# CCE SAMPLE QUESTION PAPER

## FIRST TERM (SA–I)
## MATHEMATICS
*(With Solutions)*
## CLASS X

*Time Allowed* : 3 to 3½ Hours] [*Maximum Marks* : 80

**General Instructions :**

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

### Section 'A'

*Question numbers 1 to 10 are of one mark each.*

**1. If $0° < x < 90°$ and $2 \sin^2 x = \frac{1}{2}$, then the value of $x$ is**

(*a*) 15° (*b*) 30°
(*c*) 45° (*d*) 60°

**Solution.** Choice (*b*) is correct.

$$2 \sin^2 x = \frac{1}{2}$$

$$\Rightarrow \quad \sin^2 x = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \sin x = \frac{1}{2} \qquad [\because \sin x > 0 \text{ for } 0° < x < 90°]$$

$$\Rightarrow \quad \sin x = \frac{1}{2} = \sin 30°$$

$$\Rightarrow \quad x = 30°$$

**2. The value of $\text{cosec}^2 30° \sin^2 45° - \sec^2 60°$ is**

(*a*) –1 (*b*) 0
(*c*) –2 (*d*) 2

**Solution.** Choice (*c*) is correct.

$\text{cosec}^2 30° \sin^2 45° - \sec^2 60°$

$$= (2)^2 \left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2 \qquad \left[\because \text{cosec } 30° = 2, \sin 45° = \frac{1}{\sqrt{2}} \text{ and } \sec 60° = 2\right]$$

$= \frac{4}{2} - 4$
$= 2 - 4$
$= -2$

**3. If $\tan 2A = \cot (A - 18°)$, where $2A$ is an acute angle, then the value of $A$ is**

(*a*) 24° (*b*) 12°
(*c*) 36° (*d*) 48°

**Solution.** Choice (*c*) is correct.

Given, $\tan 2A = \cot (A - 18°)$

$\Rightarrow \cot (90° - 2A) = \cot (A - 18°)$ $[\because \tan\theta = \cot(90° - \theta)]$

$\Rightarrow 90° - 2A = A - 18°$

$\Rightarrow 90° + 18° = 2A + A$

$\Rightarrow 108° = 3A$

$\Rightarrow$ $\mathbf{A = 36°}$

**4. If $\sec x + \tan x = p$, then $\sec x$ is equal to**

(*a*) $\frac{p^2 - 1}{p}$ (*b*) $\frac{p^2 - 1}{2p}$

(*c*) $\frac{p^2 + 1}{p}$ (*d*) $\frac{p^2 + 1}{2p}$

**Solution.** Choice (*d*) is correct.

Given, $\sec x + \tan x = p$ ...(1)

$\Rightarrow \frac{1}{\sec x + \tan x} = \frac{1}{p}$

$\Rightarrow \frac{\sec x - \tan x}{(\sec x - \tan x)(\sec x + \tan x)} = \frac{1}{p}$

$\Rightarrow \frac{\sec x - \tan x}{\sec^2 x - \tan^2 x} = \frac{1}{p}$

$\Rightarrow \frac{\sec x - \tan x}{1} = \frac{1}{p}$

$\Rightarrow \sec x - \tan x = \frac{1}{p}$ ...(2)

Adding (1) and (2), we get

$(\sec x + \tan x) + (\sec x - \tan x) = p + \frac{1}{p}$

$\Rightarrow 2\sec x = \frac{p^2 + 1}{p}$

$\Rightarrow$ $\mathbf{\sec x = \frac{p^2 + 1}{2p}}$

**5. The decimal expansion of $\frac{98}{125}$ will terminate after how many places of decimal ?**

(*a*) 1 (*b*) 2

(c) 3 (d) 4

**Solution.** Choice (c) is correct.

$$\frac{98}{125} = \frac{98 \times 8}{125 \times 8} = \frac{784}{1000} = 0.784$$

Thus, the decimal expansion of $\frac{98}{125}$ will terminate after **3** places of decimal.

**6. The largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively is**

(a) 11 (b) 17

(c) 34 (d) 51

**Solution.** Choice (b) is correct.

It is given that on dividing 398, 436 and 542 by the largest required number, the remainders are 7, 11 and 15 respectively. This means that

$398 - 7 = 391$, $436 - 11 = 425$ and $542 - 15 = 527$ are exactly divisible by the required largest number.

Apply Euclid's division lemma to the numbers 527 and 425, we get

$$527 = 425 \times 1 + 102$$
$$425 = 102 \times 4 + 17$$
$$102 = 17 \times 6 + 0$$

Clearly, HCF of 527 and 435 is 17.

Again, apply Euclid's division lemma to the numbers 391 and 17, we get

$$391 = 23 \times 17 + 0$$

Thus, HCF of 391 and 17 is 17.

Hence, HCF of 391, 425 and 527 is **17**.

**7. If $x = a$, $y = b$ is the solution of the equations $x + y = 50$ and $4x + 5y = 225$, then the values of $a$ and $b$ are respectively**

(a) 10 and 40 (b) 25 and 25

(c) 23 and 27 (d) 20 and 30

**Solution.** Choice (b) is correct.

Since $x = a$ and $y = b$ is the solution of the equations, therefore

$$a + b = 50 \quad \ldots(1)$$

and

$$4a + 5b = 225 \quad \ldots(2)$$

From (1), $a = 50 - b$

Substituting $a = 50 - b$ in (2), we get

$$4(50 - b) + 5b = 225$$
$$\Rightarrow \quad 200 - 4b + 5b = 225$$
$$\Rightarrow \quad b = 225 - 200$$
$$\Rightarrow \quad b = 25$$

Substituting $b = 25$ in (1), we get

$$a = 50 - 25 = 25$$

Hence the values of $a$ and $b$ are **25** and **25** respectively.

**8. In the given data :**

| ***Classes*** | **65 – 85** | **85 – 105** | **105 – 125** | **125 – 145** | **145 – 165** | **165 – 185** | **185 – 205** |
|---|---|---|---|---|---|---|---|
| ***Frequency*** | **4** | **5** | **13** | **20** | **14** | **7** | **4** |

**The difference between the upper limit of the median class and the lower limit of the modal class is**

(*a*) 0 (*b*) 10

(*c*) 20 (*d*) 30

**Solution.** Choice (*c*) is correct.

| *Classes* | 65 – 85 | 85 – 105 | 105 – 125 | 125 – 145 | 145 – 165 | 165 – 185 | 185 – 205 |
|---|---|---|---|---|---|---|---|
| *Frequency* | 4 | 5 | 13 | 20 | 14 | 7 | 4 |
| *Cumulative Frequency* | 4 | 9 | 22 | 42 | 56 | 63 | 67 |

Here $n = 67$ and $\frac{n}{2} = \frac{67}{2} = 33.5$

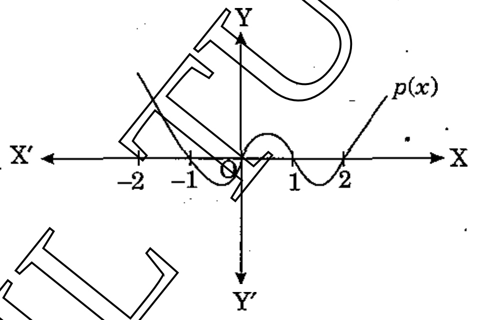
Now, 125 – 145 is the class whose cumulative frequency 42 is just greater than $\frac{n}{2} = 33.5$

Thus, 125 – 145 is the median class. The upper limit of the median class is 145.

Since the maximum frequency is 20, therefore the modal class is 125 – 145. Thus, the lower limit of the modal class 125 – 145 is 125.

Hence, the difference between the upper limit of median class, *i.e.*, 145 and the lower limit of the modal class, *i.e.*, 125 is **20** (= 145 – 125).

**9. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is**



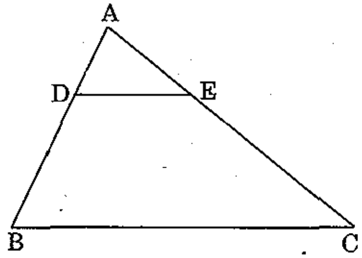

(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 1

**Solution.** Choice (*a*) is correct.

The number of zeroes is 4 as the graph intersects the *x*-axis in four points, *viz.*, – 1, 0, 1, 2.

**10. In figure, $\Delta ABC$, *D* and *E* are points on the sides *AB* and *AC* respectively such that $DE \parallel BC$. If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, then *AC* is equal to**



(*a*) 10 cm (*b*) 20 cm
(*c*) 15 cm (*d*) 30 cm

**Solution.** Choice (*b*) is correct.

In figure, $DE \parallel BC$

$\therefore$ By BPT, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{AC - AE}$$

$$\Rightarrow \quad \frac{6}{9} = \frac{8}{AC - 8}$$

$$\Rightarrow \quad 6AC - 48 = 72$$

$$\Rightarrow \quad 6AC = 120$$

$$\Rightarrow \quad \mathbf{AC = 20\ cm.}$$

## Section 'B'

*Question numbers 11 to 18 carry 2 marks each.*

**11. Express 3825 as the product of prime factor.**

**Solution.** We have

$$\begin{aligned} 3825 &= 5 \times 765 \\ &= 5 \times 5 \times 153 \\ &= 5 \times 5 \times 3 \times 51 \\ &= 5 \times 5 \times 3 \times 3 \times 17 \\ &= \mathbf{3^2 \times 5^2 \times 17} \end{aligned}$$

**12. If $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(x) = x^2 - ax + b$, then find the value of $\alpha^2 + \beta^2$.**

**Solution.** Since $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $p(x)$, therefore,

$$\alpha + \beta = -\left(\frac{-a}{1}\right) = a$$

$$\alpha\beta = \frac{b}{1} = b$$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \quad \alpha^2 + \beta^2 = (a)^2 - 2b$

$\Rightarrow \quad \boldsymbol{\alpha^2 + \beta^2 = a^2 - 2b}$

**13. For what value of '*k*' will the following pair of linear equations have infinitely many solutions?**

$$\boldsymbol{kx + 3y = k - 3}$$
$$\boldsymbol{12x + ky = k}$$

**Solution.** The given pair of linear equations can be written as

$$kx + 3y - (k - 3) = 0$$
$$12x + ky - k = 0$$

Here, $\frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}, \frac{c_1}{c_2} = \frac{-(k-3)}{-k} = \frac{k-3}{k}$

For a pair of linear equations to have infinitely many solutions :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$$

Consider, $\frac{k}{12} = \frac{3}{k}$

$$\Rightarrow \quad k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

Again, consider, $\frac{3}{k} = \frac{k-3}{k}$

$$\Rightarrow \quad 3k = k^2 - 3k$$

$$\Rightarrow \quad 6k = k^2$$

$$\Rightarrow \quad 6k - k^2 = 0$$

$$\Rightarrow \quad k(6-k) = 0$$

$$\Rightarrow \quad k = 0 \quad \text{or} \quad k = 6$$

Thus, the value of $k$, that satisfies both the solutions, is $\boldsymbol{k = 6}$. For this value, the pair of linear equations has infinitely many solutions.

**14. Prove that :**

$$\sqrt{\frac{\textbf{cosec } A - 1}{\textbf{cosec } A + 1}} + \sqrt{\frac{\textbf{cosec } A + 1}{\textbf{cosec } A - 1}} = \textbf{2 sec } A$$

**Solution.** We have

$$\text{L.H.S.} = \sqrt{\frac{\text{cosec } A - 1}{\text{cosec } A + 1}} + \sqrt{\frac{\text{cosec } A + 1}{\text{cosec } A - 1}}$$

$$= \frac{(\sqrt{\text{cosec } A - 1})(\sqrt{\text{cosec } A - 1}) + (\sqrt{\text{cosec } A + 1})(\sqrt{\text{cosec } A + 1})}{\sqrt{\text{cosec } A + 1}\sqrt{\text{cosec } A - 1}}$$

$$= \frac{(\text{cosec } A - 1) + (\text{cosec } A + 1)}{\sqrt{\text{cosec}^2 A - 1}} \qquad [\because \sqrt{a+b} \times \sqrt{a-b} = a - b]$$

$$= \frac{2 \text{ cosec } A}{\sqrt{1 + \cot^2 A - 1}}$$

$$= \frac{2 \text{ cosec } A}{\sqrt{\cot^2 A}}$$

$$= \frac{2 \text{ cosec } A}{\cot A}$$

$$= \frac{2}{\sin A} \times \frac{\sin A}{\cos A}$$

$$= \frac{2}{\cos A}$$

$= 2 \sec A$

$=$ R.H.S.

***Or***

**If *ABC* is a right angle triangle, right-angled at *C*. If $\angle A = 30°$ and *AB* = 50 units, find the remaining two sides and $\angle B$ of $\Delta ABC$.**

**Solution.** We have

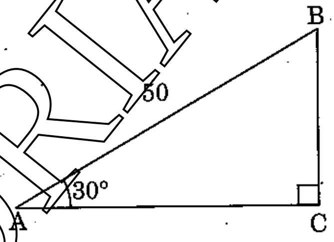
$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad 30° + \angle B + 90° = 180° \qquad [\because \angle A = 30° \text{ and } \angle C = 90°]$$

$$\Rightarrow \quad \angle B + 120° = 180°$$

$$\Rightarrow \quad \angle B = 180° - 120°$$

$$\Rightarrow \quad \angle B = 60°$$




Now, $\sin 30° = \dfrac{BC}{AB}$

$$\Rightarrow \quad \frac{1}{2} = \frac{BC}{50}$$

$$\Rightarrow \quad BC = \frac{50}{2} = 25 \text{ units}$$

and $\cos 30° = \dfrac{AC}{AB}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AC}{50}$$

$$\Rightarrow \quad AC = \frac{50\sqrt{3}}{2} = 25\sqrt{3} \text{ units}$$

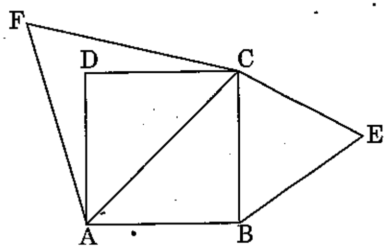
Hence, $AC = 25\sqrt{3}$ **units,** $BC = 25$ **units** and $\angle B = 60°$**.**

**15. Prove that the area of an equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.**

**Solution.** Let *ABCD* be a square and *BCE* and *ACF* are two equilateral triangles described on its side *BC* and its diagonal *AC* respectively.

Since we have to prove that

$$(\text{area } \Delta BCE) = \frac{1}{2} \times \text{area } (\Delta ACF)$$




Let the side of the square be $a$, then

$$BC = a = AB = AD = CD$$

and

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{a^2 + a^2}$$

$$= \sqrt{2a^2}$$

$$= \sqrt{2}\, a \qquad \ldots(1)$$

Now, we have

$$\Delta BCE \sim \Delta ACF \qquad [\text{All equilateral triangles are similar as each angle is of } 60°]$$

$\therefore \quad \dfrac{\text{area }(\Delta BCE)}{\text{area }(\Delta ACF)} = \dfrac{BC^2}{AC^2}$ [The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides]

$$= \frac{a^2}{(\sqrt{2}a)^2} \quad \text{[using (1)]}$$

$$= \frac{a^2}{2a^2}$$

$$\Rightarrow \quad \frac{\text{area }(\Delta BCE)}{\text{area }(\Delta ACF)} = \frac{1}{2}$$

Hence, **area ($\Delta BCE$) = $\frac{1}{2}$ × area ($\Delta ACF$)**

**16. A life insurance agent found the following data for distribution of ages of 100 policy holders, when the policies are given only to persons having age 18 years but less than 60 years.**

| *Age (in years)* | 0 – 20 | 20 – 25 | 25 – 30 | 30 – 35 | 35 – 40 | 40 – 45 | 45 – 50 | 50 – 55 | 55 – 60 |
|---|---|---|---|---|---|---|---|---|---|
| *No. of policy holders* | 2 | 4 | 18 | 21 | 33 | 11 | 3 | 6 | 2 |

**Write the above distribution as less than type cumulative frequency distribution.**

**Solution.** **Cumulative Frequency Distribution**

| *Age (in years)* | *No. of policy holders (cf)* |
|---|---|
| Less than 20 | 2 |
| Less than 25 | 6 (2 + 4) |
| Less than 30 | 24 (6 + 18) |
| Less than 35 | 45 (24 + 21) |
| Less than 40 | 78 (45 + 33) |
| Less than 45 | 89 (78 + 11) |
| Less than 50 | 92 (89 + 3) |
| Less than 55 | 98 (92 + 6) |
| Less than 60 | 100 (98 + 2) |

**17. Find the modal marks of the following distribution of marks obtained by 70 students.**

| *Marks obtained* | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 |
|---|---|---|---|---|---|---|---|---|
| *Number of students* | 5 | 18 | 30 | 45 | 40 | 15 | 10 | 7 |

**Solution.** Since the class 30 – 40 has the maximum frequency, therefore 30 – 40 is the modal class.

$\therefore \quad l = 30, h = 10, f_1 = 45, f_0 = 30, f_2 = 40$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{45 - 30}{2 \times 45 - 30 - 40} \times 10$$

$$= 30 + \frac{15}{90 - 70} \times 10$$

$$= 30 + \frac{15}{20} \times 10$$

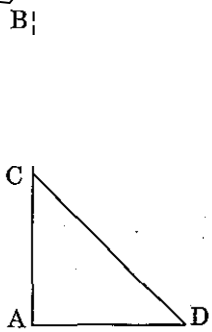
$$= 30 + 7.5$$

$$= 37.5$$

Hence the **modal marks = 37.5.**

**18. A vertical pillar $AB$ is bent at $C$ at height of 2.4 metres and its upper end $B$ touches the ground at a distance of 1.8 metres from the end $A$ on the ground. Find the height of the pillar $AB$.**

**Solution.** $AB$ is the pillar. At the point $C$ it is bent so that its upper end touches the ground at $D$, such that




$$AD = 1.8 \text{ metres}$$

Also $\quad AC = 2.4$ metres

$\therefore$ From right-angled triangle $CAD$,

$$CD^2 = AC^2 + AD^2$$

$$= (2.4)^2 + (1.8)^2$$

$$= 5.76 + 3.24 = 9$$

$\Rightarrow \quad CD = \sqrt{9} = 3$

$\therefore \quad AB = AC + CB = AC + CD$

$$= 2.4 + 3$$

$$= \textbf{5.4 metres.}$$

## Section 'C'

*Question numbers 19 to 28 carry 3 marks each.*

**19. Show that $n^2 - 1$ is divisible by 8, if $n$ is an odd positive integer.**

**Solution.** We know that any odd positive integer is of the form $4m + 1$ or $4m + 3$ for some integer $m$.

**When $n = 4m + 1$,** then

$$n^2 - 1 = (4m + 1)^2 - 1$$

$$= (16m^2 + 8m + 1) - 1$$

$$= 16m^2 + 8m$$

$$= 8m(2m + 1)$$

$\Rightarrow \quad n^2 - 1$ is divisible by 8 $\qquad$ [$\because$ $8m(2m + 1)$ is divisible by 8]

**When $n = 4m + 3$,** then

$$n^2 - 1 = (4m + 3)^2 - 1$$

$$= (16m^2 + 24m + 9) - 1$$

$$= 16m^2 + 24m + 8$$

$$= 8(2m^2 + 3m + 1)$$

$\Rightarrow \quad n^2 - 1$ is divisible by 8 $\qquad$ [$\because$ $8(2m^2 + 3m + 1)$ is divisible by 8]

Hence, **$n^2 - 1$ is divisible by 8.**

**20. Prove that $\sqrt{5}$ is an irrational number.**

**Solution.** Let us assume, to the contrary, that $\sqrt{5}$ is rational.

That is, we can find integers $a$ and $b$ ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

Suppose $a$ and $b$ have a common factor other than 1, then we can divide by the common factor, and assume that $a$ and $b$ are coprime.

So, $\quad b\sqrt{5} = a$

Squaring on both sides, and rearranging, we get

$5b^2 = a^2 \Rightarrow 5$ divides $a^2 \Rightarrow 5$ divides $a$ $\quad$ [If $r$ (prime) divides $a^2$, then $r$ divides $a$]

Let $a = 5m$, where $m$ is an integer.

Substituting $a = 5m$ in $5b^2 = a^2$, we get

$5b^2 = 25m^2 \Rightarrow b^2 = 5m^2$

$\Rightarrow 5$ divides $b^2$ and so 5 divides $b$. $\quad$ [If $r$ (prime) divides $b^2$, then $r$ divides $b$]

Therefore, $a$ and $b$ have at least 5 as a common factor and the conclusion contradicts the hypothesis that $a$ and $b$ have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is **irrational.**

***Or***

**Show that $5 + 3\sqrt{2}$ is an irrational number.**

**Solution.** Let us assume, to contrary, that $5 + 3\sqrt{2}$ is rational.

That is, we can find coprime $a$ and $b$ ($b \neq 0$) such that

$$5 + 3\sqrt{2} = \frac{a}{b}$$

Therefore, $\quad 3\sqrt{2} = \frac{a}{b} - 5$

Rearranging this equation, we get

$$3\sqrt{2} = \frac{a - 5b}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a - 5b}{3b}$$

Since $a$ and $b$ are integers, $\frac{a - 5b}{3b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 + 3\sqrt{2}$ is rational.

So, we conclude that $5 + 3\sqrt{2}$ is **irrational.**

**21. A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.**

**Solution.** Let the ages of $A$ and $B$ be $x$ and $y$ years respectively, then

$$x - y = \pm 2 \qquad \ldots(1)$$

$D$'s age = Twice the age of $A$ = $2x$ years

It is given that $B$ is twice as old as his sister $C$.

$\Rightarrow$ $C$'s age = Half the age of $B$ = $\frac{y}{2}$ years

Then, $2x - \frac{y}{2} = 40$ ...(2)

**When $x - y = 2$ ... (1$a$) and $2x - \frac{y}{2} = 40$**

Multiplying (2) by 2, we get

$$4x - y = 80 \quad ...(3)$$

Subtracting (1$a$) from (3), we get

$$(4x - y) - (x - y) = 80 - 2$$

$\Rightarrow$ $3x = 78$

$\Rightarrow$ $x = 26$

Substituting $x = 26$ in (1$a$), we get

$$26 - y = 2 \Rightarrow y = 26 - 2 = 24$$

**When $x - y = -2$ ... (1$b$) and $2x - \frac{y}{2} = 40$.**

Multiplying (2) by 2, we get

$$4x - y = 80 \quad ...(3)$$

Subtracting (1$b$) from (3), we get

$$(4x - y) - (x - y) = 80 + 2$$

$\Rightarrow$ $3x = 82$

$\Rightarrow$ $x = \frac{82}{3} = 27\frac{1}{3}$

Substituting $x = 27\frac{1}{3}$ in (1$b$), we get

$$\frac{82}{3} - y = -2$$

$\Rightarrow$ $y = \frac{82}{3} + 2$

$\Rightarrow$ $y = \frac{82 + 6}{3} = \frac{88}{3} = 29\frac{1}{3}$

Hence, **$A$'s age = 26 years** and **$B$'s age = 24 years**

***or***

**$A$'s age = $27\frac{1}{3}$ years** and **$B$'s age = $29\frac{1}{3}$ years.**

***Or***

**Solve the following pair of equations :**

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

**Solution.** We have :

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \text{...(1)}$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad \text{...(2)}$$

Multiplying equation (1) by 3 and adding in equation (2), we get

$$\left(\frac{15}{x-1} + \frac{3}{y-2}\right) + \left(\frac{6}{x-1} - \frac{3}{y-2}\right) = 6 + 1$$

$$\Rightarrow \quad \frac{21}{x-1} = 7 \quad \Rightarrow \quad x - 1 = 21 \div 7 \Rightarrow x - 1 = 3 \Rightarrow x = 4$$

Substituting $x = 4$ in equation (2), we get

$$\frac{6}{4-1} - \frac{3}{y-2} = 1$$

$$\Rightarrow \quad 2 - \frac{3}{y-2} = 1$$

$$\Rightarrow \quad \frac{3}{y-2} = 2 - 1 \quad \Rightarrow \quad \frac{3}{y-2} = 1$$

$$\Rightarrow \quad y - 2 = 3 \div 1 = 3 \quad \Rightarrow \quad y = 3 + 2 = 5$$

Hence, $\boldsymbol{x = 4, y = 5}$ is the solution of the given pair of equations.

**22. If $\alpha$ and $\beta$ are zeroes of a quadratic polynomial, such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find a quadratic polynomial having $\alpha$ and $\beta$ as its zeroes.**

**Solution.** Given, $\alpha + \beta = 24$ ...(1)

and $\alpha - \beta = 8$ ...(2)

Adding (1) and (2), we get

$$(\alpha + \beta) + (\alpha - \beta) = 24 + 8$$

$$\Rightarrow \quad 2\alpha = 32$$

$$\Rightarrow \quad \alpha = 16$$

Substituting $\alpha = 16$ in (1), we get

$$16 + \beta = 24$$

$$\Rightarrow \quad \beta = 24 - 16 = 8$$

Let $S$ and $P$ denote the sum and product of a required quadratic polynomial $p(x)$ then,

$$S = \alpha + \beta = 16 + 8 = 24 \text{ and } P = \alpha\beta = (16)(8) = 128$$

$$p(x) = k[x^2 - Sx + P], \text{ where } k \text{ is non-zero constant}$$

or $\boldsymbol{p(x) = k[x^2 - 24x + 128]}$, where $k$ is non-zero constant.

**23. If $x = a \sec\theta + b \tan\theta$ and $y = a \tan\theta + b \sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.**

**Solution.** We have

$$x = a \sec\theta + b \tan\theta \quad \text{...(1)}$$

and $y = a \tan\theta + b \sec\theta$ ...(2)

Squaring (1) and (2), we get

$$x^2 = a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta \quad \text{...(3)}$$

and $\quad y^2 = a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta \quad \ldots(4)$

Subtracting (4) from (3), we get

$$x^2 - y^2 = (a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta) - (a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta)$$

$\Rightarrow \quad x^2 - y^2 = a^2(\sec^2 \theta - \tan^2 \theta) + b^2(\tan^2 \theta - \sec^2 \theta)$

$\Rightarrow \quad x^2 - y^2 = a^2(1 + \tan^2 \theta - \tan^2 \theta) + b^2(\tan^2 \theta - 1 - \tan^2 \theta)$

$\Rightarrow \quad x^2 - y^2 = a^2(1) + b^2(-1)$

$\Rightarrow \quad \boldsymbol{x^2 - y^2 = a^2 - b^2}$

**24. Prove that :**

$$\frac{\cos A}{1 - \sin A} + \frac{\sin A}{1 - \cos A} + 1 = \frac{\sin A \cos A}{(1 - \sin A)(1 - \cos A)}$$

**Solution.** We have,

$$\text{L.H.S.} = \frac{\cos A}{1 - \sin A} + \frac{\sin A}{1 - \cos A} + 1$$

$$= \frac{\cos A\,(1 - \cos A) + \sin A\,(1 - \sin A) + (1 - \sin A)(1 - \cos A)}{(1 - \sin A)(1 - \cos A)}$$

$$= \frac{\cos A - \cos^2 A + \sin A - \sin^2 A + 1 - \sin A - \cos A + \sin A \cos A}{(1 - \sin A)(1 - \cos A)}$$

$$= \frac{(\cos A + \sin A) - (\cos^2 A + \sin^2 A) + 1 - (\cos A + \sin A) + \sin A \cos A}{(1 - \sin A)(1 - \cos A)}$$

$$= \frac{(\cos A + \sin A) - 1 + 1 - (\cos A + \sin A) + \sin A \cos A}{(1 - \sin A)(1 - \cos A)}$$

$$= \frac{\sin A \cos A}{(1 - \sin A)(1 - \cos A)}$$

$= \text{R.H.S.}$

**25. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.**

**Solution. Given :** A quadrilateral $ABCD$ whose diagonals $AC$ and $BD$ intersect each other at $O$ such that

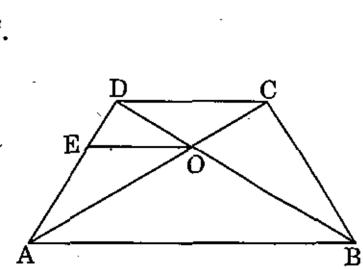
$$\frac{AO}{OC} = \frac{BO}{OD}$$

**To prove :** Quadrilateral $ABCD$ is a trapezium, *i.e.*, $AB \parallel DC$.

**Construction :** Draw $OE \parallel BA$, meeting $AD$ in $E$.

**Proof :** In $\triangle ABD$, we have

$OE \parallel BA$




So, $\dfrac{AE}{ED} = \dfrac{BO}{OD} \quad \ldots(1) \quad$ [By BPT]

But $\dfrac{AO}{OC} = \dfrac{BO}{OD} \quad \ldots(2) \quad$ [given]

From (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{OC}$$

Thus, in $\Delta DCA$, $O$ and $E$ are points on $AC$ and $AD$ respectively such that

$$\frac{AE}{ED} = \frac{AO}{OC}$$

Therefore, by the converse of BPT, we have

$$EO \parallel DC$$

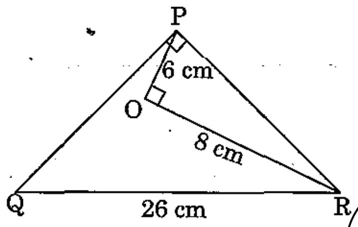
But $\quad OE \parallel BA$

$\therefore \quad DC \parallel BA$

$\Rightarrow \quad AB \parallel DC$

Hence, $ABCD$ is a **trapezium.**

**26. Calculate area ($\Delta PQR$) from figure :**




**Solution.** $PQR$ is a right-angled triangle, $M$ is any point inside $\Delta PQR$ and $PMR$ is a right-angled triangle.

$QR = 26$ cm

$PM = 6$ cm (given)

$RM = 8$ cm

In right-angled triangle $PMR$, we have

$PR^2 = PM^2 + RM^2$ [using Pythagoras Theorem]

$\Rightarrow \quad PR^2 = (6)^2 + (8)^2$

$\Rightarrow \quad PR^2 = 36 + 64$

$\Rightarrow \quad PR^2 = 100$

$\Rightarrow \quad PR = 10$ cm ...(1)

In right-angled $\Delta PQR$, we have

$QR^2 = PQ^2 + PR^2$ [using Pythagoras Theorem]

$\Rightarrow \quad PQ^2 = QR^2 - PR^2$

$\Rightarrow \quad PQ^2 = (26)^2 - (10)^2$

$\Rightarrow \quad PQ^2 = 676 - 100$

$\Rightarrow \quad PQ^2 = 576$

$\Rightarrow \quad PQ = 24$ cm ...(2)

Now, area $(\Delta PQR) = \frac{1}{2} PR \times PQ$

$= \left(\frac{1}{2} \times 10 \times 24\right) \text{cm}^2$ [using (1) and (2)]

$=$ **120 cm$^2$.**

27. **In a retail market, fruit vendor were selling mangoes kept in packing boxes. These boxes contained varying numbers of mangoes. The following was the distribution of mangoes according to the number of boxes.**

| *Number of mangoes* | *Number of boxes* |
|---|---|
| 50 – 52 | 15 |
| 53 – 55 | 110 |
| 56 – 58 | 135 |
| 59 – 61 | 115 |
| 62 – 64 | 25 |

**Find the mean number of mangoes kept in a packing box, using step-deviation method.**

**Solution.** Let the assumed mean be $a = 57$ and $h = 3$.

| *Number of mangoes* | *No. of boxes* ($f_i$) | *Class-mark* ($x_i$) | $u_i = \frac{x_i - 57}{3}$ | $f_i u_i$ |
|---|---|---|---|---|
| 50 – 52 | 15 | 51 | – 2 | – 30 |
| 53 – 55 | 110 | 54 | – 1 | – 110 |
| 56 – 58 | 135 | 57 | 0 | 0 |
| 59 – 61 | 115 | 60 | 1 | 115 |
| 62 – 64 | 25 | 63 | 2 | 50 |
| *Total* | $n = \Sigma f_i = 400$ | | | $\Sigma f_i u_i = 25$ |

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$= 57 + \frac{25}{400} \times 3$$

$$= 57 + \frac{3}{16}$$

$$= 57 + 0.1875$$

$$= 57.1875$$

Hence, the mean number of mangoes in a box ≈ **57.19.**

***Or***

**The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the value of *p*.**

| ***Daily pocket allowance (in ₹)*** | **11 – 13** | **13 – 15** | **15 – 17** | **17 – 19** | **19 – 21** | **21 – 23** | **23 – 25** |
|---|---|---|---|---|---|---|---|
| ***Number of children*** | **7** | **6** | **9** | **13** | ***p*** | **5** | **4** |

**Solution.**

| *Daily pocket allowance (in ₹)* | *No. of children* ($f_i$) | *Class-mark* ($x_i$) | $f_ix_i$ |
|---|---|---|---|
| 11 – 13 | 7 | 12 | 84 |
| 13 – 15 | 6 | 14 | 84 |
| 15 – 17 | 9 | 16 | 144 |
| 17 – 19 | 13 | 18 | 234 |
| 19 – 21 | $p$ | 20 | $20p$ |
| 21 – 23 | 5 | 22 | 110 |
| 23 – 25 | 4 | 24 | 96 |
| *Total* | $n = \Sigma f_i = 44 + p$ | | $\Sigma f_ix_i = 752 + 20p$ |

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \text{(given) } 18 = \frac{752 + 20p}{44 + p}$$

$$\Rightarrow \quad 18(44 + p) = 752 + 20p$$

$$\Rightarrow \quad 792 + 18p = 752 + 20p$$

$$\Rightarrow \quad 20p - 18p = 792 - 752$$

$$\Rightarrow \quad 2p = 40$$

$$\Rightarrow \quad \mathbf{p = 20}$$

**28. Find the median of the following data :**

| ***Classes*** | **0 – 20** | **20 – 40** | **40 – 60** | **60 – 80** | **80 – 100** | **100 – 120** | **120 – 140** |
|---|---|---|---|---|---|---|---|
| ***Frequency*** | **6** | **8** | **10** | **12** | **6** | **5** | **3** |

**Solution.** The cumulative frequency distribution table with the given frequency becomes :

| *Classes* | *Frequency (f)* | *Cumulative Frequency (cf)* |
|---|---|---|
| 0 – 20 | 6 | 6 |
| 20 – 40 | 8 | 14 |
| 40 – 60 | 10 | 24 (*cf*) |
| 60 – 80 | 12 (*f*) | 36 |
| 80 – 100 | 6 | 42 |
| 100 – 120 | 5 | 47 |
| 120 – 140 | 3 | 50 |
| *Total* | $n = \Sigma f_i = 50$ | |

Now, 60 – 80 is the class whose cumulative frequency 36 is greater than $\frac{n}{2} = 25$. Therefore, 60 – 80 is the median class. Thus, the lower limit ($l$) of the median class is 60.

Using the formula :

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 60 + \frac{25 - 24}{12} \times 20$$

$$= 60 + \frac{20}{12}$$

$$= 60 + \frac{5}{3}$$

$$= 60 + 1.67$$

$$= \mathbf{61.67.}$$

## Section 'D'

*Question numbers 29 to 34 carry 4 marks each.*

**29. Find the zeroes of the polynomial $f(x) = x^3 - 5x^2 - 16x + 80$, if its two zeroes are equal in magnitude but opposite in sign.**

**Solution.** Let $\alpha$, $\beta$, $\gamma$ be the zeroes of the given polynomial $f(x)$, then

$$\alpha + \beta + \gamma = -\left(\frac{-5}{1}\right) = 5 \quad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{16}{1} = -16 \quad \ldots(2)$$

$$\alpha\beta\gamma = -\left(\frac{80}{1}\right) = -80 \quad \ldots(3)$$

It is given that two zeroes are equal in magnitude but opposite in sign, therefore

Let $\beta = -\alpha$, then

$$\alpha + \beta = 0 \quad \ldots(4)$$

From (1) and (4), we get

$$0 + \gamma = 5 \Rightarrow \gamma = 5 \quad \ldots(5)$$

From (3) and (5), we get

$$\alpha\beta(5) = -80$$

$$\Rightarrow \alpha\beta = -16 \quad \ldots(6)$$

Substituting $\beta = -\alpha$ in (6), we get

$$\alpha(-\alpha) = -16$$

$$\Rightarrow -\alpha^2 = -16$$

$$\Rightarrow \alpha^2 = 16$$

$$\Rightarrow \alpha = \pm 4$$

Thus, $\beta = -\alpha = \pm(\pm 4) = \mp 4$.

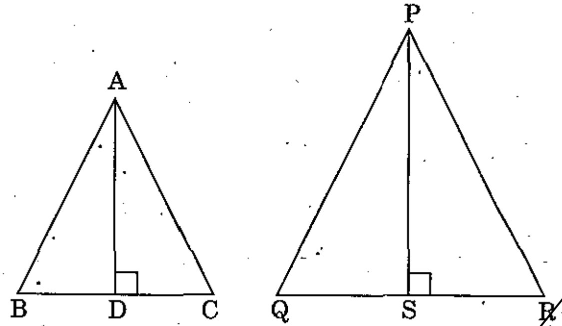
Hence, the zeroes are **4, – 4, 5.**

**30. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.**

**Solution. Given :** $\Delta ABC$ and $\Delta PQR$ such that $\Delta ABC \sim \Delta PQR$.

**To prove :** $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$.

**Construction :** Draw $AD \perp BC$ and $PS \perp QR$.




**Proof :** $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC \times AD}{QR \times PS}$ ...(1)

Now, in $\Delta ADB$ and $\Delta PSQ$, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, $\Delta ADB$ and $\Delta PSQ$ are equiangular and hence, they are similar.

Consequently, $\dfrac{AD}{PS} = \dfrac{AB}{PQ}$ ...(2)

[If $\Delta$s are similar, the ratio of their corresponding sides is same]

But $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\Rightarrow \quad \dfrac{AD}{PS} = \dfrac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS}$$

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC}{QR} \times \dfrac{BC}{QR}$ [using (3)]

$\Rightarrow \quad \dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{BC^2}{QR^2}$ ...(4)

As $\Delta ABC \sim \Delta PQR$, therefore

$\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ...(5)

Hence, $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$ [From (4) and (5)]

*Or*

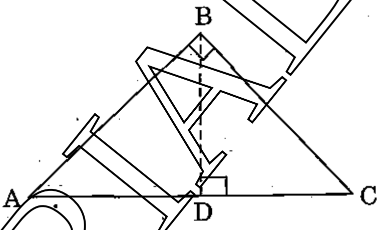
**Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.**

**Solution.** Given : A right triangle $ABC$, right angled at $B$.

**To prove :** $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

*i.e.,* $AC^2 = AB^2 + BC^2$

**Construction :** Draw $BD \perp AC$.




**Proof :** $\Delta ADB \sim \Delta ABC$.

[If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]

So, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ [Sides are proportional]

$\Rightarrow \quad AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$ [Sides are proportional]

$\Rightarrow \quad CD.AC = BC^2$ ...(2)

Adding (1) and (2) , we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$\Rightarrow \quad AC.AC = AB^2 + BC^2$

Hence, $\mathbf{AC^2 = AB^2 + BC^2}$

**31. Evaluate :**

$$\frac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38°.\sec^2 52° - \sin^2 45°$$

**Solution.** We have

$$\frac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°$$

$$= \frac{\sec^2(90° - 36°) - \cot^2 36°}{\text{cosec}^2(90° - 33°) - \tan^2 33°} + 2\sin^2 38° \sec^2(90° - 38°) - \sin^2 45°$$

$$= \frac{\text{cosec}^2 36° - \cot^2 36°}{\sec^2 33° - \tan^2 33°} + 2\sin^2 38° \,\text{cosec}^2 38° - \sin^2 45°$$

[$\because \sec(90° - \theta) = \text{cosec}\,\theta$ and $\text{cosec}(90° - \theta) = \sec\theta$]

$$= \frac{(1 + \cot^2 36°) - \cot^2 36°}{(1 + \tan^2 33°) - \tan^2 33°} + 2\sin^2 38°.\frac{1}{\sin^2 38°} - \sin^2 45°$$

[$\because \text{cosec}^2\theta = 1 + \cot^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta$ ]

$$= \frac{1}{1} + 2.(1) - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1 + 2 - \frac{1}{2}$$

$$= \frac{5}{2}$$

***Or***

**Prove that :**

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta.$$

**Solution.** We have

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{(\sin\theta/\cos\theta)}{(\sin\theta-\cos\theta)/\sin\theta} + \frac{(\cos\theta/\sin\theta)}{(\cos\theta-\sin\theta)/\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)(\sin\theta\cos\theta)} \qquad [\because\ a^3-b^3=(a-b)(a^2+b^2+ab)]$$

$$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= \tan\theta + \cot\theta + 1$

$=$ R.H.S.

**32. Prove that :**

$$\tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$$

**Solution.** We have,

$$\begin{aligned}
\text{L.H.S.} &= \tan^2 A - \tan^2 B \\
&= \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B} \\
&= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B} \\
&= \frac{(1-\cos^2 A)\cos^2 B - \cos^2 A(1-\cos^2 B)}{\cos^2 A \cos^2 B} \\
&= \frac{\cos^2 B - \cos^2 A \cos^2 B - \cos^2 A + \cos^2 A \cos^2 B}{\cos^2 A \cos^2 B} \\
&= \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} \\
&= \frac{(1-\sin^2 B)-(1-\sin^2 A)}{\cos^2 A \cos^2 B} \\
&= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} \\
&= \text{R.H.S.}
\end{aligned}$$

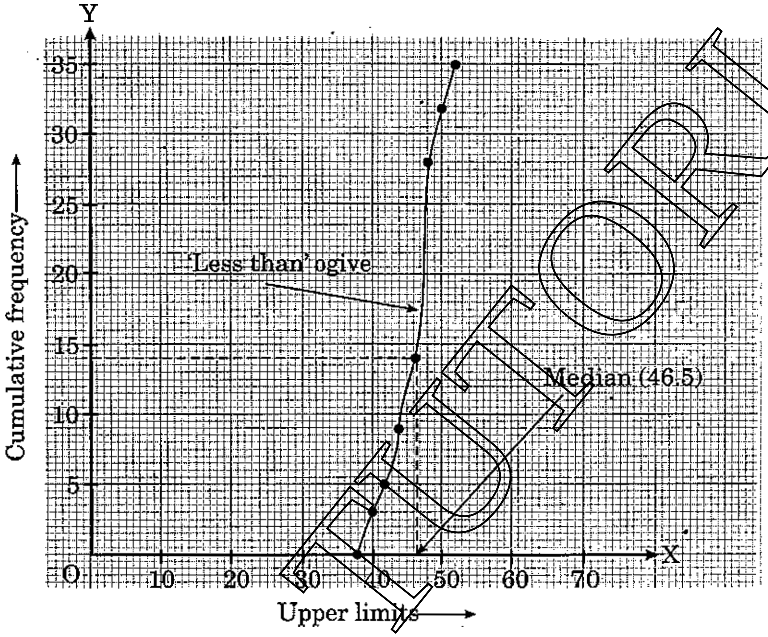
**33. During the medical check-up of 35 students of a class, their weights were recorded as follows :**

| *Weight (in kg)* | *Number of students* |
|---|---|
| **Less than 38** | **0** |
| **Less than 40** | **3** |
| **Less than 42** | **5** |
| **Less than 44** | **9** |
| **Less than 46** | **14** |
| **Less than 48** | **28** |
| **Less than 50** | **32** |
| **Less than 52** | **35** |

**Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.**

**Solution.** We consider the **cumulative frequency**, distribution given in the table of given question and draw its ogive (of the less than type).

Here 38, 40, 42, ..........,52 are the upper limits of the respective class intervals less than 38, 38 – 40, 40 – 42, ..............,50 – 52. To represent the data in the table graphically, we mark the upper limits of class intervals on the horizontal axis ($x$-axis) and their corresponding cumulative frequencies on the vertical axis ($y$-axis), choosing a convenient scale. Now plot the points corresponding to the ordered pairs given by (upper limit, corresponding cumulative frequency), *i.e.*, (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32), (52, 35) on a graph paper and join them by a free hand smooth curve. The curve we get is called a **cumulative frequency curve**, or an **ogive** (of the less than type) (see figure).




Locate $\frac{n}{2} = \frac{35}{2} = 17.5$ on the $y$-axis (see figure). From this point, draw a line parallel to $x$-axis cutting the curve at a point. From this point, draw a perpendicular on the $x$-axis. The point of intersection of this perpendicular with the $x$-axis determines the median weight of the data (see figure).

To calculate the median weight, we need to find the class intervals and their corresponding frequencies. Observe that from the given distribution, we find that there are no students with weight less than 38, *i.e.*, the frequency of class interval below 38 is 0. Now, there are 3 students with weight less than 40 and 0 student with weight less than 38. Therefore, the number of students with weight in the interval 38 – 40 is 3 – 0 = 3. Similarly, the number of students with weight in the interval 40 – 42 is 5 – 3 = 2.

Similarly, the frequency of 42 – 44 is 9 – 5 = 4, for 44 – 46, it is 14 – 9 = 5 and so on. So, the frequency distribution table with the given cumulative frequencies becomes :

**Table**

| *Class Intervals* | *Frequency* | *Cumulative Frequency* |
|---|---|---|
| Below 38 | 0 | 0 |
| 38 – 40 | 3 | 3 |
| 40 – 42 | 2 | 5 |
| 42 – 44 | 4 | 9 |
| 44 – 46 | 5 | 14 |
| 46 – 48 | 14 | 28 |
| 48 – 50 | 4 | 32 |
| 50 – 52 | 3 | 35 |

Here, $\frac{n}{2} = \frac{35}{2} = 17.5$. Now 46 – 48 is the class whose cumulative frequency is 28 is greater than $\frac{n}{2}$, *i.e.*, 17.5.

$\therefore$ 46 – 48 is the **median class.**

From the table; $f = 14$, $cf = 14$, $h = 2$

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{14} \times 2$$

$$= 46 + \frac{1}{2}$$

$$= 46 + 0.5$$

$$= 46.5$$

So, about half the students have weight less than **46.5 kg**, and the other half have weight more than **46.5 kg**.

**34. Represent the following system of linear equations graphically. From the graph, find the points where the lines intersect $x$-axis :**

$$\mathbf{2x - y = 2,\ 4x - y = 8.}$$

**Solution.** We have

$2x - y = 2$ and $4x - y = 8$

$\Rightarrow$ $y = 2x - 2$ and $y = 4x - 8$

Table of $y = 2x - 2$

| $x$ | 0 | 1 | 3 |
|---|---|---|---|
| $y$ | –2 | 0 | 4 |
| | *A* | *B* | *C* |

Table of $y = 4x - 8$

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | –4 | 0 | 4 |
| | *D* | *E* | *C* |

Take $XOX'$ and $YOY'$ as the axes of coordinates. Plotting the points $A(0, -2)$, $B(1, 0)$, $C(3, 4)$ and joining them by a line, we get a line '$l$' which is the graph of $2x - y = 2$.

Further, plotting the points $D(1, -4)$, $E(2, 0)$, $C(3, 4)$ and joining them by a line, we get a line '$m$' which is the graph of $4x - y = 8$.

From the graph of the two equations, we find that the two lines $l$ and $m$ intersect each other at the point $C(3, 4)$.

$\therefore$ $\quad$ $\boldsymbol{x = 3, y = 4}$ is the solution.

The first line $2x - y = 2$ meets the $x$-axis at the point $\boldsymbol{B(1, 0)}$. The second line $4x - y = 8$ meets the $x$-axis at the point $\boldsymbol{E(2, 0)}$.