3)^2}$
$= \sqrt{16+16} = \sqrt{32}$

$BD = \sqrt{(-6-6)^2 + (-7-5)^2}$
$= \sqrt{144+144} = \sqrt{288}$

$CD = \sqrt{(-6+2)^2 + (-7-1)^2}$
$= \sqrt{16+64} = \sqrt{80}$

$AD = \sqrt{(-6-2)^2 + (-7+3)^2}$
$= \sqrt{64+16} = \sqrt{80}$.

$\therefore AB = BC = CD = AD, AC \neq BD.$
$\therefore$ *ABCD* is a rhombus but not a square.



The area of a triangle is 5. Two of its vertices area (2, 1) and (3, -2). The third vertex lies on y = x + 3. Find the third vertex.

Let the third vertex be (x_3, y_3) area of triangle $= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

As $x_1 = 2\, y_1 = 1\,;\, x_2 = 3,\, y_2 = -2\,;$ Area of Δ = 5 sq. unit $\Rightarrow$ $5 = \frac{1}{2}\left|2(-2-y_3) + 3(y_3 - 1) + x_3(1+2)\right|$

$\Rightarrow$ $10 = |3x_3 + y_3 - 7|$ $\Rightarrow$ $3x_3 + y_3 - 7 = \pm 10$

Taking positive sign

$3x_3 + y_3 - 7 = 10 \Rightarrow$ $3x\therefore + y_3 = 17$ .....(i)

Taking negative sing

$\Rightarrow$ $3x_3 + y_3 - 7 = -10$

$\Rightarrow$ $3x\therefore + y_3 = -3$ .....(ii)

Given that (x_3, y_3) lies on $y = x + 3$

So, $-x\therefore + y_3 = 3$ ....(iii)

Solving eq. (i) & (iii)

$x_3 = \frac{7}{2}$, $y_3 = \frac{13}{2}$

Solving eq. (ii) & (iii)

$x_3 = \frac{-3}{2}$, $y_3 = \frac{3}{2}$

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

In what ratio does the X-axis divide the line segment joining the points (2, -3) and (5, 6) ?

Let the required ratio be k : 1. Then the coordinates of the point of division are $\left(\frac{5\lambda+2}{k+1}, \frac{6\lambda-3}{k+1}\right)$. But, it is a point on X-axis on which y-coordinate of every point is zero.

$\therefore$ $\frac{6\lambda - 3}{k+1} = 0$ $\Rightarrow$ $k = \frac{1}{2}$ Thus, the required ratio is $\frac{1}{2} : 1$ or $1 : 2$.

P(x, y), A (5, 1) and B (-1, 5) are the given points. AP = BP (Given) $\therefore$ $AP^2 = BP^2$

or $AP^2 - BP^2 = 0$ or $\{(x-5)^2 + (y-1)\}^2 - \{(x+1)^2 + (y-5)^2\} = 0$

or $x^2 + 25 - 10x + y^2 + 1 - 2y - x^2 - 1 - 2x - y^2 - 25 + 10y = 0$ or $-12x + 8y = 0$ or $3xx = 2y$.