

Mathematics (Standard)- Theory

Time allowed: 3 hours

Maximum marks : 80

General instruction

- (i) This question paper comprise four sections – A, B, C and D this question paper carries 40 questions . All question are compulsory.
- (ii) Section A: Q. No.1 to 20 comprises of 20 questions of one marks each.
- (iii) Section B :Q. No.21 to 26 comprises of 6 questions of two marks each.
- (iv) Section C :Q. No.27 to 34 comprises of 8 questions of three marks each.
- (v) Section D :Q. No.35 to 40 comprises of 6 questions of four marks each.
- (vi) There is no overall choice in the question paper . However, an internal choice has been provided in 2 questions of one marks each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choice in such questions.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is not permitted.

Section A

Q. Nos. 1 to 10 are multiple choice type questions of 1 mark each. Select the correct option.

1. The HCF and the LCM of 12, 21,15 respectively are
- | | |
|------------|-------------|
| (a) 3, 140 | (b) 12, 420 |
| (c) 3, 420 | (d) 420, 3 |

Answer: (c)

$$12 = 2^2 \times 3$$

$$21 = 7 \times 3 \text{ H.C.F.} = 3$$

$$15 = 5 \times 3 \text{ L.C.M.} = 24 \times 7 \times 5 = \times 3 = 420$$

2. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an AP, is 4
- | | |
|--------|---------|
| (a) 6 | (b) -6 |
| (c) 18 | (d) -18 |

Answer: (a)

$$2b = a + c$$

$$2(x + 10) = 2x + 3x + 2$$

$$18 = 3x \Rightarrow x = 6$$

3. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is
- (a) -2 (b) $\neq 2$
(c) 3 (d) 2

Answer: (b)

$$x + y - y = 0$$

$$2x + 1c - 3 = 0$$

$$\frac{1}{2} = \frac{1}{1c} \Rightarrow \frac{-4}{-3}$$

4. The first term of an AP is p and the common difference is q , then its 10th term is
- (a) $q + 9p$ (b) $p - 9q$
(c) $p + 9q$ (d) $2p + 9q$

Answer: (c)

$$10^{\text{th}} \text{ term} = p + (10 - 1)q = p + 9q$$

5. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6 , is
- (a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$
(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Answer: (a)

$$\alpha + \beta = -5, \alpha\beta = 6$$

$$x^2 + 5x + 6 = 0$$

6. The distance between the points $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$, is
- (a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

Answer: (c)

$$d = \sqrt{(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2} = \sqrt{a^2 + b^2}$$

7. The total number of factors of a prime number is
- (a) 1 (b) 0
(c) 2 (d) 3

Answer: (c)

Correct option is (c)

8. If the point P(k, 0) divides the line segment joining the points A(2, -2) and B(-7, 4) in the ratio 1 : 2, then the value of k is
- (a) 1 (b) 2
(c) -2 (d) -1

Answer: (d)

Diagram

$$K = \frac{1 \times 7 + 2 \times 2}{1 + 2} = \frac{-3}{3} = -1$$

9. The value of p, for which the points A(3,1), B(5, p) and C(7, -5) are collinear, is
- (a) -2 (b) 2
(c) -1 (d) 1

Answer: (b)

A(3, 1), B(5, P) & C(7, -5)

10. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
- (a) 10 (b) -10
(c) -7 (d) -2

Answer: (b)

$$x^2 + 3x + K = 0$$

$$\alpha = 2, \beta$$

$$2 \times \beta = -3$$

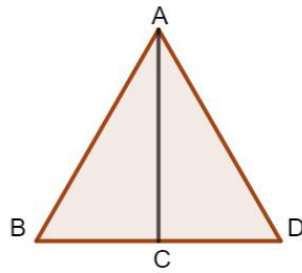
$$2 + \beta = -3$$

$$K = -1 \leftarrow \frac{K}{2} = -3$$

In Q. No, s. 11 to 15, fill in the blanks. Each question is of 1 m.

11. ABC is an equilateral triangle of side 2a, then length of one of its altitude is_____.

Answer: $\sqrt{3}a$



We have,

$$AB = 2a$$

$$BC = \frac{1}{2} \times 2a = a$$

In $\triangle ACB$, By Pythagoras theorem

$$AB^2 = AC^2 + BC^2$$

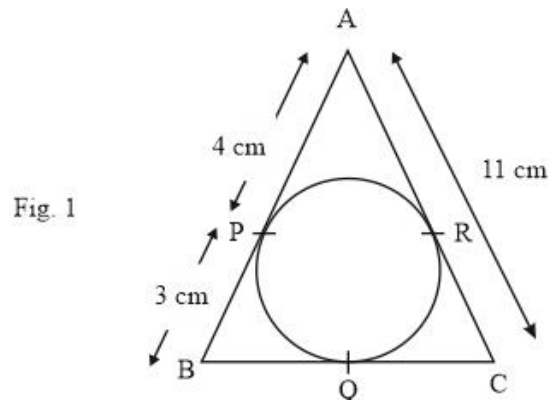
$$\Rightarrow (2a)^2 = AC^2 + a^2$$

$$\Rightarrow 4a^2 = AC^2 + a^2$$

$$\Rightarrow AC^2 = 3a^2$$

$$\Rightarrow AC = \sqrt{3} a$$

12. In Fig. 1, $\triangle ABC$ is circumscribing a circle, the length of BC is ____ cm.



Answer: 10 cm

$$AP = AR = 4 \text{ cm}$$

$$CR = AC - AR = 11 - 4 = 7 \text{ cm}$$

$$CR = CQ = 7 \text{ cm}$$

$$BQ = PB = 3 \text{ cm}$$

$$BC = BQ + CQ$$

$$3 + 7 = 10$$

13. The value of $\left(\sin^2 \theta + \frac{1}{1 + \tan^2 \theta} \right) =$ _____.

OR

The Value of $(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta) =$ _____.

Answer: 1

$$\sin^2 \theta + \frac{1}{\sec^2 \theta} = 1$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

OR

$$\begin{aligned} & (\sec^2 \theta)(1 - \sin^2 \theta) \\ & = \sec^2 \theta \times \cos^2 \theta \end{aligned}$$

$$14. \left(\frac{\sin 35^\circ}{\cos 55^\circ} \right)^2 + \left(\frac{\cos 43^\circ}{\sin 47^\circ} \right) - 2 \cos 60^\circ = \underline{\hspace{2cm}}.$$

Answer: 1

$$\frac{\sin^2 35^\circ}{\cos^2 55^\circ} + \frac{\cos^2 43^\circ}{\sin^2 47^\circ} - 2 \times \frac{1}{2} = 1 + 1 - 1 = 1$$

15. ABC and BDE are two equilateral triangles such the D is the mid-point of BC. Ratio of the areas of triangles ABC and ADE is_____.

Answer: 4

$$\frac{\text{area } \Delta ABC}{\text{ar}(\Delta BDE)} = \frac{\frac{S_3}{4} a^2}{\frac{S_3}{4} \left(\frac{A}{2} \right)^2} = 4$$

Q. Nos. 16 to 20 short answer type questions of 1 mark each.

16. A die is thrown once. What is probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Answer: $\frac{1}{3}$

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$P(\text{Number of less than 3}) = \frac{2}{6} = \frac{1}{3}$$

OR

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Probability of losing = 1 - Probability of winning

$$= 1 - 0.07 = 1 - \frac{7}{100} = \frac{93}{100} = .93$$

17. If the mean of the first n natural number is 15, then find n.

Answer: 29

$$\frac{1+2+\dots+x}{x} = 15 \Rightarrow \frac{x(x+1)}{x^2} = 15$$

$$x + 1 = 30$$

$$x = 29$$

18. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Answer: 3

$$V_1 = \frac{1}{3}\pi(3r)^2h$$

$$V_2 = \frac{1}{3}\pi r^2 3h = 3$$

19. Find the angle of elevation of the sun at that moment?

The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$.

Answer: 30°

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

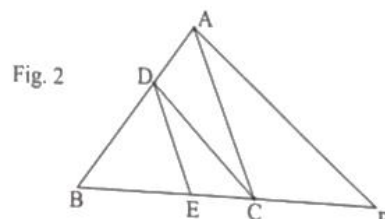
20. A die is thrown once. What is the probability of getting an even prime number?

Answer: $\frac{1}{6}$

$$S = \{1, 2, 3, 4, 5, 6\}, E = \{2\}$$

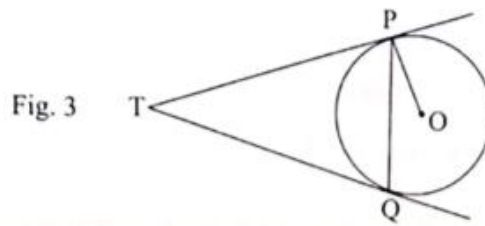
$$\text{Probability} = \frac{1}{6}$$

21. In Fig. 2 DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$



OR

In Fig, 3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2 \angle OPQ$.



Solution:

$\because DE \parallel AC$, By basic proportionality theorem

$$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC} \quad [1]$$

Also, $DC \parallel AP$

$$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP} \quad [2]$$

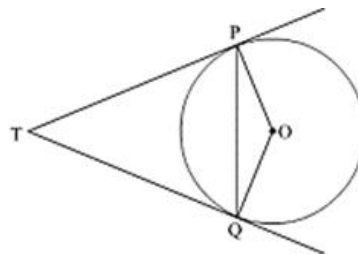
From [1] and [2], we get

$$\Rightarrow \frac{BE}{EC} = \frac{BC}{CP}$$

Hence, proved!

OR

Let TP and TQ are two tangents of a circle at points P and Q respectively with center O.



To prove: $\angle PTQ = 2\angle OPQ$

Let $\angle PTQ = \theta$

As lengths of tangents drawn from an external point to the circle are equal, therefore $TP = TQ$.

$\therefore \triangle PQT$ is an isosceles triangle.

$$\therefore \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta$$

Also, tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - (90^\circ - \frac{1}{2}\theta)$$

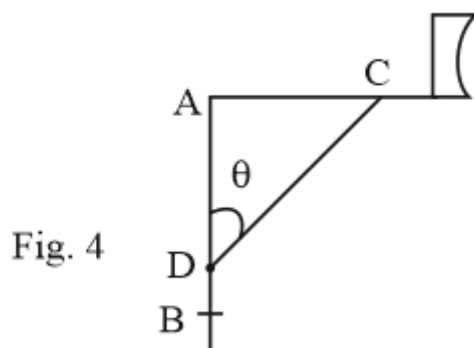
$$= \frac{1}{2}\theta$$

$$= \frac{1}{2}\angle PTQ$$

Thus, $\angle PTQ = 2\angle OPQ$.

Hence, proved.

22. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in fig. 4. If AC = 1.5 m long and CD = 3 m, find (i) $\tan \theta$ (ii) $\sec \theta + \operatorname{cosec} \theta$.



Solution:

We have,

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{CD}$$

$$\Rightarrow \sin \theta = 1.5/3 = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

$$(i) \quad \tan \theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$(ii) \quad \sec \theta + \operatorname{cosec} \theta = \sec 30^\circ + \operatorname{cosec} 30^\circ = \frac{2}{\sqrt{3}} + 2$$

$$= 2 \left(\frac{1}{\sqrt{3}} + 1 \right) = \frac{2(1 + \sqrt{3})}{\sqrt{3}}$$

23. If a number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Solution:

Total number of outcomes = 7

The numbers for which $x^2 \leq 4$ are -1, -2, 0, 1, 2

Favorable outcomes = 5

$$P(x^2 \leq 4) = \frac{5}{7}$$

24. Find the mean of the following distribution:

Class:	3-5	5-7	7-9	9-11	11-13
Frequency:	5	10	10	7	8

OR

Find the mode of the following data:

Class:	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency:	6	8	10	12	6	5	3

Solution:

Class	Frequency (f _i)	Class Mark (x _i)	f _i x _i
3-5	5	4	20
5-7	10	6	60
7-9	10	8	80
9-11	7	10	70
11-13	8	12	96
	Σf _i = 40		Σf _i x _i = 323

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{323}{40} = 8.075$$

OR

Class	Frequency
0-20	6
20-40	8
40-60	10
60-80	12
80-100	6
100-120	5
120-140	3

Model class = 60-80

Lower limit of model class, l = 60

Frequency of model class, f₁ = 12

Frequency of class preceding model class, f₀ = 10

Frequency of class following model class, f₂ = 6

Height of model class, h = 10

We know,

$$\begin{aligned} \text{Mode} &= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \\ &= 60 + \frac{12 - 10}{24 - 10 - 6} \times 10 \\ &= 60 + 0.4 \\ &= 60.4 \end{aligned}$$

25. Find the sum of first 20 terms of the following AP :

1,4,7,10,_____ .

Solution:

We have,

First term, $a = 1$

Common difference, $d = 3$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

Sum of 20 terms

$$\begin{aligned} S_{20} &= 10(2(1) + 19(3)) \\ &= 10(2 + 57) \\ &= 590 \end{aligned}$$

26. The perimeter of a sector of a circle of radius 5.2 cm is 16.4cm. Find the area of the sector.

Solution:

$$\text{Perimeter of sector} = 2r + \frac{\theta}{180}\pi r$$

We have to find $\frac{\theta}{360}\pi r^2$

$$\Rightarrow 16.4 = 2(5.2) + \frac{\theta}{180}\pi r$$

$$\Rightarrow 16.4 - 10.4 = \frac{\theta}{180}\pi r$$

$$\Rightarrow 6 = \frac{\theta}{180}\pi r$$

Multiplying both side by $(r/2)$, we have

$$\Rightarrow \frac{6r}{2} = \frac{\theta}{360}\pi r^2$$

$$\Rightarrow \text{Area of sector} = \frac{6(5.2)}{2} = 15.6 \text{ cm}^2$$

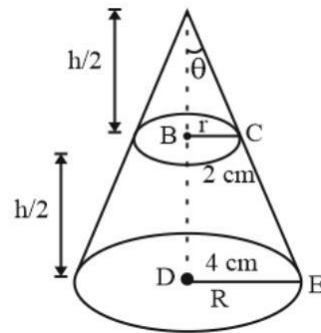
Section-C

Q. Nos. 27 to 34 carry 3 marks each.

27. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-points of its height and parallel to its base. Compare the volume of the two parts.

Solution:

volume of the cone = $\frac{1}{3}\pi r^2 h$, r = radius, h = Height)



When cone is divided in two parts , upper part is cone and bottom part is frustum .

In $\triangle ABC$, $\tan\theta = BC/AB$

In $\triangle ADE$

$$\tan\theta = \frac{DE}{AD}$$

$$\Rightarrow \frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{BC}{\frac{h}{2}} = \frac{4}{h}$$

$$\Rightarrow BC = \frac{4}{2} = 2\text{cm}$$

$$\frac{\text{Volume of upper part}}{\text{Volume of lower part}} = \frac{\frac{1}{3}\pi r^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi\left(\frac{h}{2}\right)(r^2 + R^2 + rR)}$$

$$= \frac{r^2}{r^2 + R^2 + rR} = \frac{2^2}{2^2 + 4^2 + 2(4)}$$

$$= \frac{4}{4+16+8}$$

$$= \frac{1}{7}$$

28. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

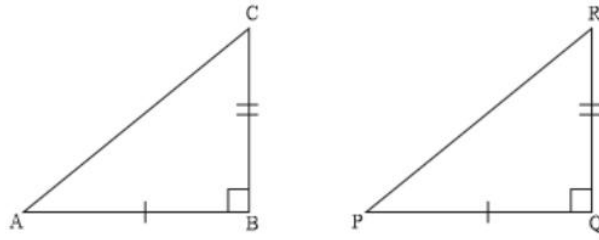
Solution:

Consider $\triangle ABC$ such that $AC^2 = AB^2 + BC^2$

To prove : $\angle B = 90^\circ$

Construction : We construct another $\triangle PQR$ right angled at Q such that $PQ = AB$ and $QR = BC$

In ΔPQR , by Pythagoras theorem, we have



$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow PR^2 = AB^2 + BC^2$$

$$\Rightarrow PR^2 = AC^2 \left[\because AC^2 = AB^2 + BC^2 \right]$$

$$\Rightarrow PR = AC$$

In ΔABC & ΔPQR

$$AB = PQ \text{ [by construction]}$$

$$BC = QR \text{ [by construction]}$$

$$AC = PR \text{ [Proved above]}$$

$$\Delta ABC \cong \Delta PQR$$

$$\angle B = \angle Q = 90^\circ \text{ [by CPCT]}$$

Hence proved

29. Find the area of triangle PQR formed by the points $P(-5,7)$, $Q(-4,-5)$ and $R(4,5)$

OR

If the point $C(-1,2)$ divides internally the line segment joining $A(2,5)$ and $B(x, y)$ in the ratio 3: 4, find the coordinates of B.

Solution:

Area of triangle

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\frac{1}{2} |-5(-5 - 5) + (-4)(5 - 7) + 4(7 + 5)|$$

$$\frac{1}{2} |50 + 8 + 48|$$

$$= 25 + 4 + 24$$

$$= 53 \text{ sq. units}$$

OR

$$(x_1, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$(-1, 2) = \frac{3(x) + 4(2)}{3+4}, \frac{3y + 4(5)}{3+4}$$

$$\frac{3x+8}{7} = -1$$

$$\Rightarrow 3x+8 = -7$$

$$\Rightarrow 3x = -15$$

$$\Rightarrow x = -5$$

$$\frac{3y+20}{7} = 2$$

$$\Rightarrow 3y+20 = 14$$

$$\Rightarrow 3y = -6$$

$$\Rightarrow y = -2$$

$$(-5, -2)$$

30. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Solution: $ax^2 + bx + c$, let its zeroes be α and β we have to find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Now, to form equation we need to find $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ and $\left(\frac{1}{\alpha\beta}\right)$

\therefore Required quadratic equation will be

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} \quad (i)$$

Now,

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} \Rightarrow \frac{1}{\alpha\beta} = \frac{a}{c}$$

$$\begin{aligned} &= \frac{-b}{\frac{c}{a}} = \frac{-b}{\frac{c}{a}} \\ &= \frac{-b}{\frac{c}{a}} \end{aligned}$$

$$\therefore \text{required equation} = x^2 + \frac{b}{c}x + \frac{a}{c}$$

$$= \frac{1}{c}(cx^2 + bx + a)$$

$$\begin{aligned}
 g(x) &= x - 1 - x^2 \\
 &= -x^2 + x - 1 \\
 f(x) &= 3x^2 - x^3 - 3x + 5 \\
 &= -x^3 + 3x^2 - 3x + 5 \\
 &\quad \begin{array}{r}
 -x^2 + x - 1 \overline{) -x^3 + 3x^2 - 3x + 5} \\
 \underline{-x^3 + x^2 - x} \\
 (+) \quad (-) \quad (+) \\
 \\
 \underline{2x^2 - 2x + 5} \\
 \underline{2x^2 - 2x + 5} \\
 (-) \quad (+) \quad (-) \\
 \underline{} \\
 3
 \end{array}
 \end{aligned}$$

According to division algorithm

$$F(x) = g(x) \cdot q(x) + r(x)$$

$$q(x) = -x^2 + x - 1$$

$$q(x) = x - 2$$

$$r(x) = 3$$

$$\text{RHS} = (-x^2 + x - 1)(x - 2) + 3$$

$$= -x^3 + 2x^2 + x^2 - 2x - x + 2 + 3$$

$$= -x^3 + 3x^2 - 3x + 5$$

$$= \text{LHS}$$

Hence verified

31. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

$$2y - x = 8$$

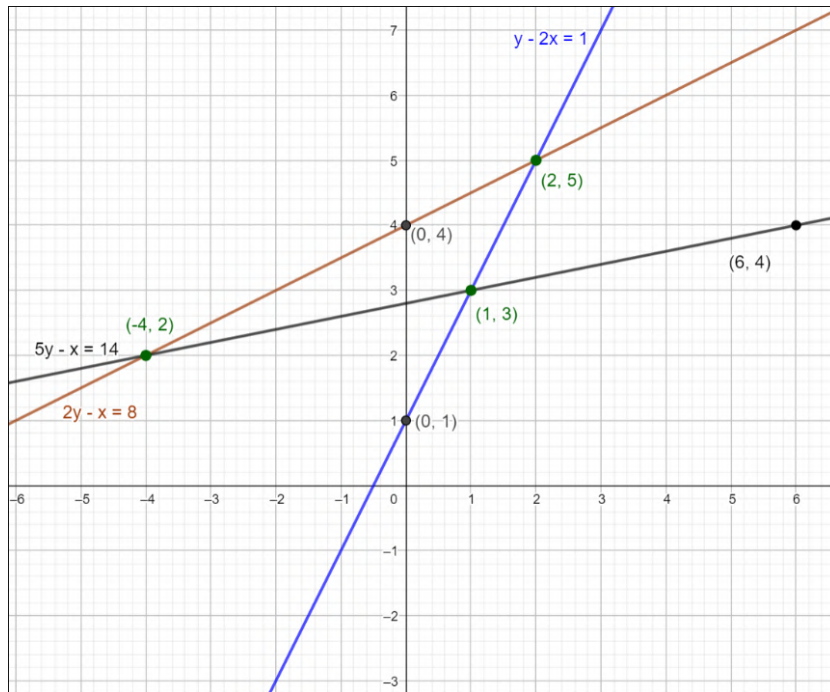
x	0	2
y	4	5

$$5y - x = 14$$

x	1	6
y	3	4

$$y - 2x = 1$$

x	0	1
y	1	3



Vertices of triangle are $(2, 5)$, $(1, 3)$ and $(-4, 2)$

OR

$x = 4$ is a zero of $(x) = x^3 - 3x^2 - 10x + 24$

$(x - 4)$ is a factor of $x^3 - 3x^2 - 10x + 24$

Now

$$x^3 - 3x^2 - 10x + 24$$

$$= x^3 - 4x^2 + x^2 - 4x - 6x + 24$$

$$= x^2(x - 4) + x(x - 4) - 6(x - 4)$$

$$= (x - 4)(x^2 + x - 6)$$

$$= (x - 4)(x^2 + 3x - 2x - 6)$$

$$= (x - 4)\{x(x + 3) - 2(x + 3)\}$$

$$= (x - 4)(x - 2)(x + 3)$$

\therefore Zero are $x = 4$, $x = 2$ and $x = -3$

32. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Solution:

Let the speed of train be 'x' km/h.

$$\text{Time taken to cover 480 km} = \frac{480}{x} \left[\because \text{time} = \frac{\text{distance}}{\text{Speed}} \right]$$

Given,

Time taken with speed (x - 8) = Time taken originally + 3 hours

$$\frac{480}{x-8} = \frac{480}{x} + 3$$

$$\Rightarrow 480 \left(\frac{1}{x-8} - \frac{1}{x} \right) = 3$$

$$\Rightarrow 160 \left(\frac{x-x+8}{x(x-8)} \right) = 3$$

$$\Rightarrow 160 \times 8 = x^2 - 8x$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

$$\Rightarrow x^2 - 40x + 32x - 1280 = 0$$

$$\Rightarrow x(x-40) + 32(x-40) = 0$$

$$\Rightarrow (x-40)(x+32) = 0$$

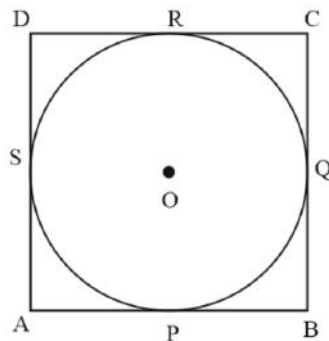
$$\Rightarrow x = 40 \text{ or } x = -32 \text{ [Negative speed not possible]}$$

$$\therefore \text{Speed} = 40 \text{ km/h.}$$

33. Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Consider a parallel gram ABCD circumscribing a circle with center O.



To Prove: ABCD is a rhombus or $AB = BC = CD = AD$

We know, tangents from an external point to a circle are equal

$$AP = AS$$

$$BP = BQ$$

$$DR = DS$$

$$CR = CQ$$

Adding above equations, we get

$AP + BP + CR + DR = AS + DS + BQ + CQ$
 $AB + CD = AD + BC$
 Now, In a parallelogram $AB = CD$ and $BC = AD$
 $\Rightarrow AB + AB = BC + BC$
 $\Rightarrow AB = BC$
 $\Rightarrow AB = BC = CD = AD$
 Hence, Proved!

34. Prove that : $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$.

Solution: $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$

We have,

$$\begin{aligned}
 \sin^4 \theta + \cos^4 \theta &= (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta \\
 &= 1 - 2\sin^2 \theta \cos^2 \theta \\
 \sin^6 \theta + \cos^6 \theta &= (\sin^2 \theta + \cos^2 \theta)(\sin^4 \theta - \sin^2 \theta \cos^2 \theta + \cos^4 \theta) \\
 &= (\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta) \\
 &= (1 - 2\sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta) \\
 &= (1 - 3\sin^2 \theta \cos^2 \theta) \\
 \therefore \text{LHS} &= 2(1 - 3\sin^2 \theta \cos^2 \theta) - 3(1 - 2\sin^2 \theta \cos^2 \theta) + 1 \\
 &= 2 - 6\sin^2 \theta \cos^2 \theta - 3 + 6\sin^2 \theta \cos^2 \theta + 1 \\
 &= 0 \\
 &= \text{RHS.}
 \end{aligned}$$

Section D

Q. Nos. 35 to 40 carry 4 marks each.

35. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40-45	45-50	50-55	55-60	60-65	65-70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

OR

The median of the following data is 525. Find the values of x and y, if total

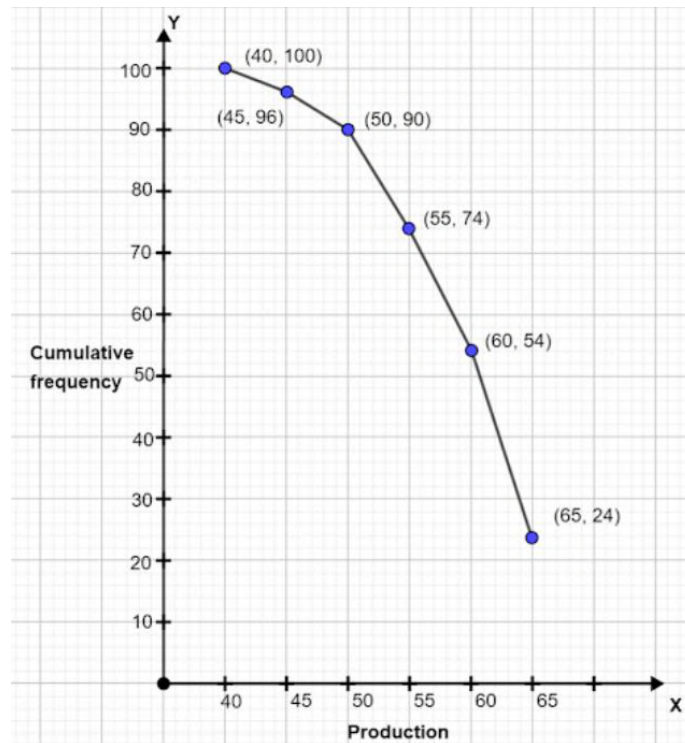
Class:	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency:	2	5	x	12	17	20	y	9	7	4

Solution:

Production	No. of Farms
40 - 45	4
45 - 50	6
50 - 55	16
55 - 60	20
60 - 65	30
65 - 70	24

Production yield (More than type)	C.F
More than 40	100
More than 45	$100 - 4 = 96$
More than 50	$96 - 6 = 90$
More than 55	$90 - 16 = 74$
More than 60	$74 - 20 = 54$
More than 65	$54 - 30 = 24$

Plot points (40, 100) (45, 96) (50, 90) (55, 74) (60,54) (65, 24) on a graph



OR

Class-interval	Frequency	Cumulative Frequency
0 – 100	2	2
100 – 200	5	2 + 5 = 7
200 – 300	x	7 + x
300 – 400	12	7 + x + 12 = 19 + x
400 – 500	17	19 + x + 17 = 36 + x (F)
500 – 600	20(f)	36 + x + 20 = 56 + x
600 – 700	y	56 + x + y
700 – 800	9	56 + x + y + 9 = 65 + x + y
800 – 900	7	65 + x + y + 7 = 72 + x + y
900 – 1000	4	72 + x + y + 4 = 76 + x + y

Given Median = 525

Then, median Class = 500-600

the lower limit (l) = 500

cumulative frequency of the class preceding 500-600(cf) = 36 + x

frequency of the median class 500-600 = 20,

class size (h) = 100

Total frequencies (n) = 100

So, 76 + x + y = 100

$\Rightarrow x + y = 100 - 76$

$\Rightarrow x + y = 24 \dots(i)$

and $\frac{n}{2} = \frac{100}{2} = 50$

Using the formula, $\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$, we have

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$\Rightarrow 525 - 500 = \frac{14 - x}{20} \times 100$$

$$\Rightarrow 25 = (14 - x) \times 5$$

$$\Rightarrow 5 = 14 - x$$

$$\Rightarrow x = 9$$

Putting the value of x in eq. (i), we get

$$\Rightarrow 9 + y = 24$$

$$\Rightarrow y = 24 - 9$$

$$\Rightarrow y = 15$$

36. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

Solution:

AB \rightarrow Flag staff

BC \rightarrow Tower of height (h)

DC \rightarrow km (say)

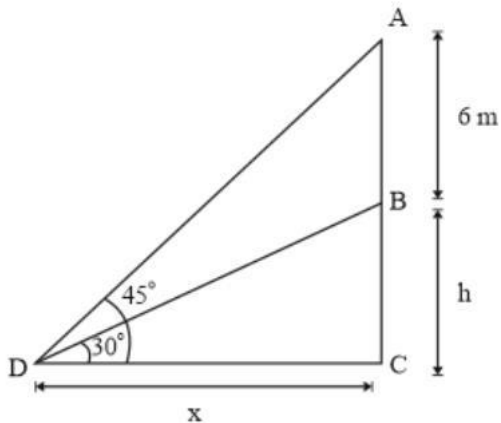
In ΔBDC

$$\tan 30^\circ = \frac{BC}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3} \text{ -----(i)}$$

In $\triangle ADC$



$$\tan 45^\circ = \frac{AC}{DC}$$

$$1 = \frac{AC}{DC}$$

$$1 = \frac{AB+BC}{DC}$$

$$1 = \frac{6+h}{x}$$

$$x = 6 + h \text{ -----(ii)}$$

From Eq. (i) and (ii)

$$h\sqrt{3} = 6+h$$

$$h(\sqrt{3} - 1) = 6$$

$$h = \frac{6}{\sqrt{3}-1} \text{ or } \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6(\sqrt{3}+1)}{2}$$

$$h = 3(\sqrt{3}+1)$$

37. Show that the square of any positive integer cannot be of the form $(5q+2)$ or $(5q+3)$ for any integer q .

OR

Prove that one of every three consecutive positive integers is divisible by 3 .

Solution:

By Euclid's division algorithm, By Euclid's Lemma, $b = a \times q + r$, $0 \leq r < a$

Here, b is any positive integer.

Let a be an arbitrary positive integer. Then corresponding to the positive integers a and 5 , there exist non-negative integers m and r such that

$$a = 5m + r, \text{ where } 0 \leq r < 5$$

Squaring both the sides using $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow a^2 = (25m^2 + r^2 + 10mr)$$

$$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2$$

Where $0 \leq r < 5$

Case I When $r = 0$ we get

$$a^2 = 5(5m^2)$$

$$\Rightarrow a^2 = 5q$$

where $q = 5m^2$ is an integer.

Case II When $r = 1$ we get

$$a^2 = 5(5m^2 + 2m) + 1$$

$$\Rightarrow a^2 = 5q + 1$$

where $q = (5m^2 + 2m)$ is an integer.

Case III When $r = 2$ we get

$$\Rightarrow a^2 = 5(5m^2 + 4m) + 4$$

$$\Rightarrow a^2 = 5q + 4$$

Where $q = (5m^2 + 4m)$ is an integer.

Case IV When $r = 3$ we get

$$\Rightarrow a^2 = 5(5m^2 + 6m) + 9 = 5(5m^2 + 6m) + 5 + 4$$

$$\Rightarrow a^2 = 5(5m^2 + 6m + 1) + 4 = 5q + 4$$

where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V when $r = 4$ we get

$$\Rightarrow a^2 = 5(5m^2 + 8m) + 16 = 5(5m^2 + 8m) + 15 + 1$$

$$\Rightarrow a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$$

where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

OR

By Euclid's division algorithm, By Euclid's Lemma, $b = a \times q + r$, $0 \leq r < a$

Here, b is any positive integer.

Let a be an arbitrary positive integer. Then corresponding to the positive integers a and 5 , there exist non-negative integers m and r such that

$$a = 5m + r, \text{ where } 0 \leq r < 5$$

Squaring both the sides using $(a + b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow a^2 = (25m^2 + r^2 + 10mr)$$

$$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2$$

Where $0 \leq r < 5$

Case I When $r = 0$ we get

$$a^2 = 5(5m^2)$$

$$\Rightarrow a^2 = 5q$$

where $q = 5m^2$ is an integer.

Case II When $r = 1$ we get

$$a^2 = 5(5m^2 + 2m) + 1$$

$$\Rightarrow a^2 = 5q + 1$$

where $q = (5m^2 + 2m)$ is an integer.

Case III When $r = 2$ we get

$$\Rightarrow a^2 = 5(5m^2 + 4m) + 4$$

$$\Rightarrow a^2 = 5q + 4$$

Where $q = (5m^2 + 4m)$ is an integer.

Case IV When $r = 3$ we get

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where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V when $r = 4$ we get

$$\Rightarrow a^2 = 5(5m^2 + 8m) + 16 = 5(5m^2 + 8m) + 15 + 1$$

$$\Rightarrow a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$$

where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

38. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7: 15 . Find the numbers.

OR

$$\text{Solve: } 1+4+7+10+\dots+x = 287$$

Solution:

Let the four consecutive terms of AP be

$$(a - 3d), (a - d), (a + d), (a + 3d)$$

$$\text{Case I : } a - 3d + a - d + a + d + a + 3d = 32$$

$$4a = 32$$

$$a = 8$$

CASE II :

$$\frac{(a-3d)(a+3d)}{(a-d)} = \frac{7}{15}$$

$$\frac{(8-3d)(8+3d)}{(8-d)(8+d)} = \frac{7}{15}$$

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$15(64-9d^2) = 7(64-d^2)$$

$$960-135d^2 = 448d^2$$

$$960-448 = 135d^2 - 7d^2$$

$$512 = 128d^2$$

$$\Rightarrow d^2 = \frac{512}{128} = 4$$

$$d = \pm 2$$

When $d = 2$ & $a = 8$

$$a - 3d = 8 - 3(2) = 2$$

$$a - d = 8 - 2 = 6$$

$$a + d = 8 + 2 = 10$$

$$a + 3d = 8 + 3(2) = 14$$

when $d = -2$ & $a = 8$

$$a - 3d = 8 - 3(-2) = 14$$

$$a - d = 8 - (-2) = 10$$

$$a + d = 8 + (-2) = 6$$

$$a + 3d = 8 + 3(-2) = 2$$

OR

Let the number of terms be 'n'

Given, sum = 287

$$\Rightarrow \frac{n}{2}(2a + (n-1)d) = 287$$

$$\Rightarrow \frac{n}{2}(2 + (n-1)3) = 287$$

$$\Rightarrow n(2 + 3n - 3) = 574$$

$$\Rightarrow n(3n - 1) = 574$$

$$\Rightarrow 3n^2 - n - 574 = 0$$

$$\Rightarrow 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow 3n(n - 14) + 41(n - 14) = 0$$

$$\Rightarrow (3n + 41)(n - 14) = 0$$

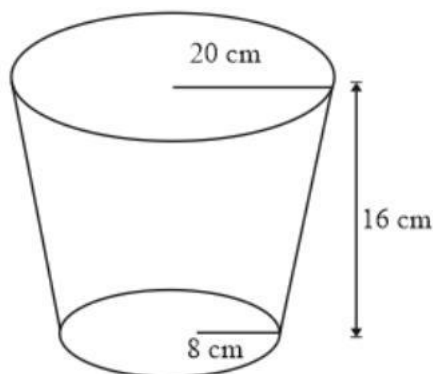
$$\Rightarrow n = 14 \text{ or } -41/3 \text{ [Not possible]}$$

$$\text{Now, } 14^{\text{th}} \text{ term} = a + 13d = 1 + 13(3) = 1 + 39 = 40$$

39. A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre. (Use $\pi = 3.14$)

Solution:

$$r_1 = 8\text{ cm} , r_2 = 20\text{ cm} , h = 16\text{ cm}$$



$$\text{Volume of bucket} = \frac{1}{3}\pi(r_1^2 + r_2^2 + r_1r_2)h$$

$$\frac{1}{3} \times 3.14 \times [(8)^2 + (20)^2 + (8 \times 20)] \times 16$$

$$\frac{1}{3} \times 3.14 [64 + 400 + 160] \times 16$$

$$= 10,449.92 \text{ cm}^3$$

$$= 10,449.92 \times 10^{-6} \times \text{m}^3 \left\{ \because 1\text{m}^3 = 10^6\text{cm}^3 \right\}$$

$$= 10,449.92 \times 10^{-6} \times 10^3 \text{ L} \left\{ \because 1\text{m}^3 = 10^3\text{L} \right\}$$

$$= 10,449.92 \times 10^{-3} \text{ L}$$

$$= 10.449 \text{ L}$$

Now , Cost of filling the milk = Rs 40 /L

$$\therefore \text{Cost of filling } 10.449 \text{ L of milk} = 10.449 \times 40$$

$$= \text{Rs } 417.96$$

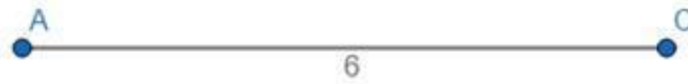
$$= \text{Rs } 418 \text{ (approx.)}$$

40. Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the first triangle.

Solution:

Let the triangle with sides 4 cm, 5 cm and 6 cm be ΔABC

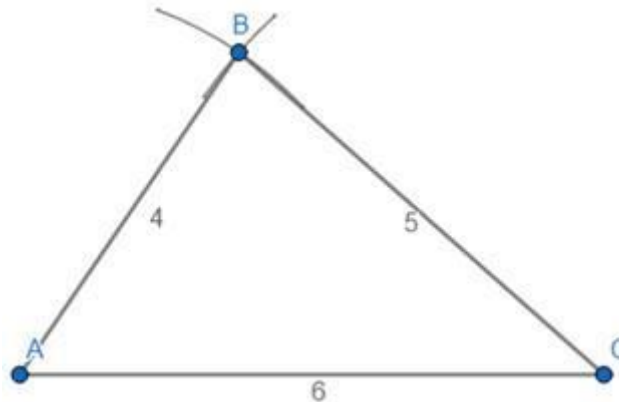
Step1: construct segment AC of 6 cm



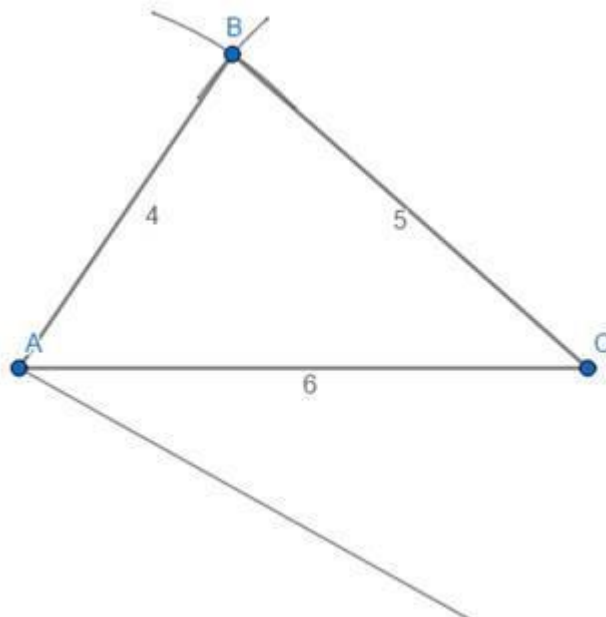
Step2: take distance 4 cm in compass keep the needle of the compass on point A and mark an arc above AC



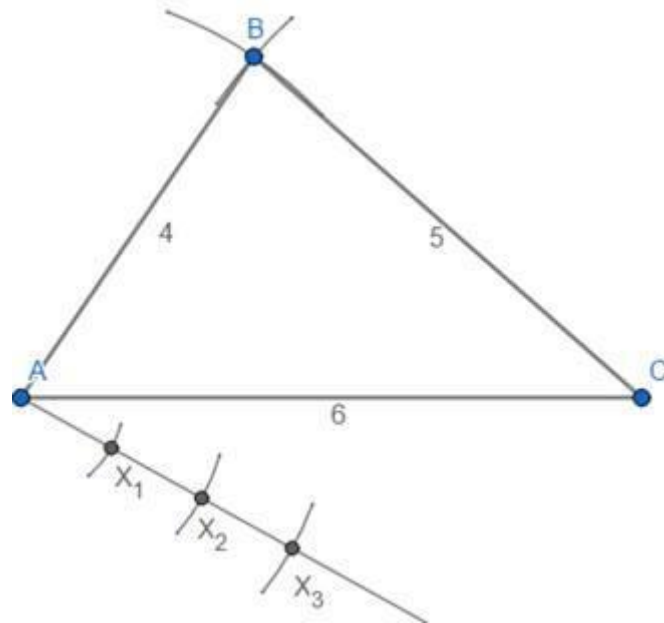
Step3: take distance 5 cm in compass keep the needle of the compass on point C and mark an arc intersecting the arc drawn in step2. Mark intersection point as B join AB and AC



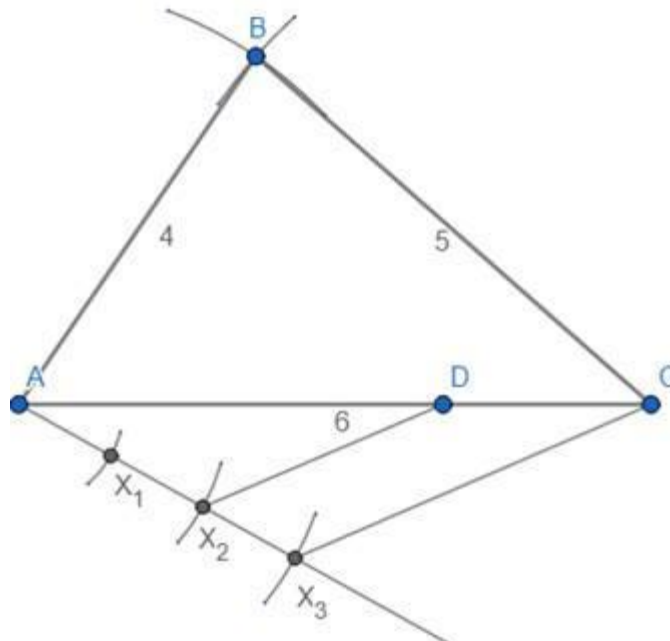
Step4: draw a ray from point A below AC at any angle



Step5: take any distance in compass and keeping the needle of the compass on point A cut an arc on ray constructed in step4 and name that point X_1 . Keeping the distance in compass same keep the needle of the compass on point X_1 and cut an arc on the same ray and mark that point as X_2 . Draw 3 such parts (greater of 2 and 3 in $2/3$), i.e. by repeating this process mark points upto X_3



Step6: join X_3 and C and from X_2 (because X_2 is the second point 2 being smaller in $2/3$) construct line parallel to X_3C and mark the intersection point with AC as D



Step7: construct line parallel to BC from point D and mark the intersection point with AB as E thus $\triangle ADE \sim \triangle ACB$ is ready

