SECOND TERM (SA-II)
MATHEMATICS
(*With Solutions*)
CLASS X

Time Allowed : 3 Hours] *[Maximum Marks : 80*

General Instructions :

(*i*) *All questions are compulsory.*
(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*
(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*
(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*
(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. The roots of the equation $x^2 - 3x - m(m+3) = 0$, where m is a constant, are

(*a*) $m, m+3$ (*b*) $-m, m+3$
(*c*) $m, -(m+3)$ (*d*) $-m, -(m+3)$

Solution. Choice (*b*) is correct.

$$x^2 - 3x - m(m+3) = 0$$
$$\Rightarrow \quad x^2 - [(m+3) - m]x - m(m+3) = 0$$
$$\Rightarrow \quad x^2 - (m+3)x + mx - m(m+3) = 0$$
$$\Rightarrow \quad x[x-(m+3)] + m[x-(m+3)] = 0$$
$$\Rightarrow \quad [x-(m+3)][x+m] = 0$$
$$\Rightarrow \quad \text{Either } x = m+3 \quad \text{or} \quad x = -m$$

Hence, $(m+3)$ and $-m$ are the roots of the given equation.

2. If the common difference of an A.P. is 3, then $a_{20} - a_{15}$ is

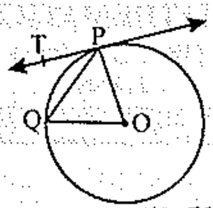
(*a*) 5 (*b*) 3
(*c*) 15 (*d*) 20

Solution. Choice (*c*) is correct.

Here, $d = 3$

$$\therefore \quad a_{20} - a_{15} = [a + (20-1)d] - [a + (15-1)d]$$
$$= (a + 19d) - (a + 14d)$$
$$= 5d$$
$$= 5 \times 3$$
$$= \mathbf{15.}$$

3. In figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If $\angle POQ = 70°$, then $\angle TPQ$ is equal to



(*a*) 55° (*b*) 70°
(*c*) 45° (*d*) 35°

Solution. Choice (*d*) is correct.

In ΔPOQ, we have

$\angle POQ + \angle OPQ + \angle OQP = 180°$ [Sum of three $\angle s$ of $\Delta = 180°$]

$\Rightarrow 70° + 2\angle OPQ = 180°$ [$\because OP = OQ$ (each = radius) $\Rightarrow \angle OQP = \angle OPQ$]

$\Rightarrow 2\angle OPQ = 180° - 70°$

$\Rightarrow 2\angle OPQ = 110$

$\Rightarrow \angle OPQ = 55°$

Since OP is the radius of a circle and TP is a tangent at P.

$\therefore OP \perp TP$

$\Rightarrow \angle OPT = 90°$

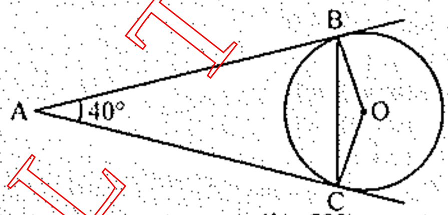
$\Rightarrow \angle OPQ + \angle TPQ = 90°$

$\Rightarrow 55° + \angle TPQ = 90°$

$\Rightarrow \angle TPQ = 90° - 55°$

$\Rightarrow$ **$\angle TPQ = 35°$**

4. In figure, AB and AC are tangents to the circle with centre O such that $\angle BAC = 40°$. Then $\angle BOC$ is equal to



(*a*) 40° (*b*) 50°
(*c*) 140° (*d*) 150°

Solution. Choice (*c*) is correct.

Since the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre, *i.e.*,

$\angle BAC + \angle BOC = 180°$

But $\angle BAC = 40°$ (given)

$\Rightarrow \angle BOC = 180° - 40° = $ **140°**

5. The perimeter (in cm) of a square circumscribing a circle of radius a cm, is

(*a*) $8a$ (*b*) $4a$
(*c*) $2a$ (*d*) $16a$

Solution. Choice (*a*) is correct.

Since the square circumscribing a circle of radius a cm, therefore the diameter of a circle is equal to the side of a square.

But the diameter of a circle = 2 × Radius of a circle

$$= (2 \times a) \text{ cm}$$
$$= 2a \text{ cm}$$

∴ Side of a square = $2a$ cm

Perimeter of a square = 4 × side of a square

$$= 4 \times 2a \text{ cm}$$
$$= 8a \text{ cm.}$$

6. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is

(*a*) 4.2 (*b*) 2.1

(*c*) 8.4 (*d*) 1.05

Solution. Choice (*b*) is correct.

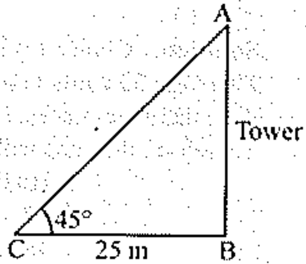
The base of the largest right circular cone will be inscribed in a face of the cube and height will be equal to the edge of the cube.

Radius of the base of the cone $(r) = \frac{4.2}{2} = \mathbf{2.1}$ **cm.**

7. A tower stands vertically on the ground. From a point on the ground which is 25 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 45°. Then the height (in meters) of the tower is

(*a*) $25\sqrt{2}$ (*b*) $25\sqrt{3}$

(*c*) 25 (*d*) 12.5



Solution. Choice (c) is correct.

In right ΔABC, we have

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{AB}{25 \text{ m}}$$

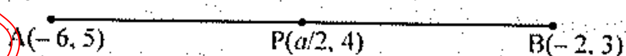
$$\Rightarrow \quad AB = 25 \text{ m.}$$

8. If $P\left(\frac{a}{2}, 4\right)$ is the mid-point of the line-segment joining the points $A(-6, 5)$ and $B(-2, 3)$, then the value of a is

(*a*) −8 (*b*) 3

(*c*) −4 (*d*) 4

Solution. Choice (*a*) is correct.



If $P\left(\frac{a}{2}, 4\right)$ is the mid-point of the line-segment joining the points $A(-6, 5)$ and $B(-2, 3)$, then

$$\frac{(-6)+(-2)}{2} = \frac{a}{2} \quad \text{and} \quad \frac{5+3}{2} = 4$$

$\Rightarrow \quad -\frac{8}{2} = \frac{a}{2}$ and $4 = 4$

$\Rightarrow \quad a = -8$

9. If A and B are the points $(-6, 7)$ and $(-1, -5)$ respectively, then the distance $2AB$ is equal to

(a) 13 (b) 26

(c) 169 (d) 238

Solution. Choice (b) is correct.

Distance between the points $A(-6, 7)$ and $B(-1, -5)$ is AB, *i.e.*,

$$AB = \sqrt{(-1+6)^2 + (-5-7)^2}$$

$$\Rightarrow \quad AB = \sqrt{25 + 144}$$

$$\Rightarrow \quad AB = \sqrt{169}$$

$$\Rightarrow \quad AB = 13$$

$\therefore$ Distance $2AB = 2 \times 13 = 26$ **units.**

10. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the card will not be an ace is

(a) $\frac{1}{13}$ (b) $\frac{1}{4}$

(c) $\frac{12}{13}$ (d) $\frac{3}{4}$

Solution. Choice (c) is correct.

There are 52 cards in a well-shuffled deck of cards.

Also there are 4 aces, *i.e.*, two red and two black aces.

Therefore, the card will not be an ace card

= Total number of cards – The ace will be red or black

= 52 – 4

= 48 cards

Let A denote the event, that the card will not be an ace.

So, the number of outcomes favourable to event A are 48.

$$\therefore \quad P(A) = \frac{48}{52} = \frac{12}{13}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find the value of m so that the quadratic equation $mx(x-7) + 49 = 0$ has two equal roots.

Solution. The given quadratic equation is :

$$mx(x-7) + 49 = 0$$

$$\Rightarrow \quad mx^2 - 7mx + 49 = 0$$

Here $a = m$, $b = -7m$ and $c = 49$

For equal roots,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow \quad (-7m)^2 - 4(m)(49) = 0$$

$\Rightarrow \quad 49m^2 - 196m = 0$

$\Rightarrow \quad 49m(m-4) = 0$

$\Rightarrow \quad$ Either $m = 0$ or $m = 4$

$\Rightarrow \quad m = 4$ as $m \neq 0$

Hence the value of m is 4.

12. Find how many two-digit numbers are divisible by 6.

Solution. Two digit number are :

10, 11, 12, ..., 99

$\therefore$ Two-digit numbers which are divisible by 6 are :

12, 18, ..., 96.

Here, $a = 12$, $d = 18 - 12 = 6$, $t_n = 96$ (last term)

$\therefore \quad t_n = 96$

$\Rightarrow \quad a + (n-1)d = 96$

$\Rightarrow \quad 12 + (n-1)6 = 96$

$\Rightarrow \quad (n-1)6 = 96 - 12$

$\Rightarrow \quad (n-1)6 = 84$

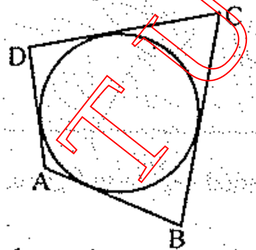
$\Rightarrow \quad n - 1 = 84 \div 6$

$\Rightarrow \quad n - 1 = 14$

$\Rightarrow \quad n = 14 + 1 = 15$

Hence, 15 two-digit numbers are divisible by 6.

13. In figure, a circle touches all the four sides of a quadrilateral *ABCD* whose sides are *AB* = 6 cm, *BC* = 9 cm and *CD* = 8 cm. Find the length of side *AD*.



Solution. We know that when a circle touches all the four sides of a quadrilateral *ABCD*, then

$AB + CD = BC + AD$

$\Rightarrow \quad 6 + 8 = 9 + AD$

$\Rightarrow \quad 14 = 9 + AD$

$\Rightarrow \quad AD = 14 - 9$

$\Rightarrow \quad AD = 5$ cm

14. Draw a line segment *AB* of length 7 cm. Using ruler and compasses, find a point *P* on *AB* such that $\frac{AP}{AB} = \frac{3}{5}$.

Solution. We have

$$\frac{AP}{AB} = \frac{3}{5} \text{ (given)}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{AP+PB}{AP} = \frac{3+2}{3}$$

$$\Rightarrow \quad 1+\frac{PB}{AP} = 1+\frac{2}{3}$$

$$\Rightarrow \quad \frac{PB}{AP} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{2}$$

Steps of Construction :

Steps 1. Draw a line segment $AB = 7$ cm.

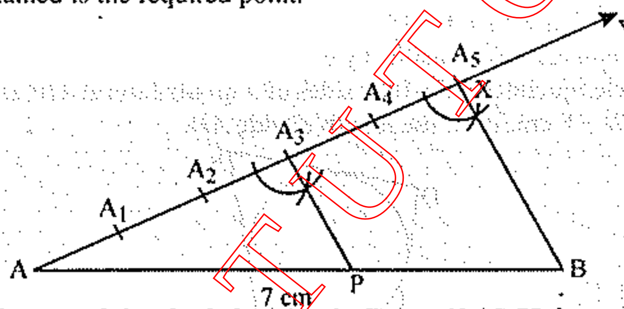
Steps 2. Draw a ray AY making an acute angle with AB.

Steps 3. Locate 5 points A_1, A_2, A_3, A_4 and A_5 on AY so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$. Join A_5B.

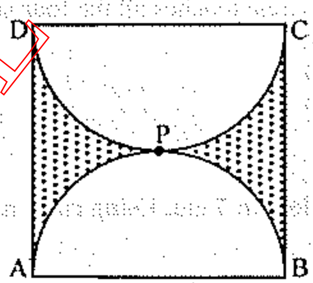
Step 4. With A_5 as centre mark an arc cutting A_5B at X.

Step 5. Through A_3 draw a line A_3P parallel to A_5B making an acute angle equal to AA_5B at A_3 intersecting AB at a point P.

The point P so obtained is the required point.



15. Find the perimeter of the shaded region in figure, if *ABCD* is a square of side 14 cm and *APB* and *CPD* are semi-circles. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution. Since $ABCD$ is a square of side 14 cm therefore.

$\therefore$ Diameter of the semi-circle = 14 cm

$\therefore$ Radius of the semi-circle = 7 cm

Perimeter of shaded region

= Length AD + Length BC + Circumference of semi-circle APB

+ Circumference of semi-circle DPC

$= 14 \text{ cm} + 14 \text{ cm} + \pi r + \pi r$

$= 14 \text{ cm} + 14 \text{ cm} + \frac{22}{7} \times 7 + \frac{22}{7} \times 7$

$= 14 \text{ cm} + 14 \text{ cm} + 22 \text{ cm} + 22 \text{ cm}$

$=$ **72 cm.**

16. Two cubes each of volume 27 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid.

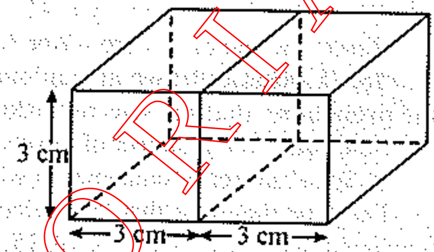
Solution. Let x be the each edge and V be the volume of a cube

$\therefore \quad V = x^3$

$\Rightarrow \quad$ (given) $27 \text{ cm}^3 = x^3$

$\Rightarrow \quad x^3 = 27 = (3)^3 \text{ cm}^3$

$\Rightarrow \quad x = 3 \text{ cm}$



The dimensions of the cuboid formed when two edges of two cubes joined are :

Length of the cuboid $(l) = (3 + 3) = 6$ cm

Breadth of the cuboid $(b) = 3$ cm

Height of the cuboid $(h) = 3$ cm

Surface area of the cuboid formed

$= 2(lb + bh + hl)$

$= 2[6 \times 3 + 3 \times 3 + 3 \times 6] \text{ cm}^2$

$= [2(18 + 9 + 18)] \text{ cm}^2$

$= [2 \times 45] \text{ cm}^2$

$=$ **90 cm^2**

Or

A cone of height 20 cm and radius of base 5 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

Solution. Height of a cone $(h) = 20$ cm

Radius of a cone $(r) = 5$ cm

Let R be the radius of a sphere.

It is given that a child reshapes cone in the form of a sphere.

$\therefore$ Volume of a sphere = Volume of a cone

$\Rightarrow \quad \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad 4R^3 = r^2 h$

$\Rightarrow \quad 4R^3 = (5)^2 \times 20$

$\Rightarrow \quad R^3 = \frac{500}{4} = 125$

$\Rightarrow \quad R^3 = (5)^3$ cm

$\Rightarrow \quad R = 5$ cm

Hence, the diameter of the sphere $= 2R = (2 \times 5)$ cm $=$ **10 cm.**

17. Find the values of y for which the distance between the points $A(3, -1)$ and $B(11, y)$ is 10 units.

Solution. Here, $A(3, -1)$ and $B(11, y)$ be the given points. Then

$AB = 10$ units (given)

$\Rightarrow \quad \sqrt{(11-3)^2+(y+1)^2} = 10$

$\Rightarrow \quad (8)^2 + (y+1)^2 = (10)^2$

$\Rightarrow \quad (y+1)^2 = (10)^2 - (8)^2$

$\Rightarrow \quad y^2 + 2y + 1 = 100 - 64$

$\Rightarrow \quad y^2 + 2y - 35 = 0$

$\Rightarrow \quad y^2 + 7y - 5y - 35 = 0$

$\Rightarrow \quad y(y+7) - 5(y+7) = 0$

$\Rightarrow \quad (y+7)(y-5) = 0$

$\Rightarrow \quad y + 7 = 0 \quad \text{or} \quad y - 5 = 0$

$\Rightarrow \quad y = -7 \quad \text{or} \quad y = 5$

Hence, the values of y are **− 7** or **5.**

18. A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. Find the probability that the selected ticket has a number which is a multiple of 5.

Solution. Total number of tickets in a bag, numbered from 1 to 40 are 40.

∴ Total number of outcomes in which one ticket is drawn are 40.

Let A be the event that "the selected ticket number which is multiple of 5". There are 8 tickets numbered multiple of 5, *i.e.*, 5, 10, 15, 20, 25, 30, 35, 40.

∴ Number of outcomes favourable to event $A = 8$

Hence, required probability $P(A) = \frac{8}{40} = \frac{1}{5}$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Find the roots of the following quadratic equation :

$$x^2 - 3\sqrt{5}x + 10 = 0$$

Solution. Given $x^2 - 3\sqrt{5}x + 10 = 0$.

Here $a = 1$, $b = -3\sqrt{5}$ and $c = 10$

$\therefore \quad D = b^2 - 4ac$

$\Rightarrow \quad D = (-3\sqrt{5})^2 - 4(1)(10)$

$\Rightarrow \quad D = 45 - 40$

$\Rightarrow \quad D = 5$

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-3\sqrt{5}) \pm \sqrt{5}}{2 \times 1}$$

$$\Rightarrow \quad x = \frac{3\sqrt{5} \pm \sqrt{5}}{2}$$

$$\Rightarrow \quad x = \frac{3\sqrt{5} + \sqrt{5}}{2} \quad \text{or} \quad x = \frac{3\sqrt{5} - \sqrt{5}}{2}$$

$\Rightarrow \quad x = 2\sqrt{5} \quad \text{or} \quad x = \sqrt{5}$

Hence, the roots of given quadratic equation are $2\sqrt{5}$ and $\sqrt{5}$.

Alternative method :

Given $\quad x^2 - 3\sqrt{5}x + 10 = 0$

$\Rightarrow \quad x^2 - 2\sqrt{5}x - \sqrt{5}x + 10 = 0$

$\Rightarrow \quad x(x - 2\sqrt{5}) - \sqrt{5}(x - 2\sqrt{5}) = 0$

$\Rightarrow \quad (x - 2\sqrt{5})(x - \sqrt{5}) = 0$

$\Rightarrow \quad x - 2\sqrt{5} = 0 \quad \text{or} \quad x - \sqrt{5} = 0$

$\Rightarrow \quad x = 2\sqrt{5} \quad \text{or} \quad x = \sqrt{5}$

Hence, the roots of the given quadratic equation are $2\sqrt{5}$ and $\sqrt{5}$.

20. Find an A.P. whose fourth term is 9 and the sum of its sixth term and thirteenth term is 40.

Solution. Let the first term and common difference of A.P. be a and d, respectively.

Let a_4, a_6 and a_{13} denote the 4th term, 6th term and 13th term of an A.P., then,

$a_4 = a + (4 - 1)d \quad [\because n = 4]$

$\Rightarrow \quad 9 = a + 3d \quad \ldots(1) \; [\because a_4 = 9 \text{ (given)}]$

It is given that the sum of its sixth term and thirteenth term is 40.

$\therefore \quad a_6 + a_{13} = 40 \text{ (given)}$

$\Rightarrow \quad [a + (6 - 1)d] + [a + (13 - 1)d] = 40$

$\Rightarrow \quad (a + 5d) + (a + 12d) = 40$

$\Rightarrow \quad 2a + 17d = 40 \quad \ldots(2)$

Multiplying (1) by 2 and subtracting from (2), we get

$(2a + 17d) - 2(a + 3d) = 40 - 2(9)$

$\Rightarrow \quad 17d - 6d = 40 - 18$

$\Rightarrow \quad 11d = 22$

$\Rightarrow \quad d = 2 \quad \ldots(3)$

From (1) and (3), we get

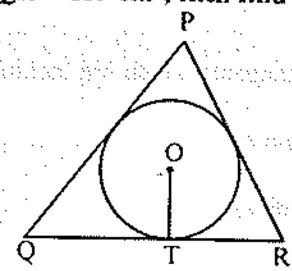
$a + 3(2) = 9$

$\Rightarrow \quad a + 6 = 9$

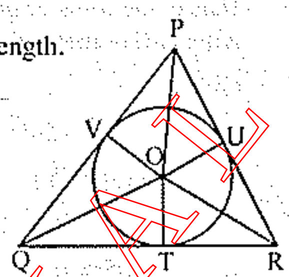
$\Rightarrow \quad a = 9 - 6 = 3$

Thus, the A.P. is **3, 5, 7, 9, ...**

21. In figure, a triangle *PQR* is drawn to circumscribe a circle of radius 6 cm such that the segments *QT* and *TR* into which *QR* is divided by the point of contact *T*, are of lengths 12 cm and 9 cm respectively. If the area of ΔPQR = 189 cm^2, then find the lengths of sides *PQ* and *PR*.



Solution. Let a triangle PQR be drawn to circumscribe a circle with centre O of radius 6 cm such that the segment QT and TR into which QR is divided by the point of contact T are of lengths 12 cm and 9 cm respectively.



Since the tangents drawn from an external point to a circle are equal in length.

$\therefore$ $PV = PU$ ...(1) [Tangents from P]

$QV = QT$ ...(2) [Tangents from Q]

$RT = RU$ ...(3) [Tangents from R]

It is given that $QT = 12$ cm and $TR = 9$ cm

Therefore from (2) and (3), we have :

$QV = 12$ cm and $RU = 9$ cm

Let $PU = PV = x$ cm, then

$PQ = PV + VQ = (x + 12)$ cm, $QR = QT + TR = (12 + 9)$ cm $= 21$ cm,

$RP = RU + PU = (9 + x)$ cm

Now join OQ, OR and OP and draw perpendiculars OU and OV on PR and PQ respectively.

From figure,

Area of ΔPQR = Area of ΔOPQ + Area of ΔOQR + Area of ΔORP

$\Rightarrow$ (given) $189 \text{ cm}^2 = \frac{1}{2} \times PQ \times$ radius of a circle $OV + \frac{1}{2} \times QR \times$ radius of a circle OT

$+ \frac{1}{2} \times RP \times$ radius of a circle OU

$\Rightarrow \quad 189 = \frac{1}{2} \times (x + 12) \times 6 + \frac{1}{2} \times 21 \times 6 + \frac{1}{2} \times (9 + x) \times 6$

$\Rightarrow \quad 378 = (x + 12) \times 6 + 21 \times 6 + (9 + x) \times 6$

$\Rightarrow \quad 63 = (x + 12) + 21 + (9 + x)$

$\Rightarrow \quad 63 = 2x + 42$

$\Rightarrow \quad 2x = 63 - 42$

$\Rightarrow \quad 2x = 21$

$\Rightarrow \quad x = 10.5$ cm

Hence, the length of sides PQ and PR are

$x + 12 = 10.5 + 12 =$ **22.5 cm** and $9 + x = 9 + 10.5 =$ **19.5 cm** respectively.

22. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60°.

Solution. Steps of Construction :

1. Take a point O on the plane of the paper and draw a circle of radius $OA = 3$ cm.
2. Extend OA to B such that $OA = AB = 3$ cm.
3. With A as centre draw a circle of radius $OA = AB = 3$ cm. Suppose it intersect the circle drawn in step 1 at the point P and Q.
4. Join BP and BQ.

Then BP and BQ are the required tangents which are inclined to each other at angle of 60° (see figure).

For justification of the construction :

In ΔOAP, we have

$OA = OP = 3$ cm (= Radius)

Also, $AP = 3$ cm (= Radius of circle with centre A).

ΔOAP is equilateral

$\Rightarrow \quad \angle PAO = 60°$

$\Rightarrow \quad \angle BAP = 120°$

In ΔBAP, we have

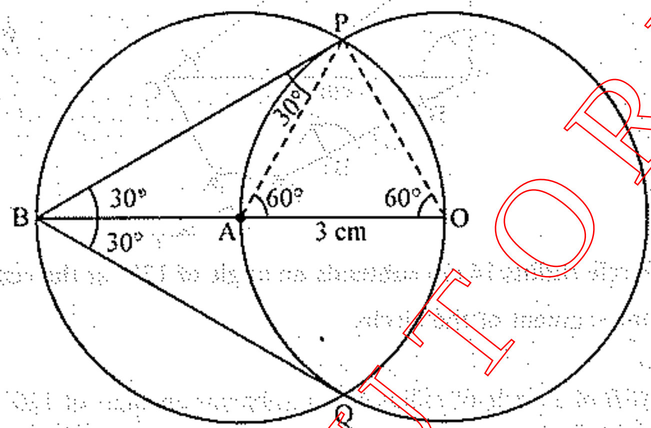
$$AB = AP \text{ and } \angle BAP = 120°$$

$\therefore \quad \angle ABP = \angle APB = 30°$

Similarly we can prove that

$$\angle ABQ = \angle AQB = 30°$$

$\Rightarrow \quad \angle PBQ = 60°.$



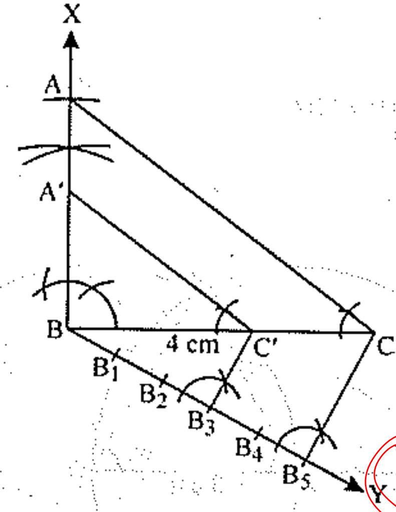
Or

Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.

Solution. Steps of Construction :

1. Draw a line segment $BC = 4$ cm.
2. At B construct $\angle CBX = 90°$.
3. With B as centre and radius 3 cm, draw an arc intersecting the line BX at A.
4. Join AC to obtain the required ΔABC.
5. Draw any ray BY making an acute angle with BC on the opposite side of the vertex A.
6. Locate 5 points $\left(\text{the greater of 3 and 5 in } \frac{3}{5}\right)$ B_1, B_2, B_3, B_4 and B_5 on BY so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
7. Join B_5C and draw a line through B_3 $\left(\text{3rd point, 3 being smaller of 3 and 5 in } \frac{3}{5}\right)$ parallel to B_5C intersecting BC at C'.

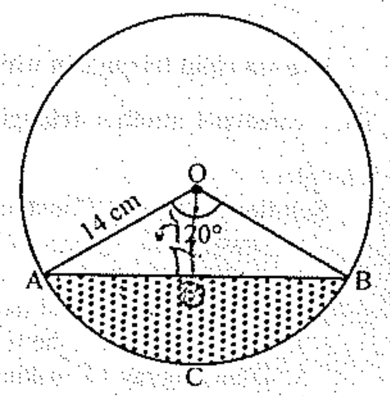
8. Draw a line through C' parallel to line CA to intersect BA at A' (see figure). Then $A'BC'$ is the required triangle.



23. A chord of a circle radius 14 cm subtends an angle of 120° at the centre. Find the area of the corresponding minor segment of the circle. **[Use $\sqrt{3} = 1.73$]**

Solution. We have

Area of a sector $OACB$ of a circle of radius 14 cm subtends an angle of 120° at the centre



$$= \frac{120°}{360°}\pi r^2$$

$$= \frac{1}{3}\pi(14)^2 \text{ cm}^2$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 196\right) \text{cm}^2$$

$$= \frac{22 \times 28}{3} \text{ cm}^2$$

$$= \frac{616}{3} \text{ cm}^2 = 205.33 \text{ cm}^2$$

Area of a triangle OAB subtends an angle 120° at O.

$$= \frac{1}{2} r^2 \sin\theta$$

$$= \frac{1}{2} \times (14)^2 \sin 120° \text{ cm}^2$$

$$= 98 \sin(90° + 30°) \text{ cm}^2$$

$$= 98 \cos 30° \text{ cm}^2$$

$$= 98 \times \frac{\sqrt{3}}{2} \text{ cm}^2$$

$= 49\sqrt{3}$ cm^2

$= (49 \times 1.73)$ cm^2

$= 84.77$ cm^2

Area of minor segment of the circle

= Area of sector *OACB* – Area of ΔOAB subtends an angle 120°

$= (205.33 - 84.77)$ cm^2

$= \mathbf{120.56}$ **cm^2**

24. An open metal bucket is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at ₹ 30 per litre. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution. The container is a frustum of a cone of height 21 cm with radii of its upper and lower ends are 20 cm and 10 cm respectively.

$\therefore$ $h = 21$ cm, $R = 20$ cm and $r = 10$ cm

Volume of the open metal bucket in the form of a frustum of a cone

$$= \frac{1}{3}\pi h[R^2 + Rr + r^2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 21[(20)^2 + (20)(10) + (10)^2] \text{ cm}^3$$

$= 22[400 + 200 + 100]$ cm^3

$= 22 \times 700$ cm^3

$= 15400$ cm^3

$= 15400$ c.c.

$$= \frac{15400}{1000} \text{ litres}$$

$= 15.4$ litres

Therefore, the quantity of milk = volume of the open metal bucket = 15.4 litres

Thus, cost of the milk @ ₹ 30 per litre

= ₹ (15.4×30)

= **₹ 462**

25. Point $P(x, 4)$ lies on the line segment joining the points $A(-5, 8)$ and $B(4, -10)$. Find the ratio in which point P divides the line segment AB. Also find the value of x.

Solution. Let the point $P(x, 4)$ divides the line segment joining the points $A(-5, 8)$ and $B(4, -10)$ in the ratio $K : 1$.

Then the coordinates of P are $\left(\frac{4K-5}{K+1}, \frac{-10K+8}{K+1}\right)$

But the coordinates of P are given as $(x, 4)$.

$$\therefore \quad \frac{4K-5}{K+1} = x \text{ and } \frac{-10K+8}{K+1} = 4$$

Consider, $\frac{-10K+8}{K+1} = 4$

$\Rightarrow \quad -10K + 8 = 4K + 4$

$\Rightarrow \quad 4K + 10K = 8 - 4$

$\Rightarrow \quad 14K = 4$

$\Rightarrow \quad K = \frac{2}{7}$

$\Rightarrow$ P divides the line segment AB in ratio $\frac{2}{7} : 1$ *i.e.*, **2 : 7**

Substituting $K = \frac{2}{7}$ in $x = \frac{4K-5}{K+1}$, we get

$$x = \frac{4\left(\frac{2}{7}\right) - 5}{\frac{2}{7} + 1}$$

$\Rightarrow \quad x = \frac{8-35}{2+7}$

$\Rightarrow \quad x = \frac{-27}{9}$

$\Rightarrow \quad$ **$x = -3$.**

26. Find the area of quadrilateral $ABCD$, whose vertices are $A(-3, -1)$, $B(-2, -4)$, $C(4, -1)$ and $D(3, 4)$.

Solution. By joining A to C, we will get two triangles ABC and ACD.

Now, the area of ΔABC

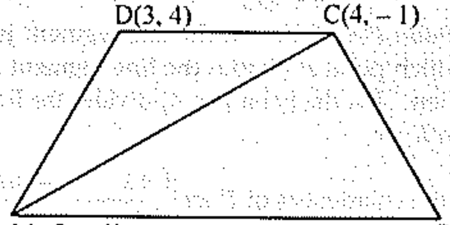
$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[(-3)(-4+1) + (-2)(-1+1) + 4(-1+4)]$$

$$= \frac{1}{2}[(-3)(-3) + (-2)(0) + 4(3)]$$

$$= \frac{1}{2}[9 + 0 + 12]$$

$$= \frac{21}{2} \text{ sq. units}$$



Also, the area of ΔACD

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[(-3)(-1-4) + 4(4+1) + 3(-1+1)]$$

$$= \frac{1}{2}[(-3)(-5) + 4(5) + 3(0)]$$

$$= \frac{1}{2}[15 + 20 + 0]$$

$$= \frac{35}{2} \text{ sq. units}$$

So, the area of the quadrilateral $ABCD$

$$= \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \frac{21}{2} + \frac{35}{2} \text{ sq. units}$$

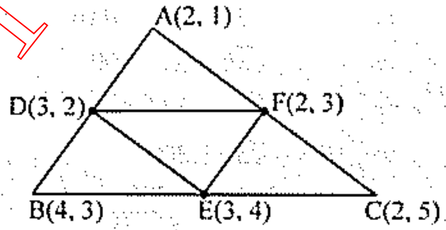
$$= \frac{56}{2} \text{ sq. units}$$

$$= \mathbf{28 \text{ sq. units.}}$$

Or

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are $A(2, 1)$, $B(4, 3)$ and $C(2, 5)$.

Solution. Let $A(2, 1)$, $B(4, 3)$ and $C(2, 5)$ be the vertices of a triangle ABC respectively. Let D, E and F be the mid-point of AB, BC and CA, then



Coordinates of $D\left(\frac{2+4}{2}, \frac{1+3}{2}\right)$ *i.e.*, $D(3, 2)$

Coordinates of $E\left(\frac{4+2}{2}, \frac{3+5}{2}\right)$ *i.e.*, $E(3, 4)$

and coordinates of $F\left(\frac{2+2}{2}, \frac{1+5}{2}\right)$ *i.e.*, $F(2, 3)$

Here, $x_1 = 3, y_1 = 2, x_2 = 3, y_2 = 4, x_3 = 2, y_3 = 3$

$$\therefore \quad \text{Area of } \Delta DEF = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[3(4-3) + 3(3-2) + 2(2-4)]$$

$$= \frac{1}{2}[3(1) + 3(1) + 2(-2)]$$

$$= \frac{1}{2}[3 + 3 - 4]$$

$$= \mathbf{1 \text{ sq. unit}}$$

27. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use $\sqrt{3} = 1.73$]

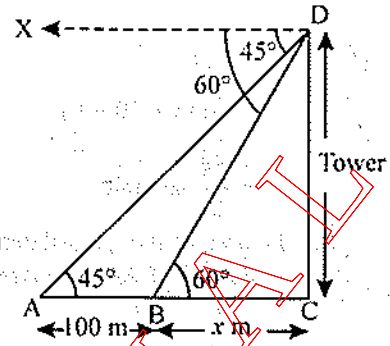
Solution. Let $CD = h$ m be a vertical tower.

Let A and B the position of the two cars in the same straight line with the base of a tower.

Let $\angle XDA = 45°$ and $\angle XDB = 60°$.

Distance between the cars $AB = 100$ m.

In right triangle DBC, we have



$$\tan 60° = \frac{DC}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad ...(1)$$

In right triangle DAC, we have

$$\tan 45° = \frac{DC}{AC}$$

$$\Rightarrow \quad 1 = \frac{h}{100 + x}$$

$$\Rightarrow \quad 100 + x = h$$

$$\Rightarrow \quad x = h - 100 \qquad ...(2)$$

From (1) and (2), we have

$$\frac{h}{\sqrt{3}} = h - 100$$

$$\Rightarrow \quad \sqrt{3}h = 3(h - 100)$$

$$\Rightarrow \quad \sqrt{3}h = 3h - 300$$

$$\Rightarrow \quad 3h - \sqrt{3}h = 300$$

$$\Rightarrow \quad h(3 - \sqrt{3}) = 300$$

$$\Rightarrow \quad h = \frac{300}{3 - \sqrt{3}}$$

$$\Rightarrow \quad h = \frac{(300)(3 + \sqrt{3})}{(3)^2 - (\sqrt{3})^2}$$

$$\Rightarrow \quad h = \frac{300(3 + \sqrt{3})}{9 - 3}$$

$$\Rightarrow \quad h = \frac{300}{6}(3 + \sqrt{3}) \text{ m}$$

$$\Rightarrow \quad h = 50(3 + 1.73) \text{ m}$$

$$\Rightarrow \quad h = 50(4.73) \text{ m}$$

$$\Rightarrow \quad h = 236.5 \text{ m}$$

Hence, the height of the tower is **236.5 m.**

28. Two dice are rolled once. Find the probability of getting such numbers on the two dice, whose product is 12.

Solution. When two dice are rolled once, then the possible outcomes of the experiment are listed in the table.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

So, the number of possible outcomes $= 6 \times 6 = 36$

Let A be the event of getting such numbers on the two dice, whose product is 12, then the outcomes favourable to A are

$$A = \{(2, 6), (6, 2), (3, 4), (4, 3)\}$$

$\therefore$ Favourable number of outcomes = 4

$\therefore$ Probability of getting such numbers on the two dice, whose product is 12 is

$$P(A) = \frac{4}{36} = \frac{1}{9}.$$

Or

A box contains 80 discs which are numbered from 1 to 80. If one disc is drawn at random from the box, find the probability that it bears a perfect square number.

Solution. Total number of discs which are numbered from 1 to 80 are 80.

$\therefore$ Total number of outcomes in which one disc can be drawn are 80.

Let A be the event that "the disc drawn at random bears a perfect square number".

The number of outcomes favourable to event $A = 8$ (viz., $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2$)

So, $$P(A) = \frac{8}{80} = \frac{1}{10}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

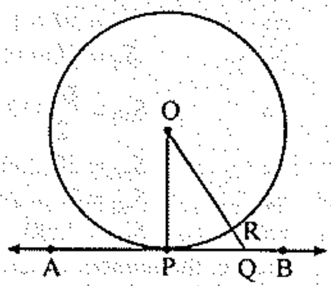
Solution. Given : A circle $C(O, r)$ and a tangent AB at a point P.

To prove : $OP \perp AB$.

Construction : Take any point Q, other than P, on the tangent AB. Joint OQ.

Suppose OQ meets the circle at R.

Proof : We know that among all line-segments joining the point O to a point on AB, the shortest one is perpendicular to AB. So, to prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.



Clearly, $OP = OR$ [Radii of the same circle]

Now, $OQ = OR + RQ$

$\Rightarrow$ $OQ > OR$

$\Rightarrow$ $OQ > OP$ [$\because OP = OR$]

$\Rightarrow$ $OP < OQ$

Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, $OP \perp AB$.

30. The first and the last terms of an A.P. are 8 and 350 respectively. If its common difference is 9, how many terms are there and what is their sum ?

Solution. Let a be the first term and d the common difference of an A.P., then $a = 8$ and $d = 9$ (given).

Let $l = a_n = 350$ (given) be the nth term of an A.P., then

$$350 = a_n = a + (n-1)d$$

$\Rightarrow$ $350 = 8 + (n-1)9$

$\Rightarrow$ $342 = (n-1)9$

$\Rightarrow$ $n - 1 = 38$

$\Rightarrow$ $n = 39$

$\therefore$ $S_n = \frac{n}{2}[a + a_n]$

$\Rightarrow$ $S_{39} = \frac{39}{2}[8 + 350]$

$\Rightarrow$ $S_{39} = \frac{39}{2}[358]$

$\Rightarrow$ $S_{39} = = 39 \times 179 = 6981$

Hence, the number of terms of an A.P. are **39** and the sum of 39 terms is **6981.**

Or

How many multiples of 4 lie between 10 and 250 ? Also find their sum.

Solution. The first term between 10 and 250, which is a multiple of 4 is 12 and second term is 16.

Clearly first term $= a = 12$

Second term $= a + d = 16$

and, common difference $(d) = 16 - 12 = 4$

Last term which is multiple of 4 is 248.

Now, we have to find the sum of n terms of the A.P.

12, 16, 20, ...,248

$$a_n = a + (n-1)d$$

$\Rightarrow$ $248 = 12 + (n-1)4$

$\Rightarrow$ $248 - 12 = (n-1)4$

$\Rightarrow$ $236 = (n-1)4$

$\Rightarrow$ $n - 1 = 236 \div 4$

$\Rightarrow$ $n - 1 = 59$

$\Rightarrow$ $n = 59 + 1 = 60$

Now, S_n = Sum of n terms of the A.P. 12, 16, 20, ..., 248

$\Rightarrow$ $S_{60} = \frac{60}{2}[12 + 248]$

$\Rightarrow$ $S_{60} = 30[260]$

$\Rightarrow$ $S_{60} = 30 \times 260$

$\Rightarrow$ $S_{60} = 7800$

Hence, the number of terms of the A.P. are **60** and the sum of 60 terms is **7800.**

31. A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution. Let the speed of the train be x km/hour.

When the speed has been 9 km/hour more, then the new speed of the train is $(x + 9)$ km/hour.

Time taken by the train with speed x km/hour for a journey of 180 km $= \frac{180}{x}$ hours.

Time taken by the train with new speed $(x + 9)$ km/hour for a journey of 180 km $= \frac{180}{(x+9)}$ hours.

It is given that the train takes 1 hour less for a journey of 180 km if the speed had been 9 km/hour more from its speed.

$$\therefore \quad \frac{180}{x} - \frac{180}{x+9} = 1$$

$$\Rightarrow \quad 180 \times \left[\frac{1}{x} - \frac{1}{x+9}\right] = 1$$

$$\Rightarrow \quad 180 \times \left[\frac{x+9-x}{x(x+9)}\right] = 1$$

$$\Rightarrow \quad 180 \times 9 = x(x+9)$$

$$\Rightarrow \quad x^2 + 9x - 1620 = 0$$

$$\Rightarrow \quad x^2 + 45x - 36x - 1620 = 0$$

$$\Rightarrow \quad x(x+45) - 36(x+45) = 0$$

$$\Rightarrow \quad (x+45)(x-36) = 0$$

$\Rightarrow$ Either $\quad x + 45 = 0 \quad$ or $\quad x - 36 = 0$

$\Rightarrow$ Either $\quad x = -45 \quad$ or $\quad x = 36$

$\Rightarrow \quad x = 36 \quad$ [$\because$ x cannot be negative]

Hence, the speed of the train is **36 km/hour.**

Or

Find the roots of the equation $\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5.$

Solution. We have,

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1, \quad x \neq \frac{3}{2}, 5$$

$$\Rightarrow \quad \frac{1}{2x-3} = 1 - \frac{1}{x-5}$$

$$\Rightarrow \quad \frac{1}{2x-3} = \frac{x-5-1}{x-5}$$

$$\Rightarrow \quad \frac{1}{2x-3} = \frac{x-6}{x-5}$$

$$\Rightarrow \quad x - 5 = (x-6)(2x-3)$$

$$\Rightarrow \quad x - 5 = 2x^2 - 12x - 3x + 18$$

$$\Rightarrow \quad 2x^2 - 16x + 23 = 0$$

$$\Rightarrow \quad x = \frac{16 \pm \sqrt{(-16)^2 - (4)(2)(23)}}{2(2)} \qquad \left[\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$\Rightarrow \quad x = \frac{16 \pm \sqrt{256 - 184}}{4}$$

$$\Rightarrow \quad x = \frac{16 \pm \sqrt{72}}{4}$$

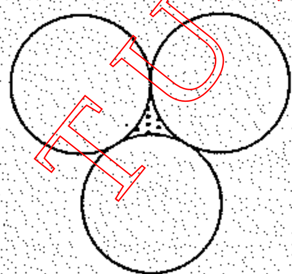
$$\Rightarrow \quad x = \frac{16 \pm 6\sqrt{2}}{4}$$

$$\Rightarrow \quad x = \frac{16}{4} \pm \frac{6}{4}\sqrt{2}$$

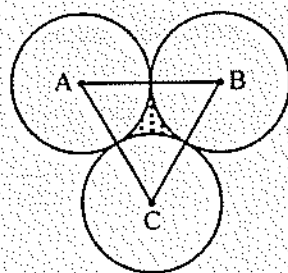
$$\Rightarrow \quad x = 4 \pm \frac{3\sqrt{2}}{2}$$

32. In figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area enclosed between these three circles (shaded region).

$\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution. Let A, B and C be the centres of three circles. Join AB, BC and CA.



Since the radius of each circle is 3.5 cm, then the sides $AB = 7$ cm, $BC = 7$ cm and $CA = 7$ cm respectively.

Area of an equilateral triangle ABC

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (7)^2 \text{ cm}^2$$

$$= \frac{49\sqrt{3}}{4} \text{ cm}^2 \qquad \ldots(1)$$

Let the area of three sectors each of angle 60° in a circle of radius 3.5 cm be **A**, then

A = 3 × Area of one sector of an angle of 60° in a circle of radius 3.5 cm.

$$= 3 \times \left[\frac{60°}{360°} \times \frac{22}{7} \times (3.5)^2\right] \text{cm}^2 \qquad \left[\because \text{Area of sector} = \frac{\theta}{360°}\pi r^2\right]$$

$$= \left[3 \times \frac{1}{6} \times \frac{22}{7} \times 3.5 \times 3.5\right] \text{cm}^2$$

$$= (11 \times 0.5 \times 3.5)\ \text{cm}^2$$

$$= (11 \times 1.75)\ \text{cm}^2$$

$$= 19.25\ \text{cm}^2$$

Now, required area of the shaded region

$$= \text{Area of } \Delta ABC - \text{A}$$

$$= \left(\frac{49\sqrt{3}}{4} - 19.25\right) \text{cm}^2$$

$$= \left(\frac{49 \times 1.73}{4} - 19.25\right) \text{cm}^2$$

$$= (49 \times 0.4325 - 19.25)\ \text{cm}^2$$

$$= (21.1925 - 19.250)\ \text{cm}^2$$

$$= 1.9425\ \text{cm}^2$$

$$= \mathbf{1.94\ cm^2}$$

33. Water is flowing at the rate of 15 km/hour through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm ?

Solution. Let the cubical pond be filled in x hours. Since water is flowing at the rate of 15 km/hour, therefore, length of the water flows through the pipe in x hours = $15x$ km = $15000x$ metres = (h) height of the pipe

Internal diameter of a pipe = 14 cm

$\therefore$ Internal radius of a pipe (r) = $\frac{14}{2}$ cm = 7 cm = $\frac{7}{100}$ m.

$\therefore$ Volume of the water flows from the cylindrical pipe in x hours

$$= \pi r^2 h$$

$$= \left(\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000x\right) \text{m}^3$$

$$= \left(22 \times \frac{1}{10} \times 7 \times 15x\right) \text{m}^3$$

$$= 11 \times 7 \times 3x\ \text{m}^3$$

$$= 231x\ \text{m}^3.$$

Also, volume of cuboidal pond = $\left(50 \times 44 \times \frac{21}{100}\right) \text{m}^3$

where l (length of a cuboidal pond) = 50 m

b (width of a cuboidal pond) = 44 m

h_1 (height of a cuboidal pond in which the level of water of pond increases) = 21 cm = $\frac{21}{100}$ m

Since the cuboidal pond is filled in x hours, therefore, the

Volume of the water flows in the cuboidal pond in x hours = Volume of a pipe

$$\Rightarrow \quad 50 \times 44 \times \frac{21}{100} = 231x$$

$$\Rightarrow \quad x = \frac{50 \times 44 \times 21}{231 \times 100} \text{ hours}$$

$$\Rightarrow \quad x = \frac{44}{11 \times 2} \text{ hours}$$

$$\Rightarrow \quad x = 2 \text{ hours}$$

Hence, the level of water in pond will rise by 21 cm in **2 hours.**

34. The angle of elevation of the top of a vertical tower from a point on the ground is 60°. From another point 10 m vertically above the first, its angle of elevation is 30°. Find the height of the tower.

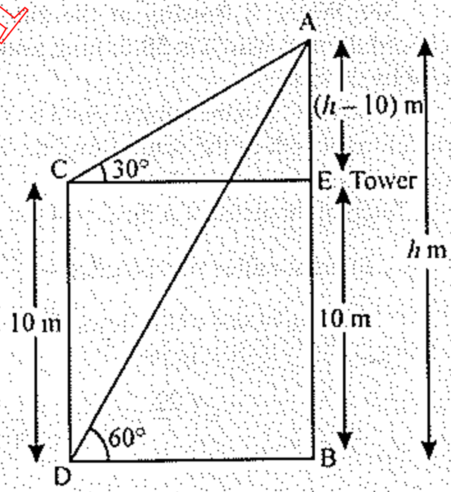
Solution. Let D be a point on the ground and C be another point 10 m vertically above the first point D.

Let $h = AB$ be the height of the tower, then

$$CD = 10 \text{ m},\ BE = CD = 10 \text{ m and } AE = AB - BE = (h - 10) \text{ m}$$

So, $\angle ADB = 60°$ and $\angle ACE = 30°$

In right triangle ADB, we have



$$\tan 60° = \frac{AB}{DB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{DB}$$

$$\Rightarrow \quad DB = \frac{h}{\sqrt{3}} \qquad \ldots(1)$$

In right triangle ACE, we have

$$\tan 30° = \frac{AE}{CE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h - 10}{DB} \qquad [\because CE = DB]$$

$$\Rightarrow \quad DB = \sqrt{3}(h - 10) \qquad \ldots(2)$$

From (1) and (2), we have

$$\frac{h}{\sqrt{3}} = \sqrt{3}(h - 10)$$

$$\Rightarrow \quad h = 3(h - 10)$$

$$\Rightarrow \quad h = 3h - 30$$

$$\Rightarrow \quad 3h - h = 30$$

$$\Rightarrow \quad 2h = 30$$

$$\Rightarrow \quad h = 15 \text{ m}$$

Hence, the height of the tower is **15 m.**

SET II

Section 'A'

Question numbers 1 to 10 are of one mark each.

9. The surface area of a solid hemisphere of radius r cm (in cm^2) is

(a) $2\pi r^2$ (b) $3\pi r^2$

(c) $4\pi r^2$ (d) $\frac{2}{3}\pi r^3$

Solution. Choice (b) is correct.

The surface area of a solid hemisphere of radius r cm (in cm^2)

$$= \text{Surface area of a hemisphere} + \text{Area of a circle at the top}$$
$$= 2\pi r^2 + \pi r^2$$
$$= 3\pi r^2$$

10. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the card is not a red king, is

(a) $\frac{1}{13}$ (b) $\frac{12}{13}$

(c) $\frac{1}{26}$ (d) $\frac{25}{26}$

Solution. Choice (d) is correct.

As there are 2 red kings in a well-shuffled deck of 52 playing cards

$\therefore$ Number of red kings = 2

Number of not a red king = Total number cards – 2 Red kings

$$= 52 - 2$$
$$= 50$$

$\therefore$ Probability that the card is not a red king $= \frac{50}{52} = \frac{25}{26}$.

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

17. Which term of the A.P. 3, 14, 25, 36,... will be 99 more than its 25th term ?

Solution. Given A.P. is 3, 14, 25, 36, ...

Here, a = first term = 3

d = common difference

$= 14 - 3 = 11$ ...(1)

Let the nth term of the given A.P. be 99 more than its 25th term. Then

$$a_n = a_{25} + 99$$

$\Rightarrow$ $a + (n-1)d = a + (25-1)d + 99$ [$\because$ nth term $(a_n) = a + (n-1)d$]

$\Rightarrow$ $(n-1)d - 24d = 99$

$\Rightarrow$ $(n - 1 - 24)d = 99$

$\Rightarrow$ $(n - 25) \times 11 = 99$ [$\because d = 11$]

$\Rightarrow$ $n - 25 = 9$

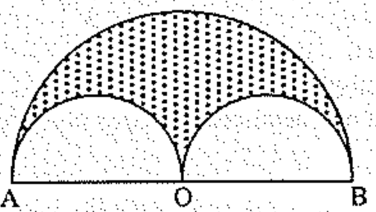
$\Rightarrow \qquad n = 9 + 25$

$\Rightarrow \qquad n = 34$

Hence, the 34th term of the given A.P. is 99 more than its 25th term.

18. In figure, a semi-circle is drawn with O as centre and AB as diameter. Semi-circles are drawn with AO and OB as diameters. If $AB = 28$ m, find the perimeter of the shaded region.

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$



Solution. Diameter of a big semi-circle = 28 cm [$AB = 28$ cm (given)]

$\Rightarrow$ Radius of a big semi-circle = 28 ÷ 2 = 14 cm

Diameters of small semi-circles is OA or $OB = 28 \div 2 = 14$ cm

$\Rightarrow$ Radius of small semi-circles = 14 ÷ 2 = 7 cm

Required perimeter of the shaded region

= Perimeter of a big semi-circle

+ Perimeter of a small semi-circles with radius $r_1 = \dfrac{OA}{2}$ and $\dfrac{OB}{2} = r_2$.

$$= \pi R + \pi r_1 + \pi r_2$$
$$= \pi[R + r_1 + r_2]$$
$$= \pi[14 + 7 + 7] \text{ cm}$$
$$= \left(\frac{22}{7} \times 28\right) \text{cm}$$
$$= 88 \text{ cm}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

27. A chord of a circle of radius 21 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. $\left[\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{3} = 1.73\right]$

Solution. We have

Area of a sector $OACB$ of a circle of radius 21 cm subtends an angle of 60° at the centre

$$= \frac{60°}{360°}\pi r^2$$
$$= \frac{1}{6} \times \frac{22}{7} \times (21)^2 \text{ cm}^2$$
$$= (11 \times 21) \text{ cm}^2$$
$$= 231 \text{ cm}^2$$

Area of a triangle OAB subtends an angle 60°

$$= \frac{1}{2} r^2 \sin\theta$$

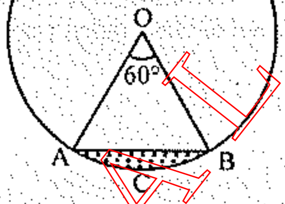
$$= \frac{1}{2} \times (21)^2 \times \sin 60° \text{ cm}^2$$

$$= 220.5 \times \frac{\sqrt{3}}{2} \text{ cm}^2$$

$$= 110.25 \times \sqrt{3} \text{ cm}^2$$

$$= 110.25 \times 1.73 \text{ cm}^2$$

$$= 190.7325 \text{ cm}^2$$



Area of minor segment of the circle

= Area of sector – Area of ΔOAB subtends an angle of 60°

$$= (231 - 190.7325) \text{ cm}^2$$

$$= \mathbf{40.2675 \text{ cm}^2}.$$

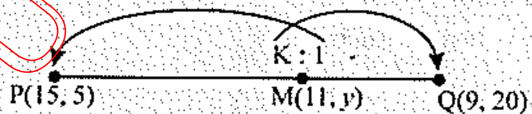
28. Point $M(11, y)$ lies on the line segment joining the points $P(15, 5)$ and $Q(9, 20)$. Find the ratio in which point M divides the line segment PQ. Also find the value of y.

Solution. Let the point $M(11, y)$ divides the line segment joining the points $P(15, 5)$ and $Q(9, 20)$ in the ratio $K : 1$.

Then the coordinates of M are $\left(\frac{9K+15}{K+1}, \frac{20K+5}{K+1}\right)$

But the coordinates of M are given as $(11, y)$.

$$\therefore \quad \frac{9K+15}{K+1} = 11 \quad \text{and} \quad \frac{20K+5}{K+1} = y$$



Consider, $\frac{9K+15}{K+1} = 11$

$\Rightarrow \quad 9K + 15 = 11K + 11$

$\Rightarrow \quad 11K - 9K = 15 - 11$

$\Rightarrow \quad 2K = 4$

$\Rightarrow \quad K = 2$

$\Rightarrow$ M divides the line segment PQ in the ratio **2 : 1.**

Substituting $K = 2$ in $y = \frac{20K+5}{K+1}$, we get

$$y = \frac{20(2)+5}{2+1}$$

$\Rightarrow \quad y = \frac{40+5}{3}$

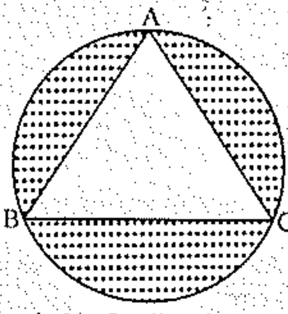
$\Rightarrow \quad y = \frac{45}{3}$

$\Rightarrow \quad \mathbf{y = 15.}$

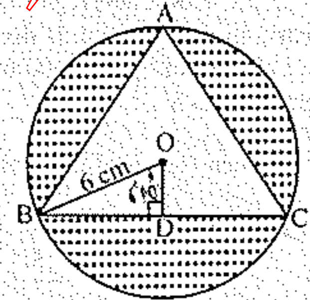
Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. In figure, an equilateral triangle has been inscribed in a circle of radius 6 cm. Find the area of the shaded region. **[Use $\pi = 3.14$]**



Solution. Let O be the centre of a circle of radius 6 cm and ABC is the equilateral triangle.
From O, draw $OD \perp BC$.
In ΔOBD, we have



$$\cos 60^\circ = \frac{OD}{OB} = \frac{B}{H}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{OD}{6} \qquad [\because OB = 6 \text{ cm}]$$

$$\Rightarrow \quad OD = 3 \text{ cm}$$

and $$\sin 60^\circ = \frac{BD}{OB}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BD}{6}$$

$$\Rightarrow \quad BD = 3\sqrt{3} \text{ cm}$$

$$\therefore \quad BC = 2BD = 2(3\sqrt{3}) = 6\sqrt{3} \text{ cm}$$

Area of the shaded region

= Area of the circle of radius 6 cm – Area of equilateral ΔABC

$$= \left[\pi \times (6)^2 - \frac{\sqrt{3}}{4} \times (6\sqrt{3})^2\right] \text{cm}^2 \qquad [\because BC = 6\sqrt{3} \text{ cm}]$$

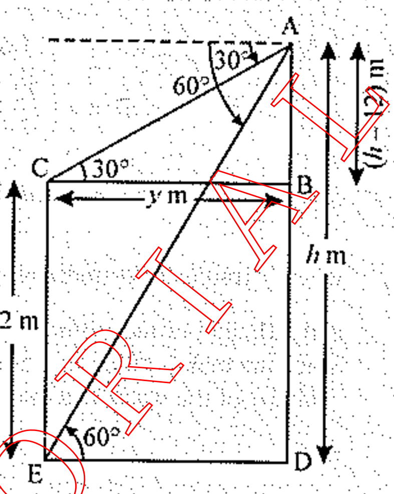
$$= \left[\frac{22}{7} \times 36 - \frac{\sqrt{3}}{4} \times 36 \times 3\right] \text{cm}^2$$

$$= 36\left[\frac{22}{7} - \frac{3\sqrt{3}}{4}\right] \text{cm}^2$$

$$= \left[\frac{792}{7} - 27\sqrt{3}\right] \text{cm}^2$$

30. The angles of depression of the top and bottom of a 12 m tall building, from the top of a multi-storeyed building are 30° and 60° respectively. Find the height of the multi-storeyed building.

Solution. Let $AD = h\ m$ be the multi-storeyed building and $EC = 12$ m be the tall building. From the top of a multi-storeyed building the angles of depression of the top and bottom of building are 30° and 60°. Let $BC = DE = y$ metres, be the distance between the bottom of the multi-storeyed building and the bottom of the building.



Then, $AB = AD - BD$

$\Rightarrow$ $AB = (h - 12)$ metres be the difference of heights between the building and multi-storeyed building.

In right triangle ABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-12}{y}$$

$$\Rightarrow \quad y = \sqrt{3}(h-12) \qquad ...(1)$$

In right triangle ADE, we have

$$\tan 60° = \frac{AD}{DE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\sqrt{3}(h-12) = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad 3(h-12) = h$$

$$\Rightarrow \quad 3h - 36 = h$$

$$\Rightarrow \quad 3h - h = 36$$

$$\Rightarrow \quad 2h = 36$$

$$\Rightarrow \quad h = 18 \text{ m}$$

Hence, the height of the multi-storeyed building is **18 m.**

31. A farmer connects a pipe of internal diameter 20 cm, from a canal into a cylindrical tank in his field, which is 10 m in diameter and 4 m deep. If water flows through the pipe at the rate of 5 km/hour, in how much time will the tank be filled ?

Solution. Let the tank be filled in x hours.

Since water is flowing at the rate of 5 km/hour, therefore, length of the water flows through the pipe in x hours

$= 5x$ km

$= 5000x$ metres $= (h)$ height of the pipe from a canal

Internal diameter of a pipe from a canal = 20 cm

$\therefore$ Internal radius of a pipe from a canal $(r) = \frac{20}{2} = 10 \text{ cm} = \frac{1}{10}$ m

$\therefore$ Volume of the water that flows from the cylindrical pipe from a canal in x hours

$$= \pi r^2 h = \left(\frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times 5000x\right) \text{m}^3$$

Also, volume of the cylindrical tank $= \left(\frac{22}{7} \times 5 \times 5 \times 4\right) \text{m}^3$

where r (radius of cylindrical tank) $= \frac{10}{2}$ m $= 5$ m

and h (height of the cylindrical tank) = 4 m (deep)

Since the tank is filled in x hours, therefore,

Volume of the water that flows in the tank in x hours = Volume of the cylindrical tank

$$\Rightarrow \quad \frac{22}{7} \times \frac{1}{10} \times \frac{1}{10} \times 5000x = \frac{22}{7} \times 5 \times 5 \times 4$$

$$\Rightarrow \quad x = \frac{5 \times 5 \times 4}{50} \text{ hours}$$

$$\Rightarrow \quad x = 2 \text{ hours}$$

Hence, the tank will be filled in **2 hours.**

SET III

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. A solid sphere of radius r is melted and recast into the shape of a solid cone of height $4r$. The radius of the base of the cone is

(*a*) r (*b*) $2r$

(*c*) $3r$ (*d*) $4r$

Solution. Choice (*a*) is correct.

Radius of a solid sphere = r

Volume of a solid sphere $= \frac{4}{3}\pi r^3$ ...(1)

Let R and H be the radius and height of the cone, then volume of the cone

$$= \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3}\pi \times R^2 \times 4r \quad \text{...(2) } [\because H = 4r \text{ (given)}]$$

From (1) and (2), we have

$$\frac{1}{3}\pi R^2 \times 4r = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R^2 \times 4r = 4r^3$$
$$\Rightarrow \quad R^2 = r^2$$
$$\Rightarrow \quad R = r$$

Hence, the radius of the cone is ***r*** **cm.**

2. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that it is not a face card is

(*a*) $\frac{12}{52}$ (*b*) $\frac{16}{52}$

(*c*) $\frac{10}{13}$ (*d*) $\frac{9}{13}$

Solution. Choice (*c*) is correct.

Number of the face cards in a well-shuffled deck of 52 playing cards

= 4 Kings + 4 Queens + 4 Jacks

= 12

Number of not face cards in a well-shuffled deck of 52 playing cards

= 52 – 12

= 40

∴ Probability that it is not a face card

$$= \frac{40}{52} = \frac{10}{13}.$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. How many natural numbers are there between 200 and 500, which are divisible by 7 ?

Solution. The first term between 200 and 500, which is divisible by 7, is 203 and the second term is 210.

Clearly, first term = a = 203

Second term = $a + d$ = 210

and common difference (d) = 210 – 203 = 7

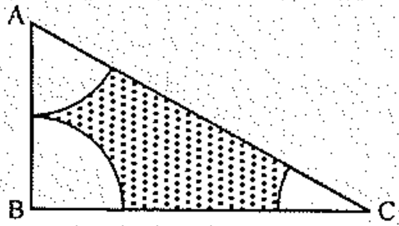
Last term which is divisible by 7 is 497.

Let the nth term be 497, then

$$a_n = a + (n-1)d$$
$$\Rightarrow \quad 497 = 203 + (n-1)7$$
$$\Rightarrow \quad 497 - 203 = (n-1)7$$
$$\Rightarrow \quad 294 = (n-1)7$$
$$\Rightarrow \quad n - 1 = 42$$
$$\Rightarrow \quad n = 43$$

Hence, 43 natural numbers are divisible by 7 between 200 and 500.

18. In figure, ABC is a triangle right-angled at B, with $AB = 14$ cm and $BC = 24$ cm. With the vertices A, B and C as centres, arcs are drawn, each of radius 7 cm. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



Solution. ABC is a triangle right-angled at B, with $AB = 14$ cm and $BC = 24$ cm.
Let A_1 be the area of a right triangle ABC, then

$$A_1 = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\Rightarrow \quad A_1 = \frac{1}{2} \times 24 \times 14 \text{ cm}^2$$

$$\Rightarrow \quad A_1 = 168 \text{ cm}^2 \quad \ldots(1)$$

Required area of the shaded region

= Area of a right triangle ABC – Area of a sector at A – Area of sector at B – Area of sector at C

$$= 168 - \frac{A}{360°} \times \pi \times (7)^2 - \frac{B}{360°} \times \pi \times (7)^2 - \frac{C}{360°} \times \pi \times (7)^2$$

$$= \left[168 - \frac{\pi \times 49}{360°} \times (A + B + C)\right] \text{cm}^2$$

$$= \left[168 - 49\pi \times \frac{180°}{360°}\right] \text{cm}^2 \quad [\because A + B + C = 180°]$$

$$= \left[168 - 49\pi \times \frac{1}{2}\right] \text{cm}^2$$

$$= \left[168 - 49 \times \frac{22}{7} \times \frac{1}{2}\right] \text{cm}^2$$

$$= [168 - 7 \times 11] \text{ cm}^2$$

$$= [168 - 77] \text{ cm}^2$$

$$= \mathbf{91 \text{ cm}^2}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. The point $A(3, y)$ is equidistant from the points $P(6, 5)$ and $Q(0, -3)$. Find the value of y.

Solution. It is given that the point $A(3, y)$ is equidistant from the points $P(6, 5)$ and $Q(0, -3)$

$\therefore \quad AP = AQ$ (given)

$$\Rightarrow \quad AP^2 = AQ^2$$

$$\Rightarrow \quad (6-3)^2 + (5-y)^2 = (0-3)^2 + (-3-y)^2$$

$$\Rightarrow \quad 9 + 25 - 10y + y^2 = 9 + 9 + y^2 + 6y$$

$$\Rightarrow \quad 6y + 10y = 9 + 25 - 9 - 9$$

$$\Rightarrow \quad 16y = 16$$

$$\Rightarrow \quad y = 1$$

Hence, the value of y is **1.**

20. Area of a sector of a circle of radius 14 cm is 154 cm². Find the length of the corresponding arc of the sector. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Solution. Radius of a circle (r) = 14 cm

Area of a sector of a circle of radius (r) = 154 cm²

$$\therefore \quad A = \frac{\theta}{360^\circ} \times \pi(14)^2 \text{ cm}^2 \quad \ldots(1)$$

Also, the length of the corresponding arc of a circle of radius (r) = l = 14 cm

$$\therefore \quad l = \frac{\theta}{360^\circ} \times 2\pi(14) \quad \ldots(2)$$

Dividing (1) by (2), we get

$$\Rightarrow \quad \frac{A}{l} = \frac{\frac{\theta}{360^\circ} \times \pi \times (14)^2}{\frac{\theta}{360^\circ} \times 2\pi(14)}$$

$$\Rightarrow \quad \frac{154}{l} = \frac{(14)^2}{2 \times 14}$$

$$\Rightarrow \quad l = \frac{154 \times 2 \times 14}{14 \times 14}$$

$$\Rightarrow \quad l = 11 \times 2 = 22 \text{ cm}$$

Hence the length of the corresponding arc of the sector is **22 cm.**

Alternative method :

Radius of a circle (r) = 14 cm

Area of a sector of a circle of radius (r) = 14 cm is 154 cm² (given)

$$\therefore \quad A = 154 \text{ cm}^2$$

Area of a sector in terms of length, *i.e.*,

$$A = \frac{1}{2} lr$$

$$\Rightarrow \quad 154 = \frac{1}{2} l \times 14$$

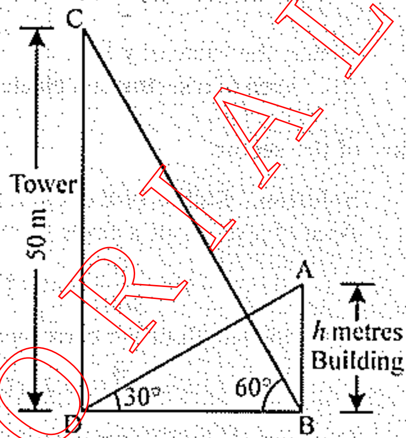
$$\Rightarrow \quad l = \frac{154 \times 2}{14}$$

$$\Rightarrow \quad l = \mathbf{22 \text{ cm.}}$$

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.



Solution. Let $AB = h$ metres be the height of the building and $CD = 50$ m be the height of the tower.

It is given that the angle of elevation of the top of a building from the foot of a tower is 30°.

$\therefore \quad \angle ADB = 30°$

Also it is given that the angle of elevation of the top of the tower from the foot of the building is 60°.

$\therefore \quad \angle DBC = 60°$

In right triangle CDB, we have

$$\tan 60° = \frac{CD}{DB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{CD}{DB}$$

$$\Rightarrow \quad DB = \frac{50}{\sqrt{3}} \text{ m} \quad \ldots(1)$$

Again, in right triangle ABD, we have

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{DB}$$

$$\Rightarrow \quad DB = \sqrt{3}\, h \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{50}{\sqrt{3}} = \sqrt{3}\, h$$

$$\Rightarrow \quad 3h = 50$$

$$\Rightarrow \quad h = 16.67 \text{ metres}$$

Hence, the height of the building is **16.67 metres.**

30. Water is flowing at the rate of 10 km/hour through a pipe of diameter 16 cm into a cuboidal tank of dimensions 22 m × 20 m × 16 m. How long will it take to fill the empty tank ?

$$\left[\text{Use } \pi = \frac{22}{7}\right]$$

Solution. Let the cuboidal tank be filled in x hours.

Since water is flowing at the rate 10 km/hour, therefore, the length of water flows through the pipe in x hours

$= 10x$ km

$= 10000x$ metres $= (h)$ height of pipe

Internal diameter of a pipe = 16 cm

$\therefore$ Internal radius of a pipe $= \frac{16}{2} = 8$ cm $= \frac{8}{100}$ metres

$\therefore$ Volume of water flows from the cylindrical pipe in x hours

$$= \pi r^2 h$$

$$= \left(\frac{22}{7} \times \frac{8}{100} \times \frac{8}{100} \times 10000x\right) \text{ m}^3$$

$$= \frac{22 \times 8 \times 8x}{7} \text{ m}^3$$

Since cuboidal tank of dimensions 22 m × 20 m × 16 m is filled in x hours, therefore, the Volume of the water flows in the cuboidal tank in x hours = Volume of the pipe

$$\Rightarrow \quad 22 \times 20 \times 16 = \pi r^2 h$$

$$\Rightarrow \quad 22 \times 20 \times 16 = \frac{22 \times 8 \times 8x}{7}$$

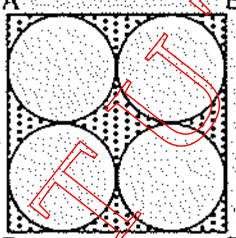
$$\Rightarrow \quad x = \frac{22 \times 20 \times 16 \times 7}{22 \times 8 \times 8} \text{ hours}$$

$$\Rightarrow \quad x = 5 \times 7 \text{ hours}$$

$$\Rightarrow \quad x = 35 \text{ hours}$$

Hence, it will take **35 hours** to fill the empty tank.

31. Find the area of the shaded region in figure, where *ABCD* is a square of side 28 cm.



Solution. Side of a square = 28 cm

$\therefore$ Area of a square $ABCD = (\text{Side})^2 = (28)^2 = 784 \text{ cm}^2$

Diameter of each circle $= \frac{\text{Side of a square}}{2} = \frac{28}{2} = 14$ cm

$\Rightarrow$ Radius (r) of each circle $= \frac{14}{2} = 7$ cm

$\therefore$ Area of one circle $= \pi r^2$

$$= \frac{22}{7} \times (7)^2 \text{ cm}^2$$

$$= (22 \times 7) \text{ cm}^2 = 154 \text{ cm}^2$$

Thus, area of the four circles = 4 times the area of one circle

$$= 4 \times 154 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Hence, area of the shaded region

= Area of a square – Area of the four circles

$= 784 \text{ cm}^2 - 616 \text{ cm}^2$

$= \mathbf{168 \text{ cm}^2}$