CCE QUESTION PAPER

MATHEMATICS

(With Solutions)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

(*vi*) *An additional 15 minutes time has been allotted to read this question paper only.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The decimal expansion of the rational number $\frac{23}{2^2 \cdot 5}$ will terminate after

(*a*) one decimal place (*b*) two decimal places

(*c*) three decimal places (*d*) more than three decimal places

Solution. Choice (*b*) is correct.

$$\frac{23}{2^2 \cdot 5} = \frac{23}{2 \cdot (2 \cdot 5)} = \frac{23}{2(10)} = \frac{11.5}{10} = 1.15$$

The decimal expansion of the rational number $\frac{23}{2^2 \cdot 5}$ will terminate after **two** decimal places.

2. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer (*b*) a natural number

(*c*) an odd integer (*d*) an even integer

Solution. Choice (*c*) is correct.

For $n = 1$, $n^2 - 1 = (1)^2 - 1 = 0 \Rightarrow n^2 - 1$ is divisible by 8

For $n = 3$, $n^2 - 1 = (3)^2 - 1 = 8 \Rightarrow n^2 - 1$ is divisible by 8

For $n = -3$, $n^2 - 1 = (-3)^2 - 1 = 8 \Rightarrow n^2 - 1$ is divisible by 8.

3. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Solution. Choice (b) is correct.

Since -3 is a zero of the polynomial $f(x) = (k-1)x^2 + kx + 1$, then

$$f(-3) = 0$$

$$\Rightarrow (k-1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow 9(k-1) - 3k + 1 = 0$$

$$\Rightarrow 6k - 9 + 1 = 0$$

$$\Rightarrow 6k - 8 = 0$$

$$\Rightarrow k = \frac{8}{6}$$

$$\Rightarrow k = \frac{4}{3}.$$

4. The lines representing the linear equations $2x - y = 3$ and $4x - y = 5$

(a) intersect at a point (b) are parallel

(c) are coincident (d) intersect at exactly two points

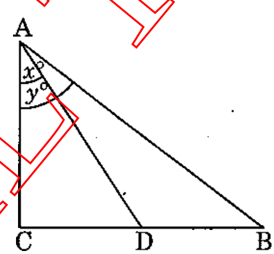
Solution. Choice (a) is correct.

For the given pair of linear equations $2x - y = 3$ and $4x - y = 5$, we have

$$\frac{2}{4} \neq \frac{-1}{-1} \neq \frac{3}{5} \text{ i.e., } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ The lines representing the given linear equations **intersect at a point.**

5. In figure, if D is mid-point of BC, the value of $\frac{\tan x°}{\tan y°}$ is



(a) $\frac{1}{3}$ (b) 1

(c) 2 (d) $\frac{1}{2}$

Solution. Choice (d) is correct.

In right ΔACD, $\tan x° = \frac{DC}{AC}$

In right ΔACB, $\tan y° = \frac{BC}{AC}$

$$\therefore \frac{\tan x°}{\tan y°} = \frac{DC/AC}{BC/AC} = \frac{DC}{BC} = \frac{DC}{2DC} = \frac{1}{2}.$$ [$\because D$ is the mid-point of BC $\therefore BC = 2DC$]

6. Construction of a cumulative frequency table is useful in determining the

(a) mean (b) median

(c) mode (d) all the above three measures

Solution. Choice (b) is correct.

7. If $x = 3\sec^2\theta - 1$, $y = \tan^2\theta - 2$ then $x - 3y$ is equal to

(a) 3 (b) 4

(c) 8 (d) 5

Solution. Choice (c) is correct.

$$x - 3y = (3\sec^2\theta - 1) - 3(\tan^2\theta - 2)$$

$$\Rightarrow \quad x - 3y = 3\sec^2\theta - 3\tan^2\theta - 1 + 6$$

$$\Rightarrow \quad x - 3y = 3(1 + \tan^2\theta) - 3\tan^2\theta + 5$$

$$\Rightarrow \quad x - 3y = 3 + 3\tan^2\theta - 3\tan^2\theta + 5$$

$$\Rightarrow \quad x - 3y = 3 + 5 = \mathbf{8}.$$

8. If $\cos\theta + \cos^2\theta = 1$, the value of $\sin^2\theta + \sin^4\theta$ is

(a) 0 (b) 1

(c) −1 (d) 2

Solution. Choice (b) is correct.

Given : $\cos\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \cos\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad \cos\theta = \sin^2\theta \qquad [\because 1 - \cos^2\theta = \sin^2\theta]$$

$$\Rightarrow \quad \cos^2\theta = \sin^4\theta \qquad \text{[Squaring both sides]}$$

$$\Rightarrow \quad 1 - \sin^2\theta = \sin^4\theta$$

$$\Rightarrow \quad \mathbf{1} = \sin^2\theta + \sin^4\theta.$$

9. If $\Delta ABC \cong \Delta RQP$, $\angle A = 80°$, $\angle B = 60°$, the value of $\angle P$ is

(a) 60° (b) 50°

(c) 40° (d) 30°

Solution. Choice (c) is correct.

Since ΔABC and ΔRQP are similar, therefore,

$$\angle A = \angle R,\ \angle B = \angle Q \text{ and } \angle C = \angle P$$

But $\angle A = 80°$ and $\angle B = 60°$ (given)

$$\therefore \quad \angle R = \angle A = 80° \text{ and } \angle Q = \angle B = 60°$$

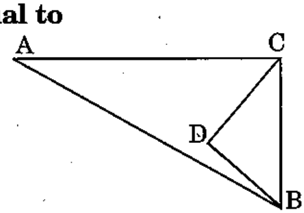
$$\therefore \quad \angle P = 180° - \angle R - \angle Q$$

$$= 180° - 80° - 60° \qquad [\because \angle R = 80° \text{ and } \angle Q = 60°]$$

$$= 180° - 140°$$

$$\Rightarrow \quad \angle P = \mathbf{40°}.$$

10. In the given figure, $\angle ACB = 90°$, $\angle BDC = 90°$, $CD = 4$ cm, $BD = 3$ cm, $AC = 12$ cm. $\cos A - \sin A$ is equal to



(a) $\frac{5}{12}$ (b) $\frac{5}{13}$

(c) $\frac{7}{12}$ (d) $\frac{7}{13}$

Solution. Choice (*d*) is correct.

In right ΔBDC,

$$BC^2 = BD^2 + CD^2 \qquad \text{[By Pythagoras Theorem]}$$
$$\Rightarrow \quad BC^2 = (3)^2 + (4)^2$$
$$\Rightarrow \quad BC^2 = 9 + 16$$
$$\Rightarrow \quad BC^2 = 25$$
$$\Rightarrow \quad BC = 5 \text{ cm}$$

In right ΔACB,

$$AB^2 = AC^2 + BC^2 \qquad \text{[By Pythagoras Theorem]}$$
$$\Rightarrow \quad AB^2 = (12)^2 + (5)^2$$
$$\Rightarrow \quad AB^2 = 144 + 25$$
$$\Rightarrow \quad AB^2 = 169$$
$$\Rightarrow \quad AB = 13 \text{ cm}$$

In right ΔACB,

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{AB} = \frac{12 \text{ cm}}{13 \text{ cm}} = \frac{12}{13}$$

and
$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{5 \text{ cm}}{13 \text{ cm}} = \frac{5}{13}$$

$$\therefore \quad \cos A - \sin A = \frac{12}{13} - \frac{5}{13} = \mathbf{\frac{7}{13}}.$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Use Euclid's division algorithm to find HCF of 870 and 225.

Solution. Given integers are 870 and 225.

Applying Euclid division algorithm to 870 and 225, we get

$$870 = 225 \times 3 + 195 \qquad \ldots(1)$$
$$225 = 195 \times 1 + 30 \qquad \ldots(2)$$
$$195 = 30 \times 6 + 15 \qquad \ldots(3)$$
$$30 = 15 \times 2 + 0 \qquad \ldots(4)$$

In equation (4), the remainder is zero. So, the last divisor or the non-zero remainder at the earliest stage, *i.e.*, in equation (3) is 15.

Therefore, HCF of 870 and 225 is **15.**

12. Solve $37x + 43y = 123$, $43x + 37y = 117$.

Solution. We have

$$37x + 43y = 123 \qquad \ldots(1)$$

and
$$43x + 37y = 117 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$(37x + 43y) + (43x + 37y) = 123 + 117$$
$$\Rightarrow \quad 80x + 80y = 240$$
$$\Rightarrow \quad 80(x + y) = 240$$
$$\Rightarrow \quad x + y = 3 \qquad \ldots(3)$$

Subtracting (1) from (2), we get

$$(43x + 37y) - (37x + 43y) = 117 - 123$$
$$\Rightarrow \quad (43x - 37x) + (37y - 43y) = -6$$

$\Rightarrow \quad 6x - 6y = -6$

$\Rightarrow \quad x - y = -1 \quad \ldots(4)$

Adding (3) and (4), we get

$$(x + y) + (x - y) = 3 + (-1)$$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$

Substituting $x = 1$ in (3), we get

$$1 + y = 3 \Rightarrow y = 2$$

Hence, $\boldsymbol{x = 1}$ **and** $\boldsymbol{y = 2}$.

Or

Solve $\boldsymbol{x + \frac{6}{y} = 6,\ 3x - \frac{8}{y} = 5}$.

Solution. The given system of linear equations are

$$x + \frac{6}{y} = 6 \quad \ldots(1)$$

$$3x - \frac{8}{y} = 5 \quad \ldots(2)$$

Multiplying (1) by 3, we get

$$3x + \frac{18}{y} = 18 \quad \ldots(3)$$

Subtracting (2) from (3), we get

$$\left(3x + \frac{18}{y}\right) - \left(3x - \frac{8}{y}\right) = 18 - 5$$

$\Rightarrow \quad \frac{18}{y} + \frac{8}{y} = 13$

$\Rightarrow \quad \frac{26}{y} = 13$

$\Rightarrow \quad \frac{2}{y} = 1$

$\Rightarrow \quad y = 2$

Substituting $y = 2$ in (1), we get

$$x + \frac{6}{2} = 6$$

$\Rightarrow \quad x + 3 = 6$

$\Rightarrow \quad x = 3$

Hence, $\boldsymbol{x = 3}$ **and** $\boldsymbol{y = 2}$.

13. α, β are the roots of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$. Find the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

Solution. Since α and β are the roots of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$, therefore,

$$\alpha + \beta = \frac{-[-(k+6)]}{1} = k + 6 \qquad ...(1)$$

$$\alpha\beta = \frac{2(2k-1)}{1} = 2(2k-1) \qquad ...(2)$$

Given that : $\alpha + \beta = \frac{1}{2}\alpha\beta$

$\Rightarrow \quad (k+6) = \frac{1}{2}[2(2k-1)]$ [using (1) and (2)]

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad 2k - k = 6 + 1$

$\Rightarrow \quad k = 7.$

14. If $\cot\theta = \frac{7}{8}$, find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$.

Solution. We have

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

$$= (\cot\theta)^2$$

$$= \left(\frac{7}{8}\right)^2 \qquad \left[\because \cot\theta = \frac{7}{8} \text{(given)}\right]$$

$$= \frac{49}{64}.$$

15. Find the median class and the modal class for the following distribution.

C.I.	**135 – 140**	**140 – 145**	**145 – 150**	**150 – 155**	**155 – 160**	**160 – 165**
f	**4**	**7**	**18**	**11**	**6**	**5**

Solution.

C.I.	*f*	*Cumulative Frequency (cf)*
135 – 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51
Total	$n = \Sigma f = 51$	

Here, $n = 51$ and $\frac{n}{2} = \frac{51}{2} = 25.5$.

Now, 145 – 150 is the class interval whose cumulative frequency 29 is greater than $\frac{n}{2} = 25.5$.

$\therefore$ **145 – 150** is the median class.

Since the maximum frequency is 18, therefore, the modal class is **145 – 150.**

16. Write the following distribution as more than type cumulative frequency distribution :

C.I.	**50 – 55**	**55 – 60**	**60 – 65**	**65 – 70**	**70 – 75**	**75 – 80**
Frequency	**2**	**6**	**8**	**14**	**15**	**5**

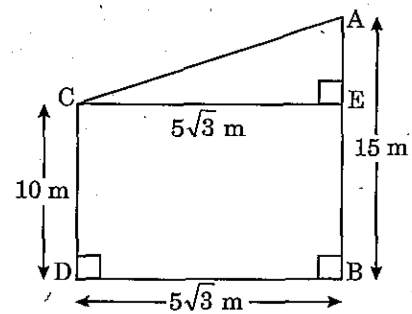
Solution. We prepare the cumulative frequency table by **more than type method** as given below :

More than type Cumulative Frequency Distribution

C.I.	*Frequency*	*More than*	*Cumulative frequency*
50 – 55	2	50	50
55 – 60	6	55	48
60 – 65	8	60	42
65 – 70	14	65	34
70 – 75	15	70	20
75 – 80	5	75	5

17. Two poles of height 10 m and 15 m stand vertically on a plane ground. If the distance between their feet is $5\sqrt{3}$ m, find the distance between their tops.

Solution. Let AB be a pole of height 15 m and CD be a pole of height 10 m standing on a plane ground. The distance between their feet is $DB = 5\sqrt{3}$ m. We have to calculate AC the distance between their tops.



Draw $CE \parallel DB$ intersecting AB at E such that $CE = DB = 5\sqrt{3}$ m.

$\therefore \quad AE = AB - BE = 15\text{ m} - CD \qquad [\because CD = EB]$

$\Rightarrow \quad AE = 15\text{ m} - 10\text{ m} = 5\text{ m}$

In right ΔAEC, we have

$AC^2 = AE^2 + CE^2$ [By Pythagoras Theorem]

$\Rightarrow \quad AC^2 = (5)^2 + (5\sqrt{3})^2$

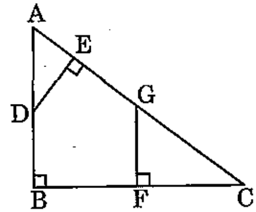
$\Rightarrow \quad AC^2 = 25 + 75$

$\Rightarrow \quad AC^2 = 100$

$\Rightarrow \quad AC = 10\text{ m}$

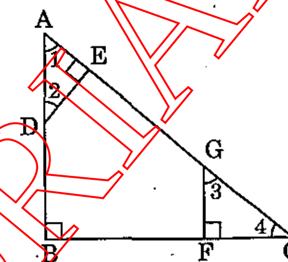
Hence, the distance between the tops of the poles is **10 m.**

18. In figure, $AB \perp BC$, $DE \perp AC$ and $GF \perp BC$. Prove that $\Delta ADE \sim \Delta GCF$.



Solution. Since $AB \perp BC$ and $GF \perp BC$, therefore $AB \parallel GF$.

$\therefore \quad \angle 1 = \angle 3$ [Corresponding $\angle s$] ...(1)



In ΔADE, we have

$\angle 1 + \angle 2 = 90°$

$\Rightarrow \quad \angle 3 + \angle 2 = 90°$ ...(2) [using (1)]

In ΔGCF, we have

$\angle 3 + \angle 4 = 90°$ ...(3)

From (2) and (3), we have

$\angle 3 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle 2 = \angle 4$ ...(4)

In ΔADE and ΔGCF, we have

$\angle 1 = \angle 3$ [Proved in (1)]

and $\angle 2 = \angle 4$ [Proved in (4)]

So, by AA similarity of criterion, we have

$\Delta ADE \sim \Delta GCF$

Alternative Method :

Since $AB \perp BC$ and $GF \perp BC$, therefore $AB \parallel GF$

$\therefore \quad \angle 1 = \angle 3$ ...(1) [Corresponding $\angle s$]

In ΔADE and ΔGCF, we have

$\angle 1 = \angle 3$ [Proved in (1)]

and $\angle AED = \angle GFC$ [Each = 90°]

So, by AA similarity criterion, we have

$\Delta ADE \sim \Delta GCF$.

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Show that $5 + \sqrt{2}$ is an irrational number.

Solution. Let us assume, to the contrary, that $5 + \sqrt{2}$ is rational *i.e.*, we can find co-prime a and b ($b \neq 0$) such that

$$5 + \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{a}{b} - 5 = \sqrt{2}$$

Rearranging the equation, we get

$$\sqrt{2} = \frac{a}{b} - 5 = \frac{a - 5b}{b}$$

Since a and b are integers, we get $\frac{a-5b}{b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5+\sqrt{2}$ is rational.

So, we conclude that $5+\sqrt{2}$ is **irrational.**

Or

Prove that $\sqrt{3}+\sqrt{5}$ is an irrational.

Solution. Let us assume, to the contrary that $\sqrt{3}+\sqrt{5}$ is a rational.

That is, we can find co-prime p and q $(q \neq 0)$ such that

$$\sqrt{3}+\sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{p}{q} - \sqrt{3} = \sqrt{5}$$

$$\Rightarrow \quad \left(\frac{p}{q} - \sqrt{3}\right)^2 = (\sqrt{5})^2 \quad \text{[Squaring both sides]}$$

$$\Rightarrow \quad \frac{p^2}{q^2} - 2\frac{p}{q}\cdot\sqrt{3} + 3 = 5$$

$$\Rightarrow \quad \frac{p^2}{q^2} - 2 = 2\frac{p}{q}\cdot\sqrt{3}$$

$$\Rightarrow \quad \frac{p^2-2q^2}{q^2} = 2\frac{p}{q}\cdot\sqrt{3}$$

$$\Rightarrow \quad \frac{p^2-2q^2}{2pq} = \sqrt{3}$$

Since, p and q are integers, $\frac{p^2-2q^2}{2pq}$ is rational, and so $\sqrt{3}$ is rational. But this contradicts the fact that $\sqrt{3}$ is irrational.

So, we conclude that $(\sqrt{3}+\sqrt{5})$ is **irrational.**

20. Show that 5^n can't end with the digit 2 for any natural number n.

Solution. We know that any positive integer ending with the digit 0, 2, 4, 6 and 8 is divisible by 2 and so its prime factorisation must contain the prime 2.

We have : 5^n

$\Rightarrow$ There is no prime in the factorisation of 5^n

$\Rightarrow$ 2 does not occur in the prime factorisation of 5^n for any natural number.

[By uniqueness of the Fundamental Theorem of Arithmetic]

Hence, 5^n can't end with the digit **2** for any natural number.

21. If α, β are the two zeroes of the polynomial $21y^2 - y - 2$, find a quadratic polynomial whose zeroes are 2α and 2β.

Solution. Since α and β are zeroes of the polynomial $21y^2 - y - 2$, therefore

$$\alpha+\beta = \frac{-(-1)}{21} = \frac{1}{21}$$

$$\Rightarrow \quad \alpha\beta = \frac{-2}{21} = -\frac{2}{21}$$

Let S and P denote respectively the sum and product of zeroes of the required polynomial, then

$$S = 2\alpha + 2\beta = 2(\alpha + \beta) = 2\cdot\frac{1}{21} = \frac{2}{21}$$

and $$P = (2\alpha)(2\beta) = 4\alpha\beta = 4\cdot\left(-\frac{2}{21}\right) = -\frac{8}{21}$$

Hence, the required polynomial $p(x)$ is given by

$$p(x) = k(x^2 - Sx + P)$$

$$\Rightarrow \quad p(x) = k\left(x^2 - \frac{2}{21}x - \frac{8}{21}\right), \text{ where } k \text{ is any non-zero real number.}$$

$$\Rightarrow \quad p(x) = k(21x^2 - 2x - 8), \text{ where } k \text{ is any non-zero real number.}$$

22. If A, B, C are interior angles of ΔABC, show that

$$\sec^2\left(\frac{B+C}{2}\right) - 1 = \cot^2\frac{A}{2}$$

Solution. If A, B, C are interior angles of ΔABC, then

$$A + B + C = 180°$$

$$\Rightarrow \quad B + C = 180° - A$$

$$\Rightarrow \quad \frac{B+C}{2} = \frac{180° - A}{2}$$

$$\Rightarrow \quad \frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\Rightarrow \quad \sec^2\left(\frac{B+C}{2}\right) = \sec^2\left(90° - \frac{A}{2}\right)$$

$$\Rightarrow \quad \sec^2\left(\frac{B+C}{2}\right) = \text{cosec}^2\frac{A}{2} \qquad [\because \sec(90° - \theta) = \text{cosec}\,\theta]$$

$$\Rightarrow \quad \sec^2\left(\frac{B+C}{2}\right) = 1 + \cot^2\frac{A}{2} \qquad [\because \text{cosec}^2\theta = 1 + \cot^2\theta]$$

$$\Rightarrow \quad \sec^2\left(\frac{B+C}{2}\right) - 1 = \cot^2\frac{A}{2}.$$

Or

Prove that : $$\frac{\cos(90° - \theta)}{1 + \sin(90° - \theta)} + \frac{1 + \sin(90° - \theta)}{\cos(90° - \theta)} = 2\,\text{cosec}\,\theta$$

Solution. We have

$$\text{L.H.S.} = \frac{\cos(90° - \theta)}{1 + \sin(90° - \theta)} + \frac{1 + \sin(90° - \theta)}{\cos(90° - \theta)}$$

$$= \frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} \qquad [\because \sin(90° - A) = \cos A \text{ and } \cos(90° - A) = \sin A]$$

$$= \frac{\sin^2\theta + (1 + \cos\theta)^2}{(1 + \cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + (1+\cos^2\theta + 2\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

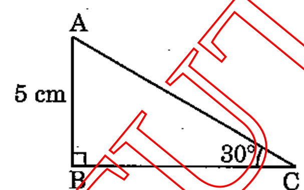
$$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$= 2 \operatorname{cosec}\theta$

$=$ R.H.S.

23. In figure, ABC is a triangle right-angled at B, $AB = 5$ cm, $\angle ACB = 30°$. Find the length of BC and AC.



Solution. We have

In ΔABC, right-angled at B and $\angle ACB = 30°$

$\therefore \quad \sin 30° = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{5\text{ cm}}{AC}$ $\left[\because AB = 5\text{ cm (given) and } \sin 30° = \frac{1}{2}\right]$

$\Rightarrow \quad$ **$AC = 10$ cm** ...(1)

and $\quad \cos 30° = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BC}{10\text{ cm}}$ $\left[\because \cos 30° = \frac{\sqrt{3}}{2} \text{ and } AC = 10 \text{ cm by (1)}\right]$

$\Rightarrow \quad BC = 10 \times \frac{\sqrt{3}}{2}$ cm

$\Rightarrow \quad$ **$BC = 5\sqrt{3}$ cm.**

24. The mean of the following frequency distribution is 25.2. Find the missing frequency x.

C.I.	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**
Frequency	**8**	***x***	**10**	**11**	**9**

Solution. **Calculation of Mean**

C.I.	Frequency (f_i)	Class-mark (x_i)	f_ix_i
0 – 10	8	5	40
10 – 20	x	15	$15x$
20 – 30	10	25	250
30 – 40	11	35	385
40 – 50	9	45	405
Total	$n = \Sigma f_i = 38 + x$		$\Sigma f_ix_i = 1080 + 15x$

From the table, $n = \Sigma f_i = 38 + x$, $\Sigma f_ix_i = 1080 + 15x$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_ix_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 25.2 = \frac{1080 + 15x}{38 + x}$$

$$\Rightarrow \quad (38 + x)(25.2) = 1080 + 15x$$

$$\Rightarrow \quad 38 \times 25.2 + 25.2x = 1080 + 15x$$

$$\Rightarrow \quad 957.6 + 25.2x = 1080 + 15x$$

$$\Rightarrow \quad 25.2x - 15x = 1080 - 957.6$$

$$\Rightarrow \quad 10.2x = 122.4$$

$$\Rightarrow \quad x = \frac{122.4}{10.2} = 12$$

Hence, the missing frequency x is **12.**

25. Find the mode of the following frequency distribution :

C.I.	**5 – 15**	**15 – 25**	**25 – 35**	**35 – 45**	**45 – 55**	**55 – 65**	**65 – 75**
Frequency	**2**	**3**	**5**	**7**	**4**	**2**	**2**

Solution. Since the class 35 – 45 has the maximum frequency, therefore 35 – 45 is the modal class.

$\therefore \quad l = 35, h = 10, f_1 = 7, f_0 = 5, f_2 = 4$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 35 + \frac{7 - 5}{2 \times 7 - 5 - 4} \times 10$$

$$= 35 + \frac{2}{14 - 9} \times 10$$

$$= 35 + \frac{2}{5} \times 10$$

$$= 35 + 4$$

$$= 39$$

Hence, the mode is **39.**

26. Nine times a two-digit number is the same as twice the number obtained by interchanging the digits of the number. If one digit of the number exceeds the other number by 7, find the number.

Solution. Let the unit's place digit be x and ten's place digit be y.

$\therefore$ Original number $= 10y + x$

The number obtained by reversing the digits $= 10x + y$

According to the first condition

Nine times a two-digit number = Twice the number obtained by interchanging the digits of the number

$$\Rightarrow \quad 9 \times (10y + x) = 2 \times (10x + y)$$
$$\Rightarrow \quad 90y + 9x = 20x + 2y$$
$$\Rightarrow \quad 90y - 2y = 20x - 9x$$
$$\Rightarrow \quad 88y = 11x$$
$$\Rightarrow \quad 8y = x \qquad \text{...(1)}$$

According to second condition

One digit of the number exceeds the other number by 7

$$\therefore \quad x - y = 7 \qquad \text{...(2)}$$

Substituting $x = 8y$ from (1) in (2), we get

$$8y - y = 7$$
$$\Rightarrow \quad 7y = 7$$
$$\Rightarrow \quad y = 1$$

Putting $y = 1$ in (1), we get

$$x = 8 \times 1 = 8$$

Hence, the original number $= 10y + x = 10 \times 1 + 8 =$ **18.**

Or

The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

Solution. Let the ratio of incomes of two persons be $9x : 7x$.

Let the ratio of expenditures of two persons be $4y : 3y$.

$\therefore$ Monthly saving of first person $= 9x - 4y$

Monthly saving of second person $= 7x - 3y$

According to the condition given, each person saves monthly ₹ 2000

$$\therefore \quad 9x - 4y = 2000 \qquad \text{...(1)}$$
$$7x - 3y = 2000 \qquad \text{...(2)}$$

Subtracting (2) from (1), we get

$$(9x - 4y) - (7x - 3y) = 2000 - 2000$$
$$\Rightarrow \quad (9x - 7x) + (-4y + 3y) = 0$$
$$\Rightarrow \quad 2x - y = 0$$
$$\Rightarrow \quad y = 2x \qquad \text{...(3)}$$

Substituting $y = 2x$ from (3) in (1), we get

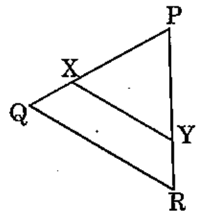
$$9x - 4(2x) = 2000$$
$$\Rightarrow \quad 9x - 8x = 2000$$
$$\Rightarrow \quad x = 2000$$

Putting $x = 2000$ in (3), we get

$$y = 2 \times 2000 = 4000$$

Hence, the monthly incomes of two persons be $9 \times 2000 =$ **₹ 18,000** and $7 \times 2000 =$ **₹ 14,000.**

27. In figure, $XY \parallel QR$, $\frac{PQ}{XQ} = \frac{7}{3}$ and $PR = 6.3$ cm. Find YR.



Solution. In ΔPQR, $XY \parallel QR$

$$\therefore \quad \frac{PX}{XQ} = \frac{PY}{YR} \qquad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{PX}{XQ} + 1 = \frac{PY}{YR} + 1$$

$$\Rightarrow \quad \frac{PX + XQ}{XQ} = \frac{PY + YR}{YR}$$

$$\Rightarrow \quad \frac{PQ}{XQ} = \frac{PR}{YR}$$

$$\Rightarrow \quad \text{(given)} \ \frac{7}{3} = \frac{PR}{YR}$$

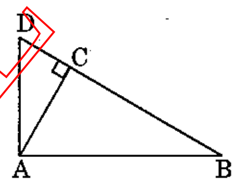
$$\Rightarrow \quad \frac{7}{3} = \frac{6.3 \text{ cm}}{YR} \qquad [\because PR = 6.3 \text{ cm (given)}]$$

$$\Rightarrow \quad YR = \frac{3 \times 6.3 \text{ cm}}{7}$$

$$\Rightarrow \quad YR = 3 \times 0.9 \text{ cm}$$

$$\Rightarrow \quad YR = \mathbf{2.7 \text{ cm}}.$$

28. In figure, ABD is a triangle in which $\angle DAB = 90°$ and $AC \perp BD$. Prove that $AC^2 = BC \times DC$.



Solution. Given : ΔABD is right triangle, right-angled at A and $AC \perp BD$.

To prove : $AC^2 = BC \times DC$

Proof : In ΔABD,

$$\angle DAC + \angle CAB = 90° \qquad \ldots(1)$$

In ΔCDA,

$$\angle DAC + \angle CDA = 90° \qquad \ldots(2)$$

$$\therefore \quad \angle DAC + \angle CAB = \angle DAC + \angle CDA \qquad \text{[From (1) and (2)]}$$

$$\Rightarrow \quad \angle CAB = \angle CDA$$

$$\Rightarrow \quad \angle CAB = \angle D \qquad \ldots(3) \ [\because \angle CDA = \angle D]$$

In ΔDCA and ΔACB, we have

$$\angle D = \angle CAB \qquad \text{[Proved in (3)]}$$

$\angle DCA = \angle BCA$ [Each = 90°]

So, by AA-criterion of similarity of triangles, we have

$$\Delta DCA \sim \Delta ACB$$

$$\frac{DC}{AC} = \frac{AC}{BC}$$

Hence, $AC^2 = BC \times DC$.

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Solve the following system of equations graphically and find the vertices of the triangle formed by these lines and the x-axis.

$$4x - 3y + 4 = 0,\ 4x + 3y - 20 = 0$$

Solution. We have

$4x - 3y + 4 = 0$

$\Rightarrow \quad 3y = 4x + 4$

$\Rightarrow \quad y = \dfrac{4(x+1)}{3}$

Table of $4x - 3y + 4 = 0$

x	5	−1	2
y	8	0	4
	A	B	C

and $4x + 3y - 20 = 0$

$\Rightarrow \quad 3y = 20 - 4x$

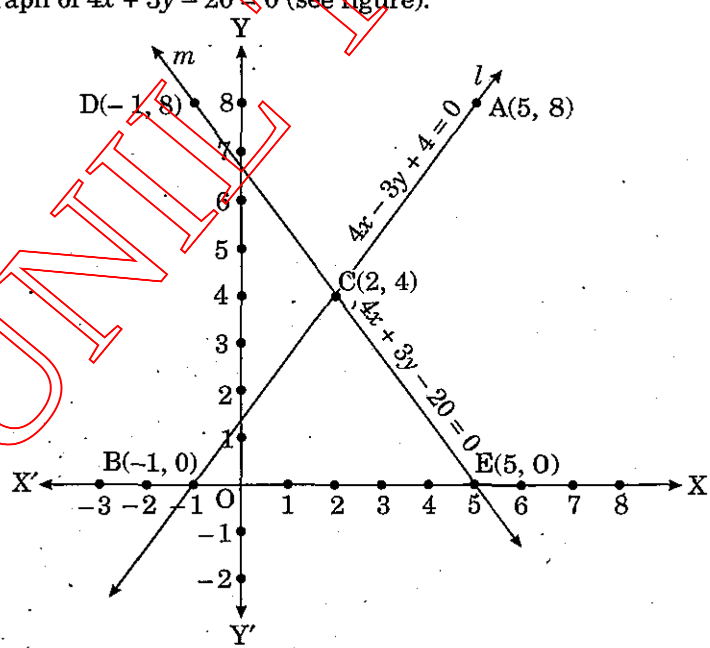
$\Rightarrow \quad y = \dfrac{20 - 4x}{3}$

Table of $4x + 3y - 20 = 0$

x	−1	5	2
y	8	0	4
	D	E	C

Take XOX' and YOY' as the axes of coordinates. Plotting the points $A(5, 8)$, $B(-1, 0)$, $C(2, 4)$ and joining them by a line, we get a line 'l' which is the graph of $4x - 3y + 4 = 0$.

Further, plotting the points $D(-1, 8)$, $E(5, 0)$, $C(2, 4)$ and joining them by a line, we get a line 'm' which is the graph of $4x + 3y - 20 = 0$ (see figure).



From the graph of the two equations, we find that the two lines l and m intersect each other at the point $C(2, 4)$.

$\therefore$ $x = 2, y = 4$ is the solution.

The first line $4x - 3y + 4 = 0$ meets the x-axis at the point $B(-1, 0)$.

The second line $4x + 3y - 20 = 0$ meets the x-axis at the point $E(5, 0)$.

Hence, the vertices of the triangle ECB formed by the given lines with the x-axis are $E(5, 0)$, $C(2, 4)$ and $B(-1, 0)$ respectively.

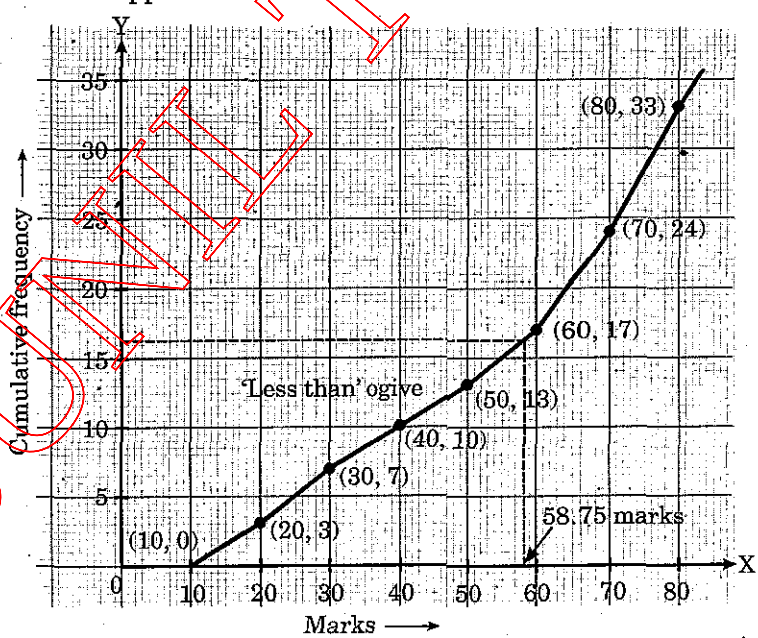
30. Draw 'less than ogive' for the following frequency distribution and hence obtain the median.

Marks obtained	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**
No. of students	**3**	**4**	**3**	**3**	**4**	**7**	**9**

Solution. We prepare the cumulative frequency table by less than type method as given below :

Marks obtained	*No. of students*	*Marks less than*	*Cumulative frequency*
10 – 20	3	20	3
20 – 30	4	30	7
30 – 40	3	40	10
40 – 50	3	50	13
50 – 60	4	60	17
60 – 70	7	70	24
70 – 80	9	80	33

Here, 20, 30, 40, 50, 60, 70, 80 are the upper limits of the respective class intervals less than 10 – 20, 20 – 30, 30 – 40, 40 – 50, 50 – 60, 60 – 70, 70 – 80. To represent the data in the table graphically, we mark the upper limits of the class intervals on the horizontal axis (x-axis) and



their corresponding cumulative frequencies on the vertical axis (y-axis), choosing a convenient scale other than the class intervals, we assume a class interval 0 – 10 prior to the first class interval 10 – 20 with zero frequency.

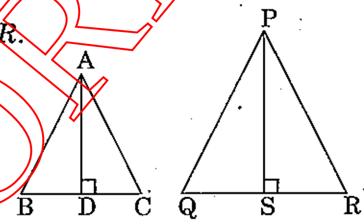
Now, plot the points (10, 0) (20, 3), (30, 7), (40, 10), (50, 13), (60, 17), (70, 24) and (80, 33) on a graph paper and join them by a free hand smooth curve. The curve we get is called **an ogive of less than type** (see figure).

Let $\frac{n}{2} = \frac{33}{2} = 16.5$ on the y-axis (see figure).

From this point, draw a line parallel to the x-axis cutting the curve at a point. From this point, draw a perpendicular to the x-axis. The point of intersection of this perpendicular with the x-axis determine the **median marks** of the data (see figure) *i.e.*, **median marks = 58.75.**

31. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.



To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.

Proof : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\frac{AB}{PQ} = \frac{BC}{QR}$ [$\because \Delta ABC \sim \Delta PQR$]

$\Rightarrow \frac{AD}{PS} = \frac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$$

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$ ...(4)

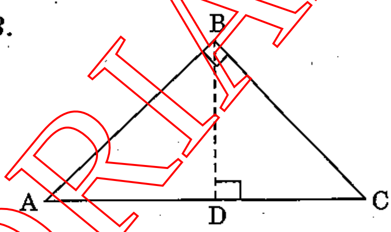
As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad \ldots(5)$$

Hence, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad \text{[From (4) and (5)]}$$

Or

Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.



Solution. Given : A right triangle ABC, right angled at B.

To prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $$AC^2 = AB^2 + BC^2$$

Construction : Draw $BD \perp AC$.

Proof : $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]

So, $$\frac{AD}{AB} = \frac{AB}{AC} \quad \text{[Sides are proportional]}$$

$\Rightarrow$ $$AD.AC = AB^2 \quad \ldots(1)$$

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $$\frac{CD}{BC} = \frac{BC}{AC} \quad \text{[Sides are proportional]}$$

$\Rightarrow$ $$CD.AC = BC^2 \quad \ldots(2)$$

Adding (1) and (2), we have

$$AD.AC + CD.AC = AB^2 + BC^2$$

$$\Rightarrow (AD + CD).AC = AB^2 + BC^2$$

$$\Rightarrow AC.AC = AB^2 + BC^2$$

Hence, $$\mathbf{AC^2 = AB^2 + BC^2}$$

32. Find all the zeroes of the polynomial $x^4 - 5x^3 + 2x^2 + 10x - 8$, if two of its zeroes are $\sqrt{2}, -\sqrt{2}$.

Solution. Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$, therefore $(x - \sqrt{2})(x + \sqrt{2}) = (x^2 - 2)$ is a factor of the given polynomial.

Now, we divide the given polynomial by $(x^2 - 2)$.

$$\begin{array}{r|l} & x^2 - 5x + 4 \\ \hline x^2 - 2 & x^4 - 5x^3 + 2x^2 + 10x - 8 \\ & x^4 \quad\quad\;\; \mp 2x^2 \\ & \underline{-\quad\quad\quad\; + \quad\quad\quad\quad} \\ & -5x^3 + 4x^2 + 10x - 8 \\ & -5x^3 \quad\quad\; + 10x \\ & \underline{+ \quad\quad\quad\quad\; - \quad\quad} \\ & \quad 4x^2 \quad\quad - 8 \\ & \quad 4x^2 \quad\quad - 8 \\ & \underline{\quad - \quad\quad\quad + \quad} \\ & \quad\quad\quad 0 \end{array}$$

$\left[\text{First term of the quotient is } \frac{x^4}{x^2} = x^2\right]$

$\left[\text{Second term of the quotient is } \frac{-5x^3}{x^2} = -5x\right]$

$\left[\text{Third term of the quotient is } \frac{4x^2}{x^2} = 4\right]$

So, $x^4 - 5x^3 + 2x^2 + 10x - 8 = (x^2 - 2)(x^2 - 5x + 4)$

$$= (x - \sqrt{2})(x + \sqrt{2})(x^2 - x - 4x + 4)$$

$$= (x - \sqrt{2})(x + \sqrt{2})[x(x - 1) - 4(x - 1)]$$

$$= (x - \sqrt{2})(x + \sqrt{2})(x - 1)(x - 4)$$

So, the zeroes of $x^2 - 5x + 4$ are given by 1 and 4.

Hence, all the zeroes of the given polynomial are $\sqrt{2}$, $-\sqrt{2}$, **1** and **4.**

33. Prove that $\dfrac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta} = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$

Solution. We have

$$\text{L.H.S.} = \frac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta}$$

$$= \frac{[(\cot\theta + \operatorname{cosec}\theta) - 1]}{1 - (\operatorname{cosec}\theta - \cot\theta)}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - 1}{(\operatorname{cosec}^2\theta - \cot^2\theta) - (\operatorname{cosec}\theta - \cot\theta)} \qquad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - 1}{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}\theta - \cot\theta)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta) - 1}{(\operatorname{cosec}\theta - \cot\theta)[(\operatorname{cosec}\theta + \cot\theta) - 1]}$$

$$= \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

$= \text{R.H.S.}$

Or

If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that

$$(m^2 - n^2)^2 = 16mn$$

Solution. Given, $\tan\theta + \sin\theta = m$ ...(1)

$\tan\theta - \sin\theta = n$ ...(2)

Adding (1) and (2), we get

$$2\tan\theta = m + n$$

$\Rightarrow$ $\tan\theta = \dfrac{m + n}{2}$

$\Rightarrow$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{2}{m + n}$ ...(3)

Subtracting (2) from (1), we get

$2\sin\theta = m - n$ ...(4)

$\Rightarrow$ $\sin\theta = \dfrac{m - n}{2}$

$$\Rightarrow \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{2}{m-n}$$

We know that

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\Rightarrow \qquad \left(\frac{2}{m-n}\right)^2 - \left(\frac{2}{m+n}\right)^2 = 1 \qquad \text{[using (3) and (4)]}$$

$$\Rightarrow \qquad \frac{4}{(m-n)^2} - \frac{4}{(m+n)^2} = 1$$

$$\Rightarrow \qquad 4\left[\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right] = 1$$

$$\Rightarrow \qquad 4\left[\frac{(m+n)^2 - (m-n)^2}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \qquad 4\left[\frac{(m^2+n^2+2mn) - (m^2+n^2-2mn)}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \qquad 4\left[\frac{2mn+2mn}{(m-n)^2(m+n)^2}\right] = 1$$

$$\Rightarrow \qquad \frac{4 \times 4mn}{(m-n)^2(m+n)^2} = 1$$

$$\Rightarrow \qquad \frac{16mn}{(m-n)^2(m+n)^2} = 1$$

$$\Rightarrow \qquad \frac{16mn}{[(m-n)(m+n)]^2} = 1$$

$$\Rightarrow \qquad \frac{16mn}{(m^2-n^2)^2} = 1$$

Hence, $\qquad \mathbf{(m^2 - n^2)^2 = 16mn}$

34. Prove that $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$**.**

Solution. We have

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \frac{1+\sin A}{\sqrt{(1-\sin A)(1+\sin A)}}$$

$$= \frac{1+\sin A}{\sqrt{1-\sin^2 A}}$$

$$= \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

= R.H.S.