CCE SAMPLE QUESTION PAPER

FIRST TERM (SA–I)

MATHEMATICS

(*With Solutions*)

CLASS X

Time Allowed : 3 to 3½ Hours] [*Maximum Marks* : 80

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If a rational number x is expressed as $x = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factor (except 1), then the decimal expansion of x is terminating if and only if q has a prime factorization of the form :

(*a*) $2^m \cdot 5^n$ (*b*) $2^m \cdot 3^n$

(*c*) $2^m \cdot 7^n$ (*d*) $5^m \cdot 3^n$

where m and n are non-negative integers.

Solution. Choice (*a*) is correct.

The prime factorization of q is of the form $2^m \cdot 5^n$, where m and n are non-negative integers, then a rational number $x = \frac{p}{q}$ has a terminating decimal.

2. If $\cot\theta + \frac{1}{\cot\theta} = 2$, then the value of $\cot^2\theta + \frac{1}{\cot^2\theta}$ is

(*a*) 3 (*b*) 4

(*c*) 2 (*d*) –4

Solution. Choice (*c*) is correct.

Given, $\cot\theta + \frac{1}{\cot\theta} = 2$

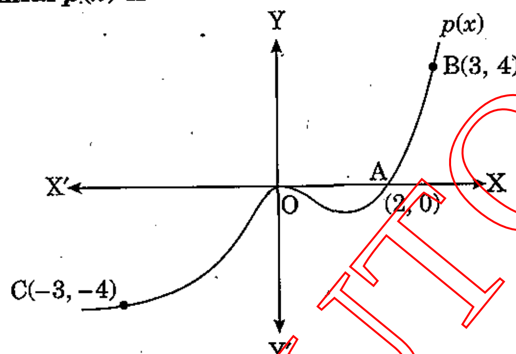
$$\Rightarrow \quad \cot^2\theta + \frac{1}{\cot^2\theta} + 2\cot\theta \cdot \frac{1}{\cot\theta} = 4 \qquad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$\Rightarrow \quad \cot^2\theta + \frac{1}{\cot^2\theta} + 2 = 4$$

$$\Rightarrow \quad \cot^2\theta + \frac{1}{\cot^2\theta} = 4 - 2$$

$$\Rightarrow \quad \cot^2\theta + \frac{1}{\cot^2\theta} = 2$$

3. In the given figure, the graph of some polynomial $p(x)$ is given. The number of zeroes of the polynomial $p(x)$ is

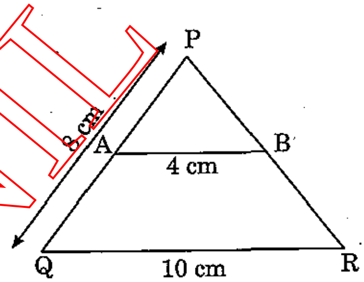


(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

Solution. Choice (*b*) is correct.

The number of zeroes are **2**, as the graph intersects the x-axis in two points, *viz.*, $O(0, 0)$ and $A(2, 0)$.

4. In figure, $AB \parallel QR$. The length of PA is



(*a*) 3.2 cm (*b*) 4.2 cm
(*c*) 4.8 cm (*d*) 3.6 cm

Solution. Choice (*a*) is correct.

In ΔPQR, we have

$$AB \parallel QR$$

$$\therefore \quad \frac{PA}{AQ} = \frac{PB}{BR} \qquad \text{[By BPT]}$$

$\Rightarrow \quad \frac{AQ}{PA} = \frac{BR}{PB}$ [Taking reciprocal of both sides]

$\Rightarrow \quad 1 + \frac{AQ}{PA} = 1 + \frac{BR}{PB}$

$\Rightarrow \quad \frac{PA + AQ}{PA} = \frac{PB + BR}{PB}$

$\Rightarrow \quad \frac{PQ}{PA} = \frac{PR}{PB}$

$\Rightarrow \quad \frac{PA}{PQ} = \frac{PB}{PR}$

Thus, in Δ's PAB and PQR, we have

$\frac{PA}{PQ} = \frac{PB}{PR}$ [From above]

and $\angle P = \angle P$ [Common]

So, by SAS-criterion of similarity of Δ's, we have

$\Delta PAB \sim \Delta PQR$

$\Rightarrow \quad \frac{PA}{PQ} = \frac{PB}{PR} = \frac{AB}{QR}$

$\Rightarrow \quad \frac{PA}{PQ} = \frac{AB}{QR}$

$\Rightarrow \quad \frac{PA}{8} = \frac{4}{10}$ [Given, $PQ = 8$ cm, $AB = 4$ cm, $QR = 10$ cm]

$\Rightarrow \quad PA = \frac{8 \times 4}{10} = \frac{32}{10} = 3.2$ cm.

5. If $A = 45°$ and $B = 30°$, then the value $\sin A \cos B + \cos A \sin B$ is

(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{3}}$

Solution. Choice (a) is correct.

$\sin A \cos B + \cos A \sin B$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

6. The value of $(\sec\theta + \tan\theta)(1 - \sin\theta)$ is

(a) $\sec\theta$ (b) $\operatorname{cosec}\theta$

(c) $\sin\theta$ (d) $\cos\theta$

Solution. Choice (*d*) is correct.

(sec θ + tan θ)(1 – sin θ)

$$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$$

$$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin\theta)$$

$$= \frac{1 - \sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta}$$

$$= \mathbf{\cos\theta}$$

7. If the HCF of 210 and 55 is expressible in the form 210 × 5 + 55 × *p*, then the value of *p* is

(*a*) – 17 (*b*) – 18

(*c*) – 19 (*d*) – 20

Solution. Choice (*c*) is correct.

Given integers are 210 and 55.

Apply Euclid's division lemma to 210 and 55, we get

$$210 = 55 \times 3 + 45 \qquad \ldots(1)$$

Since the remainder 45 ≠ 0, therefore, apply Euclid's division lemma to 55 and 45, we get

$$55 = 45 \times 1 + 10 \qquad \ldots(2)$$

Since the remainder 10 ≠ 0, therefore, apply Euclid's division lemma to 45 and 10, we get

$$45 = 10 \times 4 + 5 \qquad \ldots(3)$$

Since the remainder 5 ≠ 0, therefore, apply Euclid's division lemma to 10 and 5.

$$10 = 5 \times 2 + 0$$

The remainder at this stage is 0. So, the divisor at this stage or the remainder at the previous stage *i.e.*, 5 is the HCF of 210 and 55.

From (3), we get

$$5 = 45 - 10 \times 4$$
$$= 45 - (55 - 45 \times 1) \times 4 \qquad \text{[using (2)]}$$
$$= 45 - 55 \times 4 + 45 \times 4$$
$$= 45 \times 5 - 55 \times 4$$
$$= (210 - 55 \times 3) \times 5 - 55 \times 4 \qquad \text{[using (1)]}$$
$$= 210 \times 5 - 55 \times 15 - 55 \times 4$$
$$= 210 \times 5 - 55 \times 19$$

Note : $5 = \text{HCF} = 210 \times 5 + 55 \times p \Rightarrow 55 \times p = 5 - 210 \times 5$

$\Rightarrow$ $55 \times p = 5 - 1050 = -1045 \Rightarrow p = -1045 \div 55 = \mathbf{-19}$

8. If $\bar{x}$ is the arithmetic mean of *n* observations $x_1, x_2, \ldots, x_n$, then the arithmetic mean of $ax_1, ax_2, \ldots, ax_n$ is

(*a*) $a\bar{x}$ (*b*) $\frac{\bar{x}}{a}$

(*c*) $\frac{a}{\bar{x}}$ (*d*) None of these

Solution. Choice (a) is correct.

Mean of n observations $x_1, x_2, \ldots, x_n$ is

$$\text{(given) } \bar{x} = \frac{x_1 + x_2 + \ldots.. + x_n}{n}$$

$\Rightarrow \quad x_1 + x_2 + \ldots + x_n = n\bar{x}$ ... (1)

Mean of n observations $ax_1, ax_2, \ldots, ax_n$ is

$$= \frac{ax_1 + ax_2 + \ldots.. + ax_n}{n}$$

$$= \frac{a(x_1 + x_2 + \ldots.. + x_n)}{n}$$

$$= \frac{a(n\bar{x})}{n} \quad \text{[using (1)]}$$

$$= a\bar{x}$$

9. If the pair of linear equations

$$3x + 2y = 1$$

$$(2k + 1)x + (k + 2)y = k - 1$$

has infinitely many solution, then the value of k is

(a) 2 (b) 3

(c) 4 (d) 5

Solution. Choice (c) is correct.

Here, $\frac{a_1}{a_2} = \frac{3}{2k+1}$, $\frac{b_1}{b_2} = \frac{2}{k+2}$, $\frac{c_1}{c_2} = \frac{-1}{-(k-1)} = \frac{1}{k-1}$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{2k+1} = \frac{2}{k+2} = \frac{1}{k-1}$$

$$\Rightarrow \quad \frac{3}{2k+1} = \frac{2}{k+2} \text{ and } \frac{2}{k+2} = \frac{1}{k-1}$$

$$\Rightarrow \quad 3k + 6 = 4k + 2 \text{ and } 2k - 2 = k + 2$$

$$\Rightarrow \quad 4k - 3k = 6 - 2 \text{ and } 2k - k = 2 + 2$$

$$\Rightarrow \quad k = 4 \text{ and } k = 4$$

Hence, the given system of linear equations have an infinite number of solutions when $k = 4$.

10. If $\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$, then the value of x is

(a) 30° (b) 15°

(c) 45° (d) 60°

Solution. Choice (a) is correct.

$$\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$$

$$\Rightarrow \quad \cos x = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$\Rightarrow \qquad \cos x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\Rightarrow \qquad \cos x = 2 \times \frac{\sqrt{3}}{4}$$

$$\Rightarrow \qquad \cos x = \frac{\sqrt{3}}{2} = \cos 30°$$

$$\Rightarrow \qquad x = 30°$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Explain $5 \times 4 \times 3 \times 2 \times 1 + 3$ is a composite number.

Solution. We have

$$5 \times 4 \times 3 \times 2 \times 1 + 3$$
$$= (5 \times 4 \times 1 \times 2 \times 1 + 1) \times 3$$
$$= (40 + 1) \times 3$$
$$= 41 \times 3$$

$\Rightarrow$ $5 \times 4 \times 3 \times 2 \times 1 + 3 = 3 \times 41$ is a **composite number** as product of prime occur.

12. If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, then evaluate : $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Solution. Since α and β are the zeroes of the polynomial $p(x) = ax^2 + bx + c$, therefore

$$\alpha + \beta = -\left(\frac{b}{a}\right) = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Now, $$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\frac{c}{a}}$$

$$= \frac{(b^2 - 2ac)/a^2}{c/a}$$

$$= \mathbf{\frac{b^2 - 2ac}{ac}}$$

13. Solve for x and y

$$\left.\begin{aligned}\frac{5}{x}+\frac{1}{y}&=2\\ \frac{6}{x}-\frac{3}{y}&=1\end{aligned}\right\} x \neq 0, y \neq 0$$

Solution. The given system of linear equations are

$$\frac{5}{x}+\frac{1}{y}=2 \quad \ldots(1)$$

and $$\frac{6}{x}-\frac{3}{y}=1 \quad \ldots(2)$$

Multiplying (1) by 3, we obtain the new equation as

$$\frac{15}{x}+\frac{3}{y}=6 \quad \ldots(3)$$

Adding (2) and (3), we get

$$\left(\frac{6}{x}+\frac{15}{x}\right)+\left(-\frac{3}{y}+\frac{3}{y}\right)=1+6$$

$$\Rightarrow \quad \frac{21}{x}+0=7$$

$$\Rightarrow \quad \frac{21}{x}=7 \Rightarrow x=\frac{21}{7}=3$$

Putting $x=3$ in (1), we get

$$\frac{5}{3}+\frac{1}{y}=2$$

$$\Rightarrow \quad \frac{1}{y}=2-\frac{5}{3}$$

$$\Rightarrow \quad \frac{1}{y}=\frac{1}{3}$$

$$\Rightarrow \quad y=3$$

Hence, the solution is $\boldsymbol{x=3}$ **and** $\boldsymbol{y=3}$.

14. Prove that : $\sec^4\theta-\sec^2\theta=\tan^4\theta+\tan^2\theta$.

Solution. We have

$$\begin{aligned}\text{L.H.S.} &= \sec^4\theta-\sec^2\theta\\ &= \sec^2\theta(\sec^2\theta-1)\\ &= (1+\tan^2\theta)[(1+\tan^2\theta)-1] \qquad [\because \sec^2 A = 1+\tan^2 A]\\ &= (1+\tan^2\theta)(\tan^2\theta)\\ &= \tan^2\theta+\tan^4\theta\\ &= \text{R.H.S.}\end{aligned}$$

Or

Find acute angles A and B, if

$$\sin(A+2B)=\frac{\sqrt{3}}{2} \text{ and } \cos(A+4B)=0, A>B.$$

Solution. We have

$$\sin(A + 2B) = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ $\sin(A + 2B) = \sin 60°$

$\Rightarrow$ $A + 2B = 60°$ ...(1)

and $\cos(A + 4B) = 0$

$\Rightarrow$ $\cos(A + 4B) = \cos 90°$

$\Rightarrow$ $A + 4B = 90°$ ...(2)

Subtracting (1) from (2), we get

$$(A + 4B) - (A + 2B) = 90° - 60°$$

$\Rightarrow$ $2B = 30°$

$\Rightarrow$ $B = 15°$

Substituting $B = 15°$ in (1), we get

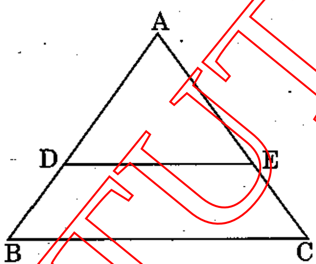
$$A + 2 \times (15°) = 60°$$

$\Rightarrow$ $A + 30° = 60°$

$\Rightarrow$ $A = 60° - 30° = 30°$

Hence, $\boldsymbol{A = 30°}$ **and** $\boldsymbol{B = 15°}$**.**

15. In figure, $DE \parallel BC$ and $BD = CE$. Prove that ΔABC is an isosceles triangle.



Solution. In ΔABC, we have

$DE \parallel BC$ [given]

$\Rightarrow$ $\dfrac{AD}{BD} = \dfrac{AE}{CE}$ [By BPT]

$\Rightarrow$ $\dfrac{AD}{BD} + 1 = \dfrac{AE}{CE} + 1$ [Adding 1 to both sides]

$\Rightarrow$ $\dfrac{AD + BD}{BD} = \dfrac{AE + CE}{CE}$

$\Rightarrow$ $\dfrac{AB}{BD} = \dfrac{AC}{CE}$

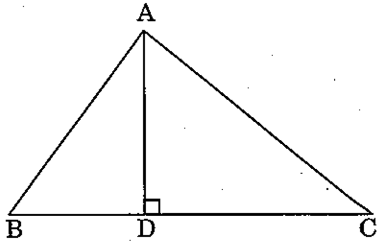
But $BD = CE$ [given]

$\therefore$ $\dfrac{AB}{BD} = \dfrac{AC}{BD}$ [using $CE = BD$]

$\Rightarrow$ $AB = AC$

$\Rightarrow$ ΔABC is an **isosceles triangle.**

16. In figure, $AD \perp BC$, if $\frac{BD}{DA} = \frac{DA}{DC}$, prove that ABC is a right triangle.



Solution. In right triangles ADB and ADC, we have

$AB^2 = AD^2 + BD^2$ ...(1) [Pythagoras Theorem]

and $AC^2 = AD^2 + DC^2$ ...(2) [Pythagoras Theorem]

Adding (1) and (2), we get

$$AB^2 + AC^2 = (AD^2 + BD^2) + (AD^2 + DC^2)$$
$$= 2AD^2 + BD^2 + DC^2$$
$$= 2BD.DC + BD^2 + DC^2$$
$$= (BD + DC)^2$$
$$= BC^2$$

$\left[\because \frac{BD}{DA} = \frac{DA}{DC} \text{ (given)} \Rightarrow BD.DC = DA^2 \Rightarrow BD.DC = AD^2\right]$

Thus, in ΔABC, we have

$$AB^2 + AC^2 = BC^2$$

Hence, ΔABC is a right triangle, right angled at A.

17. **The following distribution gives the daily income of 100 workers of a factory.**

Income (in ₹)	**0 – 100**	**100 – 200**	**200 – 300**	**300 – 400**	**400 – 500**	**500 – 600**
Number of workers	**7**	**15**	**35**	**28**	**10**	**5**

Write the above distribution as more than type cumulative frequency distribution.

Solution. Cumulative frequency distribution table of more than type

Income (in ₹)	*No. of workers (f)*	*Income*	*Cumulative frequency*
0 – 100	7	More than 0	100 (93 + 7)
100 – 200	15	More than 100	93 (78 + 15)
200 – 300	35	More than 200	78 (43 + 35)
300 – 400	28	More than 300	43 (28 + 15)
400 – 500	10	More than 400	15 (10 + 5)
500 – 600	5	More than 500	5
Total	100		

18. **If the mode of the following distribution is 57.5, find the value of x.**

Classes	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**	**80 – 90**	**90 – 100**
Frequency	**6**	**10**	**16**	x	**10**	**5**	**2**

Solution. Since the mode is given as 57.5, therefore the modal class is 50 – 60 and the lower limit (l) is 50.

$\therefore$ $l = 50, h = 10, f_1 = 16, f_0 = 10, f_2 = x$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow \quad 57.5 = 50 + \frac{16 - 10}{2 \times 16 - 10 - x} \times 10$$

$$\Rightarrow \quad 57.5 - 50 = \frac{6}{22 - x} \times 10$$

$$\Rightarrow \quad 7.5 = \frac{60}{22 - x}$$

$$\Rightarrow \quad 7.5 \times (22 - x) = 60$$

$$\Rightarrow \quad 165 - 7.5x = 60$$

$$\Rightarrow \quad 7.5x = 165 - 60$$

$$\Rightarrow \quad x = \frac{105}{7.5}$$

$$\Rightarrow \quad \mathbf{x = 14.}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Show that $2 - \sqrt{3}$ is an irrational number.

Solution. Let us assume, to the contrary, that $2 - \sqrt{3}$ is rational.
That is we can find co-prime a and b ($b \neq 0$) such that

$$2 - \sqrt{3} = \frac{a}{b}$$

$$\Rightarrow \quad 2 - \frac{a}{b} = \sqrt{3}$$

Rearranging this equation, we have

$$\sqrt{3} = 2 - \frac{a}{b} = \frac{2b - a}{b}$$

Since a and b are integers, $2 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $2 - \sqrt{3}$ is rational.

So, we conclude that $2 - \sqrt{3}$ is **irrational.**

Or

Prove that $\sqrt{3}$ is an irrational number.

Solution. Let us assume, to the contrary, that $\sqrt{3}$ is rational. Then

$$\sqrt{3} = \frac{p}{q}, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0.$$

Suppose p and q have a common factor other than 1, then we can divide by the common factor, to get

$$\sqrt{3} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are coprime.}$$

So, $\sqrt{3}b = a$

Squaring on both sides, and rearranging, we get

$3b^2 = a^2 \Rightarrow a^2$ is divisible by $3 \Rightarrow a$ is also divisible by 3

[If r (prime) divides a^2, then r divides a]

Let $a = 3m$, where m is an integer.

Substituting $a = 3m$ in $3b^2 = a^2$, we get

$3b^2 = 9m^2 \Rightarrow b^2 = 3m^2$

This means that b^2 is divisible by 3, and so b is also divisible by 3. Therefore, a and b have at least 3 as a common factor. But this contradicts the fact that a and b are coprime. This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

So, we conclude that **$\sqrt{3}$ is irrational.**

20. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Solution. Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$.

When $x = 3q$, then by squaring, we have

$x^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$, where $m = 3q^2$

When $x = 3q + 1$, then by squaring, we have

$x^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3q(3q + 2) + 1 = 3m + 1$, where $m = q(3q + 2)$

When $x = 3q + 2$, then by squaring, we have

$x^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1$

$= 3(3q^2 + 4q + 1) + 1 = 3m + 1$, where $m = 3q^2 + 4q + 1$.

Hence, the square of any positive integer, say x, is either of the form $3m$ or $3m + 1$ for some integer m.

21. Solve the following pair of equations :

$$\frac{5}{x+y} - \frac{2}{x-y} = -1$$

$$\frac{15}{x+y} + \frac{7}{x-y} = 10$$

Solution. Given equations are

$$\frac{5}{x+y} - \frac{2}{x-y} = -1 \quad \text{...(1)}$$

and
$$\frac{15}{x+y} + \frac{7}{x-y} = 10 \quad \text{...(2)}$$

Multiplying equation (1) by 3 and subtracting from (2), we get

$$\left(\frac{15}{x+y}+\frac{7}{x-y}\right)-\left(\frac{15}{x+y}-\frac{6}{x-y}\right)=10-(-3)$$

$$\Rightarrow \quad \frac{7}{x-y}+\frac{6}{x-y}=13$$

$$\Rightarrow \quad \frac{13}{x-y}=13$$

$$\Rightarrow \quad x-y=1 \qquad \text{...(3)}$$

Substituting $x - y = 1$ in (1), we get

$$\frac{5}{x+y}-\frac{2}{1}=-1$$

$$\Rightarrow \quad \frac{5}{x+y}=2-1$$

$$\Rightarrow \quad x+y=5 \qquad \text{...(4)}$$

Adding equations (3) and (4), we get

$$2x = 6 \Rightarrow x = 3$$

Substituting $x = 3$ in (4), we get

$$3 + y = 5 \Rightarrow y = 2$$

Hence, $x = 3, y = 2$ is the solution of the given pair of equations.

Or

Ram travels 760 km to his home, partly by train and partly by car. He takes 8 hours if he travels 160 km by train and the rest by the car. He takes 12 minutes more if he travels 240 km by train and the rest by car. Find the speed of the train and the car separately.

Solution. Let the speed of the train be x km/h and that of the car be y km/h.

When Ram travels 160 km by train and rest by car

Time taken by Ram to travel 160 km by train = $\frac{160}{x}$ h

Time taken by Ram to travel (760 – 160) = 600 km by car = $\frac{600}{y}$ h

∴ Time taken by Ram to cover 760 km = $\left(\frac{160}{x}+\frac{600}{y}\right)$ h

It is given that the total time taken is 8 hours.

$$\therefore \quad \frac{160}{x}+\frac{600}{y}=8$$

$$\Rightarrow \quad \frac{20}{x}+\frac{75}{y}=1 \qquad \text{...(1)}$$

When Ram travels 240 km by train and rest by car

Time taken by Ram to travel 240 km by train = $\frac{240}{x}$ h

Time taken by Ram to travel (760 – 240) = 520 km by car = $\frac{520}{y}$ h

∴ Time taken by Ram to cover 760 km = $\left(\frac{240}{x}+\frac{520}{y}\right)$ h

It is given that the Ram takes 8 hours 12 minutes for the journey.

$$\therefore \quad \frac{240}{x}+\frac{520}{y}=8\frac{12}{60}=8\frac{1}{5}$$

$$\Rightarrow \quad \frac{240}{x}+\frac{520}{y}=\frac{41}{5}$$

$$\Rightarrow \quad \frac{30}{x}+\frac{65}{y}=\frac{41}{5\times 8}=\frac{41}{40} \qquad \ldots(2)$$

Multiplying equation (1) by 3 and equation (2) by 2, we get

$$\frac{60}{x}+\frac{225}{y}=3 \qquad \ldots(3)$$

and
$$\frac{60}{x}+\frac{130}{y}=\frac{41}{20} \qquad \ldots(4)$$

Subtracting (4) from (3), we get

$$\left(\frac{60}{x}+\frac{225}{y}\right)-\left(\frac{60}{x}+\frac{130}{y}\right)=3-\frac{41}{20}$$

$$\Rightarrow \quad \frac{225}{y}-\frac{130}{y}=\frac{60-41}{20}=\frac{19}{20}$$

$$\Rightarrow \quad \frac{95}{y}=\frac{19}{20}$$

$$\Rightarrow \quad y=\frac{95\times 20}{19}$$

$$\Rightarrow \quad y=100$$

Substituting $y = 100$ in (3), we get

$$\frac{60}{x}+\frac{225}{100}=3$$

$$\Rightarrow \quad \frac{60}{x}=3-\frac{225}{100}$$

$$\Rightarrow \quad \frac{60}{x}=\frac{300-225}{100}$$

$$\Rightarrow \quad \frac{60}{x}=\frac{75}{100}$$

$$\Rightarrow \quad x=\frac{60\times 100}{75}=80$$

Hence, speed of train = **80 km/h** and speed of car = **100 km/h**.

22. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - 2x + 3$, find a quadratic polynomial whose roots are $\frac{\alpha - 1}{\alpha + 1}, \frac{\beta - 1}{\beta + 1}$.

Solution. Since α and β are the zeroes of a given quadratic polynomial $p(x)$, then

$$\alpha + \beta = -\left(\frac{-2}{1}\right) = 2$$

and $$\alpha\beta = \frac{3}{1} = 3$$

Let S and P denote the sum and product of the roots $\frac{\alpha - 1}{\alpha + 1}$ and $\frac{\beta - 1}{\beta + 1}$, then

$$S = \frac{\alpha - 1}{\alpha + 1} + \frac{\beta - 1}{\beta + 1}$$

$$\Rightarrow \quad S = \frac{(\alpha - 1)(\beta + 1) + (\alpha + 1)(\beta - 1)}{(\alpha + 1)(\beta + 1)}$$

$$\Rightarrow \quad S = \frac{(\alpha\beta + \alpha - \beta + 1) + (\alpha\beta - \alpha + \beta - 1)}{\alpha\beta + \alpha + \beta + 1}$$

$$\Rightarrow \quad S = \frac{2\alpha\beta}{\alpha\beta + (\alpha + \beta) + 1}$$

$$\Rightarrow \quad S = \frac{2(3)}{3 + 2 + 1}$$

$$\Rightarrow \quad S = \frac{6}{6} = 1$$

and $$P = \frac{\alpha - 1}{\alpha + 1} \times \frac{\beta - 1}{\beta + 1}$$

$$\Rightarrow \quad P = \frac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1}$$

$$\Rightarrow \quad P = \frac{\alpha\beta - (\alpha + \beta) + 1}{\alpha\beta + (\alpha + \beta) + 1}$$

$$\Rightarrow \quad P = \frac{3 - 2 + 1}{3 + 2 + 1}$$

$$\Rightarrow \quad P = \frac{2}{6} = \frac{1}{3}$$

∴ Required quadratic polynomial is

$$f(x) = [x^2 - Sx + P]$$

or $$f(x) = \left(x^2 - x + \frac{1}{3}\right)$$

or $$f(x) = k(3x^2 - 3x + 1)$$, where k is non-zero constant.

23. If $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and $\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = -1$, prove that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2.$$

Solution. We have

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \quad \ldots (1)$$

and
$$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = -1 \quad \ldots (2)$$

Squaring and adding (1) and (2), we get

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 1^2 + (-1)^2$$

$$\Rightarrow \left(\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\frac{xy}{ab}\sin\theta\cos\theta\right) + \left(\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\frac{xy}{ab}\sin\theta\cos\theta\right) = 2$$

$$\Rightarrow \quad \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

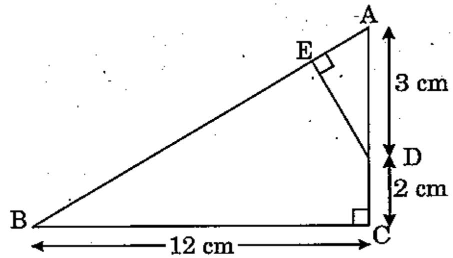
24. Prove that

$$(\tan A - \tan B)^2 + (1 + \tan A \tan B)^2 = \sec^2 A \sec^2 B.$$

Solution. We have

$$\begin{aligned}
\text{L.H.S.} &= (\tan A - \tan B)^2 + (1 + \tan A \tan B)^2 \\
&= (\tan^2 A + \tan^2 B - 2\tan A\tan B) + (1 + \tan^2 A\tan^2 B + 2\tan A\tan B) \\
&= \tan^2 A + \tan^2 B + 1 + \tan^2 A\tan^2 B \\
&= (1 + \tan^2 A) + (\tan^2 B + \tan^2 A\tan^2 B) \\
&= (1 + \tan^2 A) + \tan^2 B\,(1 + \tan^2 A) \\
&= (1 + \tan^2 A)(1 + \tan^2 B) \\
&= \sec^2 A \sec^2 B \\
&= \text{R.H.S.}
\end{aligned}$$

25. In figure, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.



Solution. In ΔABC and ΔADE, we have

$\angle ACB = \angle AED$ [Each = 90°]

$\angle BAC = \angle DAE$ [Each equal to $\angle A$]

So, by AA-criterion of similarity of triangles, we have

$\Delta ABC \sim \Delta ADE$

$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$ ...(1)

In right triangle ABC, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AB^2 = (AD + DC)^2 + BC^2$

$\Rightarrow \quad AB^2 = (3 + 2)^2 + (12)^2$

$\Rightarrow \quad AB^2 = 25 + 144 = 169 \text{ cm}^2$

$\Rightarrow \quad AB = 13 \text{ cm}$

From (1), we have

$\frac{AB}{AD} = \frac{AC}{AE}$

$\Rightarrow \quad \frac{13}{3} = \frac{AD + DC}{AE} \Rightarrow \frac{13}{3} = \frac{3+2}{AE}$ [using $AB = 13$ cm, $AD = 3$ cm, $DC = 2$ cm]

$\Rightarrow \quad AE = \mathbf{\frac{15}{13}}$ **cm**

Again, from (1), we have

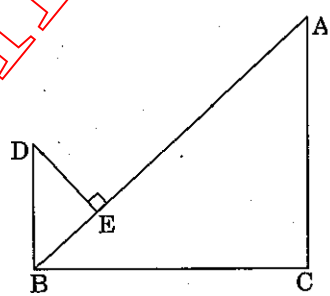
$\frac{AB}{AD} = \frac{BC}{DE}$

$\Rightarrow \quad \frac{13}{3} = \frac{12}{DE}$ [using $AB = 13$ cm, $BC = 12$ cm, $AD = 3$ cm]

$\Rightarrow \quad DE = \mathbf{\frac{36}{13}}$ **cm.**

26. In figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Solution. In figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

Since DB and AC both are perpendicular to BC, therefore $DB \parallel AC$.

$\angle DBE = \angle BAC$ ...(1) [Alternate $\angle s$ as $DB \parallel AC$]

Also $\angle DEB = \angle ACB$ ...(2) [Each = 90°]

In Δs BED and ACB, we have

$\angle DBE = \angle BAC$

and $\angle DEB = \angle ACB$ [Proved above]

So, by AA-criterion of similarity of triangles, we have

$\Delta BED \sim \Delta ACB$

$$\Rightarrow \quad \frac{BE}{DE} = \frac{AC}{BC}$$

27. The following table gives the daily income of 50 workers of a factory :

Daily Income (in ₹)	**100 – 120**	**120 – 140**	**140 – 160**	**160 – 180**	**180 – 200**
Number of workers	**12**	**14**	**8**	**6**	**10**

Find the median of the above data.

Solution. The cumulative frequency distribution table with the given frequency becomes :

Daily Income (in ₹)	*No. of workers (f)*	*Cumulative frequency (cf)*
100 – 120	12	12
120 – 140	14	26
140 – 160	8	34
160 – 180	6	40
180 – 200	10	50
Total	$n = \Sigma f_i = 50$	

Now, 120 – 140 is the class whose cumulative frequency 26 is greater than $\frac{n}{2} = 25$

Therefore, 120 – 140 is the median class. Thus, the lower limit (l) of the median class is 120.

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 120 + \frac{25 - 12}{14} \times 20$$

$$= 120 + \frac{13}{14} \times 20$$

$$= 120 + \frac{130}{7}$$

$$= 120 + 18.57$$

$$= 138.57$$

So, about half the workers have daily income less than ₹ **138.57** and other half have daily income more than ₹ **138.57.**

28. **The mean of the following frequency distribution is 62.8. Find the value p.**

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	8	p	12	7	8

Solution.

Classes	Frequency (f_i)	Class-mark (x_i)	$f_i x_i$
0 – 20	5	10	50
20 – 40	8	30	240
40 – 60	p	50	$50p$
60 – 80	12	70	840
80 – 100	7	90	630
100 – 120	8	110	880
Total	$n = \Sigma f_i = 40 + p$		$\Sigma f_i x_i = 2640 + 50p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 62.8 = \frac{2640 + 50p}{40 + p}$$

$$\Rightarrow \quad 62.8(40 + p) = 2640 + 50p$$

$$\Rightarrow \quad 2512 + 62.8p = 2640 + 50p$$

$$\Rightarrow \quad 62.8p - 50p = 2640 - 2512$$

$$\Rightarrow \quad 12.8p = 128$$

$$\Rightarrow \quad p = \frac{128}{12.8}$$

$$\Rightarrow \quad \mathbf{p = 10}$$

Or

Find the mean of the following frequency distribution, using step-deviation method.

Classes	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54	55 – 59
Frequency	14	22	16	6	5	3	4

Solution. Let the assumed mean be $a = 42$, $h = 5$.

Classes	Frequency (f_i)	Class-mark (x_i)	$u_i = \frac{x_i - 42}{5}$	$f_i u_i$
25 – 29	14	27	– 3	– 42
30 – 34	22	32	– 2	– 44
35 – 39	16	37	– 1	– 16
40 – 44	6	42	0	0
45 – 49	5	47	1	5
50 – 54	3	52	2	6
55 – 59	4	57	3	12
Total	$n = \Sigma f_i = 70$			$\Sigma f_i u_i = -79$

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 42 + \frac{(-79)}{70} \times 5$$

$$= 42 - \frac{79}{14}$$

$$= 42 - 5.64$$

$$= \mathbf{36.36}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Compute the missing frequencies, x and y in the following data if the mean is $166\frac{9}{26}$ and the sum of the observations is 52.

Classes	*Frequency*
140 – 150	5
150 – 160	x
160 – 170	20
170 – 180	y
180 – 190	6
190 – 200	2
Total	52

Now, also calculate the median.

Solution. Since the classes are of equal size, it will be more convenient to use the Step-deviation Method.

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 165}{10}$	$f_i u_i$	*cf*
140 – 150	5	145	– 2	– 10	5
150 – 160	x	155	– 1	$-x$	$5 + x = 15$
160 – 170	20	$165 = a$	0	0	$25 + x = 35$
170 – 180	y	175	1	y	$25 + x + y = 44$
180 – 190	6	185	2	12	$31 + x + y = 50$
190 – 200	2	195	3	6	$33 + x + y = 52$
Total	$n = \Sigma f_i = 33 + x + y$			$\Sigma f_i u_i = 8 - x + y$	

Assumed Mean $(a) = 165$, $h = 10$.

We have $n = 52 = \text{Total frequency} = \Sigma f_i$

$\Rightarrow \quad 33 + x + y = 52$

$\Rightarrow \quad x + y = 52 - 33$

$\Rightarrow \quad x + y = 19$ ...(1)

By Step-deviation Method, we have

$$\overline{X} = a + h \times \frac{1}{n}\Sigma f_i u_i$$

$$\Rightarrow \quad \text{(given) } 166\frac{9}{26} = 165 + 10 \times \frac{1}{52} \times (8 - x + y)$$

$$\Rightarrow \quad \frac{4325}{26} = 165 + \frac{10(8 - x + y)}{52}$$

$$\Rightarrow \quad \frac{4325}{26} - 165 = \frac{10(8 - x + y)}{52}$$

$$\Rightarrow \quad \frac{4325 - 4290}{26} = \frac{10(8 - x + y)}{52}$$

$$\Rightarrow \quad \frac{35}{26} = \frac{10(8 - x + y)}{52}$$

$$\Rightarrow \quad 35 = 5(8 - x + y)$$

$$\Rightarrow \quad 7 = 8 - x + y$$

$$\Rightarrow \quad x - y = 1 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$2x = 20 \Rightarrow x = 10$$

Substituting $x = 10$ in (1), we get

$$y = 19 - 10 = 9$$

Hence, the missing frequencies corresponding to the classes 150 – 160 and 170 – 180 are **10** and **9** respectively.

Here, $\frac{n}{2} = \frac{52}{2} = 26$

Now, 160 – 170 is the class whose cumulative frequency 35 is greater than $\frac{n}{2} = 26$.

$\therefore$ 160 – 170 is median class. Thus, the lower limit of median class is 160.

From the table, $f = 20$, $cf = 15$, $h = 10$

Using the formula :

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 160 + \frac{26 - 15}{20} \times 10$$

$$= 160 + \frac{11}{2}$$

$$= 160 + 5.5$$

$$= \mathbf{165.5.}$$

30. Solve the following system of linear equations graphically :

$$2x + y = 8$$

$$3x - 2y = 12$$

Also find the coordinates of the points where the lines meet the x-axis.

Solution. We have

$$2x + y = 8 \quad \text{and} \quad 3x - 2y = 12$$

$$\Rightarrow \quad y = 8 - 2x \qquad \Rightarrow \quad 2y = 3x - 12$$

$$\Rightarrow \quad y = \frac{3x - 12}{2}$$

Table of $y = 8 - 2x$

x	1	2	3	4
y	6	4	2	0
	A	B	C	D

Table of $y = \dfrac{3x - 12}{2}$

x	4	2	0	6
y	0	−3	−6	3
	D	E	F	G

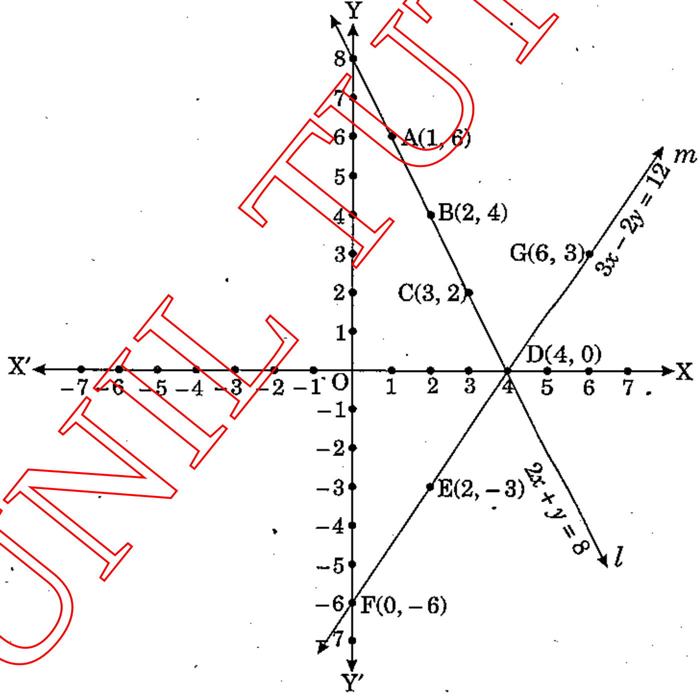
Take XOX' and YOY' as the axes of coordinate. Plotting the points $A(1, 6)$, $B(2, 4)$, $C(3, 2)$ and $D(4, 0)$ and joining them by a line, we get a line 'l' which is the graph of $2x + y = 8$.

Further, plotting the points $D(4, 0)$, $E(2, -3)$, $F(0, -6)$ and $G(6, 3)$ and joining them by a line, we get a line 'm' which is the graph of $3x - 2y = 12$.

From the graph of the two equations, we find that the two lines l and m intersect each other at the point $D(4, 0)$.

$\therefore$ **$x = 4, y = 0$ is the solution.**

The first line $2x + y = 8$ meets the x-axis at $x = 4$. The second line $3x - 2y = 12$ meets the x-axis at $x = 4$. From the graph, we observe that both the lines meet the x-axis at **$D(4, 0)$.**



31. Without using trigonometric tables, evaluate :

$\cot 12° \cot 38° \cot 52° \cot 60° \cot 78° + \tan (55° - \theta) - \cot (35° + \theta) + \cos(40° + \theta) - \sin (50° - \theta)$

Solution. We have

$$\cot 12° \cot 38° \cot 52° \cot 60° \cot 78° + \tan(55° - \theta) - \cot(35° + \theta) + \cos(40° + \theta) - \sin(50° - \theta)$$

$$= \cot 12° \cot 38° \cot(90° - 38°) \cot 60° \cot(90° - 12°) + \tan[90° - (35° + \theta)] - \cot(35° + \theta) + \cos(40° + \theta) - \sin[90° - (40° + \theta)]$$

$$= \cot 12° \cot 38° \tan 38° \cot 60° \tan 12° + \cot(35° + \theta) - \cot(35° + \theta) + \cos(40° + \theta) - \cos(40° + \theta)$$

$[\because \cot(90° - \theta) = \tan\theta, \tan(90° - \theta) = \cot\theta, \sin(90° - \theta) = \cos\theta]$

$$= (\cot 12° \tan 12°)(\cot 38° \tan 38°) \cot 60° + 0 + 0$$

$$= (1)(1)(\sqrt{3}) + 0 + 0 \qquad [\because \tan\theta \cot = 1]$$

$$= \sqrt{3}$$

Or

If $7\sin^2\theta + 3\cos^2\theta = 4$, find the value of $\sec\theta + \operatorname{cosec}\theta$.

Solution. Given,

$$7\sin^2\theta + 3\cos^2\theta = 4 \qquad \ldots(1)$$

$$\Rightarrow \quad 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$$

$$\Rightarrow \quad 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$$

$$\Rightarrow \quad 4\sin^2\theta + 3 = 4 \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$\Rightarrow \quad 4\sin^2\theta = 4 - 3 = 1$$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{4} \qquad \ldots(2)$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2} \qquad \text{[Taking positive sign]}$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = 2$$

$$\Rightarrow \quad \operatorname{cosec}\theta = 2$$

We know that, $\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$ [using (2)]

$$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2} \qquad \text{[Taking positive sign]}$$

$$\Rightarrow \quad \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}} \Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$$

Now, $\sec\theta + \operatorname{cosec}\theta$

$$= \frac{2}{\sqrt{3}} + 2$$

$$= \frac{2 + 2\sqrt{3}}{\sqrt{3}} = \frac{2(1 + \sqrt{3})}{\sqrt{3}} = \frac{2\sqrt{3}(1 + \sqrt{3})}{3}$$

$$= \frac{2(\sqrt{3} + 3)}{3} = \frac{2(1.732 + 3)}{3} = \frac{2(4.732)}{3}$$

$$= 2(1.577)$$

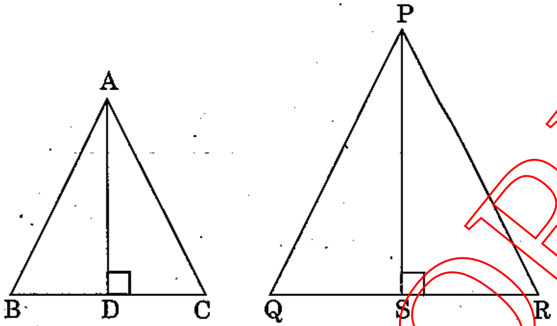
$$= \mathbf{3.154}$$

32. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.



Proof : $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently, $\dfrac{AD}{PS} = \dfrac{AB}{PQ}$ ...(2)

[If Δ's are similar, the ratio of their corresponding sides is same]

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\Rightarrow \quad \dfrac{AD}{PS} = \dfrac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$$

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC}{QR} \times \dfrac{BC}{QR}$ [using (3)]

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ ...(4)

As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \qquad \ldots(5)$$

Hence, $$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$ [From (4) and (5)]

Or

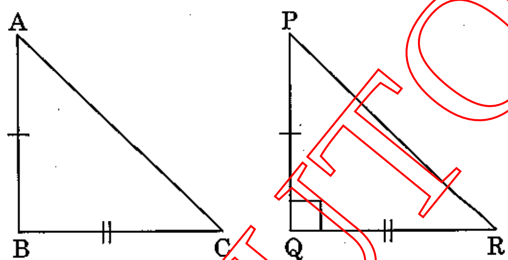
State and prove the converse of the following theorem :

In a right triangle, the square of the hypotenuse is equal to the sum of the other two sides.

Solution. Statement : In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given : A triangle ABC such that : $AC^2 = AB^2 + BC^2$

To prove : ΔABC is a right-angled at B, *i.e.*, $\angle B = 90°$.



Construction : Construct a ΔPQR such that $\angle Q = 90°$ and $PQ = AB$ and $QR = BC$. [see figure]

Proof : In ΔPQR, as $\angle Q = 90°$, we have

$PR^2 = PQ^2 + QR^2$ [By Pythagoras Theorem]

$\Rightarrow \quad PR^2 = AB^2 + BC^2$ ...(1)

[As $PQ = AB$ and $QR = BC$]

But $\quad AC^2 = AB^2 + BC^2$ ...(2)

From (1) and (2), we have

$PR^2 = AC^2$

$\Rightarrow \quad PR = AC$ ...(3)

Now in ΔABC and ΔPQR, we have

$AB = PQ$

$BC = QR$

and $\quad AC = PR$ [using (3)]

$\therefore \quad \Delta ABC \cong \Delta PQR$ [SSS congruency]

$\Rightarrow \quad \angle B = \angle Q = 90°$ [CPCT]

Hence, $\quad \boldsymbol{\angle B = 90°}$.

33. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Solution. Let the speed of the boat in still water be x km/h and speed of the stream be y km/h.

The speed of the boat downstream = $(x + y)$ km/h.

The speed of the boat upstream = $(x - y)$ km/h.

In the first case, when the boat goes 30 km upstream and 44 km downstream.

Time taken in going 30 km upstream + Time taken in going 44 km downstream

= 10 hours (given)

$$\Rightarrow \quad \frac{30}{x-y} + \frac{44}{x+y} = 10 \qquad \left[\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \quad \frac{30}{x-y} + \frac{44}{x+y} - 10 = 0 \qquad \ldots(1)$$

In the second case, when the boat goes 40 km upstream and 55 km downstream.

Time taken in going 40 km upstream + Time taken in going 55 km downstream

= 13 hours (given)

$$\Rightarrow \quad \frac{40}{x-y} + \frac{55}{x+y} = 13$$

$$\Rightarrow \quad \frac{40}{x-y} + \frac{55}{x+y} - 13 = 0 \qquad \ldots(2)$$

Using Cross-multiplication method, we get

$$\frac{\frac{1}{x-y}}{\begin{vmatrix} 44 & -10 \\ 55 & -13 \end{vmatrix}} = \frac{\frac{1}{x+y}}{\begin{vmatrix} -10 & 30 \\ -13 & 40 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 30 & 44 \\ 40 & 55 \end{vmatrix}}$$

$$\Rightarrow \quad \frac{\frac{1}{x-y}}{-572+550} = \frac{\frac{1}{x+y}}{-400+390} = \frac{1}{1650-1760}$$

$$\Rightarrow \quad \frac{\frac{1}{x-y}}{-22} = \frac{\frac{1}{x+y}}{-10} = \frac{1}{-110}$$

$$\Rightarrow \quad \frac{1}{x-y} = \frac{-22}{-110} = \frac{1}{5} \text{ and } \frac{1}{x+y} = \frac{-10}{-110} = \frac{1}{11}$$

$$\Rightarrow \quad x - y = 5 \text{ and } x + y = 11$$

Adding these equations, we get

$$2x = 16 \Rightarrow x = 8$$

Subtracting these equations, we get

$$(x+y) - (x-y) = 11 - 5 \Rightarrow 2y = 6 \Rightarrow y = 3.$$

Hence, the speed of the boat in still water is **8 km/h** and the speed of the stream is **3 km/h.**

34. If $x = \cot A + \cos A$ and $y = \cot A - \cos A$, show that $x^2 - y^2 = 4\sqrt{xy}$.

Solution. Given $\cot A + \cos A = x \qquad \ldots(1)$

and $\cot A - \cos A = y \qquad \ldots(2)$

Adding (1) and (2), we get

$$2 \cot A = x + y$$

$$\Rightarrow \quad \cot A = \frac{x+y}{2}$$

$$\Rightarrow \quad \frac{1}{\tan A} = \frac{x+y}{2}$$

$$\Rightarrow \quad \tan A = \frac{2}{x+y} \qquad \ldots(3)$$

Subtracting (2) from (1), we get

$$2\cos A = x - y$$

$$\Rightarrow \quad \cos A = \frac{x-y}{2}$$

$$\Rightarrow \quad \frac{1}{\sec A} = \frac{x-y}{2}$$

$$\Rightarrow \quad \sec A = \frac{2}{x-y} \qquad \ldots(4)$$

We know that

$$\sec^2 A - \tan^2 A = 1$$

$$\Rightarrow \quad \left(\frac{2}{x-y}\right)^2 - \left(\frac{2}{x+y}\right)^2 = 1 \qquad \text{[using (3) and (4)]}$$

$$\Rightarrow \quad \frac{4}{(x-y)^2} - \frac{4}{(x+y)^2} = 1$$

$$\Rightarrow \quad 4\left[\frac{(x+y)^2 - (x-y)^2}{(x-y)^2(x+y)^2}\right] = 1$$

$$\Rightarrow \quad 4\left[\frac{4xy}{(x-y)^2(x+y)^2}\right] = 1$$

$$\Rightarrow \quad 16xy = [(x-y)(x+y)]^2$$

$$\Rightarrow \quad 16xy = (x^2 - y^2)^2$$

$$\Rightarrow \quad 4\sqrt{xy} = x^2 - y^2 \qquad \text{[Taking +ve square root]}$$