Mathematics (Basic)- Theory

Time allowed: 3 hours **Maximum marks: 80**

General instruction

(i) This question paper comprises four sections – A, B, C and D this question paper carries 40 questions. All question are compulsory.

(ii) Section A: Q. No.1 to 20 comprises of 20 questions of one mark each.

(iii) Section B: Q. No.21 to 26 comprises of 6 questions of two marks each.

(iv) Section C: Q. No.27 to 34 comprises of 8 questions of three marks each.

(v) Section D: Q. No.35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of one mark each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choices in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

SECTION - A

Q. Nos. 1 to 10 are multiple choice questions. Select the correct option.

1. HCF of 144 and 198 is

A. 9 B. 18

C. 6 D. 12

Answer: $144 = 2^4 \times 3^2$

$198 = 2 \times 3^2 \times = 11$

$\therefore$ HCF $= 2 \times 3^2 = 18$

2. The median and mode respectively of a frequency distribution are 26 and 29. Then its mean is

A. 27.5 B. 24.5

C. 28.4 D. 25.8

Answer: Median = 26

Mode = 29

We know that ,

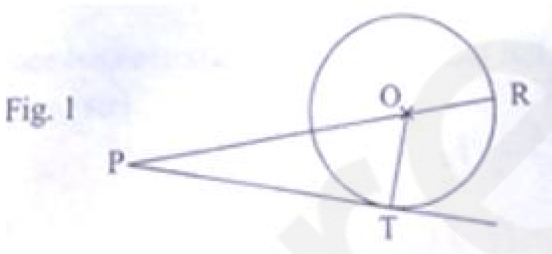
3 median = mode + 2 mean

3(26) = 29 + 2(mean)

2(mean) = 49

∴ mean = 24.5

3. In Fig. 1, on a circle of radius 7 cm, tangent PT is draw from. a point P such that PT= 24 cm. If O is the centre of the circle, then the length of PR is



A. 30 cm B. 28 cm

C. 32 cm D. 25 cm

Answer: We know that, radius to a tangent is perpendicular at the point of contact

∴ In right Δ OTP, By Pythagoras theorem $(OP)^2 = (OT)^2 + (PT)^2$

$= (7)^2 + (24)^2$

= 49 + 576

= 625

∴ OP = 25 cm

Now, PR = OP + OR = 25 + 7 = 32cm

4. 225 can be expressed as

A. 5×3^2 B. $5^2 \times 3$

C. $5^2 \times 3^2$ D. $5^3 \times 3$

Answer: $225 = 5^2 \times 3^2$

5. The probability that a number selected at random from the numbers 1,2,3,……15 is a multiple of 4 is

A. 4/15 B. 2/15

C. 1/15 D. 1/5

Answer: Total outcome = 15

Favourable outcome = 4, 8, 12, i.e 3

∴ P (number selected is a multiple of 4) = 3/15

= 1/5

6. If one zero of a quadratic polynomial ($kx^2 + 3x + k$) is 2, then the value of k is

A. $\frac{5}{6}$ B. $-\frac{5}{6}$

C. $\frac{6}{5}$ D. $-\frac{6}{5}$

Answer: Here, $p(x) = kx^2 + 3x + k$

$P(2) = k(2)^2 + 3(2) + k = 0$

$\Rightarrow 4k + 6 + k = 0$

$\Rightarrow 5k + 6 = 0$

$\Rightarrow k = -6/5$

7. $2.\overline{35}$ is

A. an integer B. a rational number

C. an irrational number D. a natural number

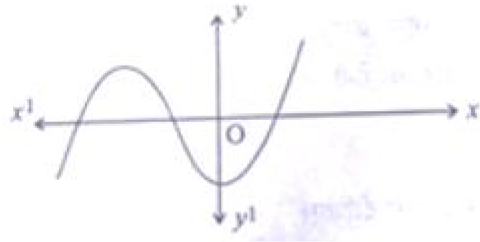
Answer: $2.\overline{35}$ = 2.35 35 35 ---------------

Since, the digits after decimals are repeating

therefore, $2.\overline{35}$ is a rational number .

8. The graph of a polynomial is shown in Fig. 2, then the number of its zeroes

Fig. 2



A. 3 B. 1

C. 2 D. 4

Answer: Since, the graph is cutting the x- axis at 3 – points, therefore , there will be three zeroes of the given polynomial.

9. Distance of point P(3,4) from x-axis is

A. 3 units B. 4 units

C. 5 units D. 1 units

Answer:

We know that, the distance of a point from x – axis is its y-coordinate.

∴ the distance of S from x-axis is 4 units

10. If the distance between the points A(4,p) and B(1,0) is 5 units, then the value (s) of p is (are)

A. 4 only B. -4 only

C. ± 4 D. 0

Answer: Given AP = 5

On squaring both sides $AP^2 = 25$

$(4 - 1)^2 + (P - O)^2 = 25$

$9 + P^2 = 25$

$P^2 = 25 - 9$

$P^2 = 16$

$\therefore P = \pm 4$

Q. Nos 11 to 15, fill in the blanks.

11. If the C (k,4) divides the line segment joining two points A (2,6) and B (5,1) in ratio 2:3, the value of k is ____________.

OR

If points A (-3, 12), B (7, 6) and C (x, 9) are collinear, then the value of x is ___________.

Answer:

$(2,6)$ ^{2} $(k,4)$ ^{3} $(5,1)$

A C B

Coordinate of C = $(k,4)=\left(\frac{6}{5},\frac{20}{5}\right)$

Equating the x coordinate of both sides

K= 6/5

OR

Given, A(-3 , 12) , B(7, 6) , C(x, 9)are collinear

∴ ar Δ(ABC) = 0

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=0$$

$$\left|1-3(6-9)+7(9-12)+x(12-6)\right|=0$$

$$\left|9-21+6x\right|=0$$

$$\left|12+6x\right|=0$$

$$2+x=0$$

$$x=2$$

12. If the equations kx - 2y = 3 and 3x + y = 5 represent two intersecting lines at unique point, then the value of k is____________.

OR

If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is _________.

Answer: Kx -2y – 3 = 0

3x + y – 5 = 0

For unique solution:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{k}{3} \neq \frac{-2}{1}$$

$$k \neq 6$$

Therefore, the value of k can be any number expect 6.

OR

$3x^2 - 4x + k = 0$

a = 3, b = -4, c = k

for equal roots,

D= b^2 - 4ac = 0

(-4^2) – 4 (3) (k) = 0

16 -12k = 0

K= 16/12 = 4/3

13. The value of $(\sin 20^o \cos 70^o \sin 70^o \cos 20^o)$ is ______.

Answer: Sin20° cos70° + sin70° cos20°

=Sin (90 – 70) cos70° + sin70° Cos(90 – 70)

=Cos70°. Cos70° + Sin70° . Sin70°

=$Cos^2$70° + $Sin^2$70°

=1

14. If tan(A+B) = √3 and tan $(A-B) = \frac{1}{\sqrt{3}}, A > B,$ then the value of A is ________.

Answer:

tan (A + B) = √3 & tan (A – B) = $\frac{1}{\sqrt{3}}$

∴ tan (A + B) = tan60° & tan (A -B) = tan30°

Equating the angles or both sides

A + B = 60°--------------------(i) & A – B = 30°----------------------(ii)

Adding eq. (i) & (ii)

$$\begin{aligned} A + B &= 60^{o} \\ A - B &= 30^{o} \\ \hline 2A &= 90^{o} \end{aligned}$$

$$\Rightarrow A = 45^{o}$$

15. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is __________.

Answer: We know that , when two triangles are similar then,

$$\frac{\text{Perimeter of first triangle}}{\text{Perimeter of second triangle}} = \frac{\text{Side of first triangle}}{\text{Side of second triangle}}$$

$$\frac{25}{15} = \frac{9}{x}$$

$$x = \frac{9 \times 15}{25} = \frac{27}{5} = 5.4\text{cm}$$

In Q. Nos. 16 to 20, answer the following

16. If 5 tanθ = 3, then what is the value of $\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}$?

Answer: Given, $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$

Dividing numerator and denominator by cosθ.

$$\left(\frac{5\tan\theta-3}{4\tan\theta+3}\right)$$

Substituting tanθ = 3/5 in above expression

$$\left(\frac{5\times\frac{3}{5}-3}{4\times\frac{3}{5}+3}\right)=0$$

17. The areas of two circles are in the ratio 9:4, then what is the ratio of their circumferences?

Answer: Given,

$$\frac{\text{Area of first triangle}}{\text{Area of second triangle}}=\frac{9}{4}$$

$$\frac{\pi r_1^2}{\pi r_2^2}=\frac{9}{4}$$

$$\frac{r_1^2}{r_2^2}=\frac{9}{4}\Rightarrow\frac{r_1}{r_2}=\frac{3}{2}$$

Now,

$$\frac{\text{Circumference of first circle}}{\text{Circumference of second circle}}=\frac{2\pi r_1}{2\pi r_2}$$

$$\frac{r_1}{r_2}=\frac{3}{2}$$

∴ ratio of their circumference is 3:2

18. If a pair of dice is thrown once, then what is the probability of getting a sum of 8?

Answer: Total outcome for a pair of dice are 36

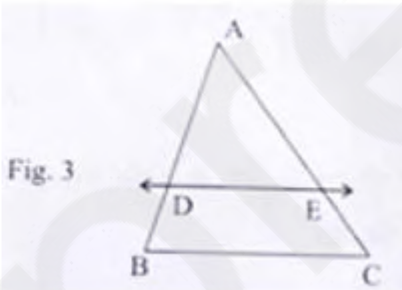
$$= \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6) \\ (2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) \\ (4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) \\ (6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

For sum equals to 8, the favourable outcome are :

(2,6) (3,5) (4,4) (5,3) and (6,2)

∴ P (getting sum equal to 8)=5/36

19. In Fig. 3, Δ ABC, DE || BC such that AD = 2.4 cm, AB = 3.2 cm and AC = 8 cm, then what is the length of AE?

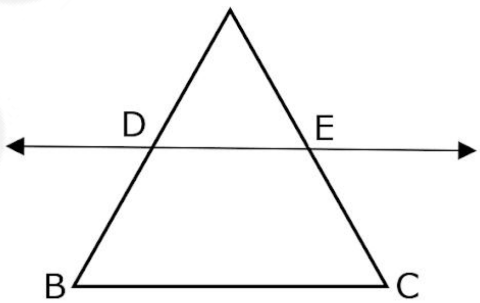


Answer:

Given, AD = 2.4 cm

AB = 3.2 cm

AC = 8cm



We know that when a line is parallel to any side of a triangle , then it divides the other two sides in equal proportion

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\therefore \frac{2.4}{3.2} = \frac{AE}{8}$$

$$\frac{3}{4} = \frac{AE}{8}$$

$$\Rightarrow AE = 6cm$$

20. The nth term of an AP is (7 – 4n), then what is its common difference?

Answer: An = 7 – 4n

$\therefore a_1 = 7 - 4(1) = 3$

$a_2 = 7 - 4(2) = -1$

$\therefore d = a_2 - a_1 = -1-3 = -4$

SECTION - B

Q. Nos. 21 to 26 carry two marks each

21. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Answer: Let the blue balls in the bag be x .

P (Blue ball) = 3P (red ball)

$$\frac{x}{x+5} = 3\left(\frac{5}{x+5}\right)$$

$$\Rightarrow x = 3 \times 5 = 15$$

$\therefore$ there are 15 blue balls in the bag

22. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta.$

OR

Prove that $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = 1$

Answer:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$$

LHS:

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$\sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}=\frac{1-\sin\theta}{\cos\theta}$$

$$\therefore\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\Rightarrow\sec\theta-\tan\theta=\text{RHS}$$

OR

$$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=1$$

LHS:

$$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\frac{1}{\tan^2\theta}}{1+\frac{1}{\tan^2\theta}}$$

$$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{1}{\tan^2\theta}\times\frac{\tan^2\theta}{\tan^2\theta+1}$$

$$\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{1}{\tan^2\theta+1}$$

$$\frac{\tan^2\theta+1}{\tan^2\theta+1}=1=\text{RHS}$$

Hence Proved

23. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

OR

Find the probability that 5 Sundays occur in the month of November of a randomly selected year.

Answer: Total outcome when two dices are thrown:

$$= \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6) \\ (2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) \\ (4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) \\ (6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

= 36 outcomes

Favourable outcome when the sum is less than 5.

(1,1) (1,2) (2,1) (1,3) (2,2) (3,1)

= 6 outcomes

∴ P (getting a sum less than 5) $= \frac{6}{36} = \frac{1}{6}$

OR

We know that in any year the number of days in November are 30.

Therefore there will be 4 complete weeks and 2 days in November .

Now, 4 complete weeks will have 4 Sundays.

The remaining 2 days can be any combination from the below :

Sunday – Monday

Monday – Tuesday

Tuesday – Wednesday

Wednesday – Thursday

Thursday – Friday

Friday – Saturday

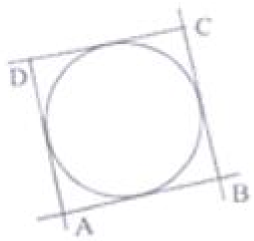
Saturday – Sunday

Hence, Total outcome = 2 i.e. (Sunday – Monday & Saturday – Sunday)

∴ P(occurring of 5 Sundays in November) = 2/7

24. In Fig. 4, a circle touches all the four sides of a quadrilateral ABCD. If AB = 6 cm, BC = 9cm, and CD = 8 cm, then find the length of AD.

Fig. 4



Answer:

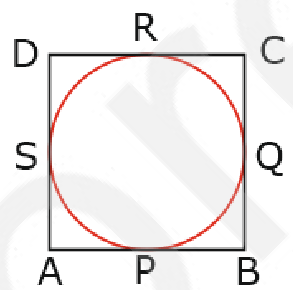
Given

AB = 6 cm

BC = 9 cm

CD = 8 cm

AD = ?



We know that tangents drawn from an external point to a circle are equal.

∴ AP = AS ...(1)

BP = BQ ...(2)

DR = DS ...(3)

CR = CQ ...(4)

Adding equation (1), (2), (3) and (4)

(AP + BP) + (DR + CR) = AS + BQ + DS + CQ

AB + DC = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

Now, 6 + 8 = AD + 9

⇒ AD = 14 – 9 = 5 cm

25. The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the areas of the sector.

Answer:

Given:

Radius = 6.5 cm

Perimeter of sector = 31 cm

$$\therefore \frac{\theta}{360} \times 2\pi \times r = 31$$

$$\frac{\theta}{360} \times 2 \times \pi \times 6.5 = 31$$

$$\theta = \frac{31 \times 360}{13\pi} = \frac{11,160}{13\pi}$$

Now, Area of sector = $\frac{\theta}{360} \pi r^2$

Substituting the value of θ in above equation

Area of sector $= \frac{11,160}{13\pi} \times \frac{1}{360} \times \pi \times 6.5 \times 6.5 = \frac{31}{2} \times 6.5$

$= 100.75 \text{ cm}^2$

26. Divide the polynomial ($4x^2 + 4x + 5$) by ($2x + 1$) and write the quotient and the remainder.

Answer:

Given:

$P(x) = 4x^2 + 4x + 5$

$G(x) = 2x+1$

Now,

$$
\begin{array}{r|l}
 & 2x+1 \\
\hline
2x+1 & 4x^2+4x+5 \\
 & 4x^2+2x \\
 & -\quad - \\
\hline
 & \quad 2x+5 \\
 & \quad 2x+1 \\
 & \quad 2x+1 \\
\hline
 & \quad\quad 4
\end{array}
$$

Hence, $q(x) = 2x + 1$

& $r(x) = 4$

SECTION -C

Q. Nos. 27 to 34 carry 3 marks each.

27. If α and β are the zeros of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$.

Answer: For given equation, we have

$\alpha + \beta = 4$ [-b/a]

$\alpha\beta = -5$ [c/a]

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (4)^2 - 2(-5)$

$= 16 + 10 = 26$

28. Draw a circle of radius 4 cm. From a point 7 cm away from the centre of circle. Construct a pair of tangents to the circles.

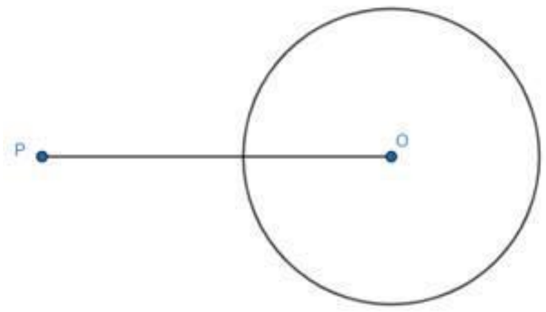
OR

Draw a line segment of 6 cm and divide it in the ratio 3:2.

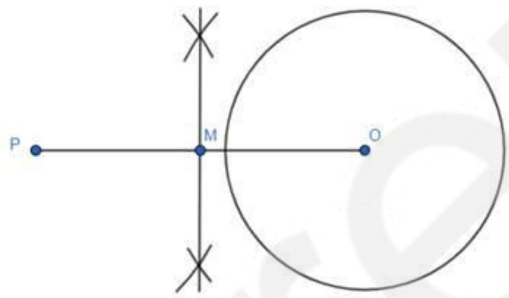
Answer: Steps of Construction:

1. Draw a line segment OP = 7 cm.

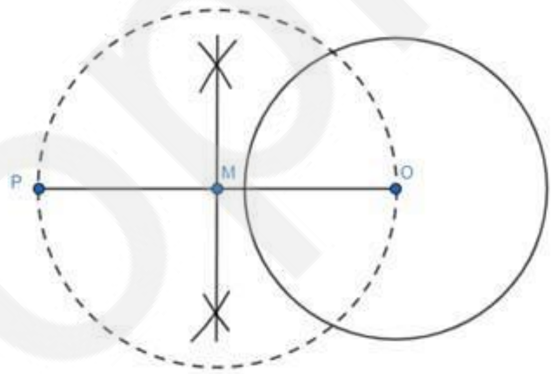
2. From the point O, draw a circle with radius = 4 cm.



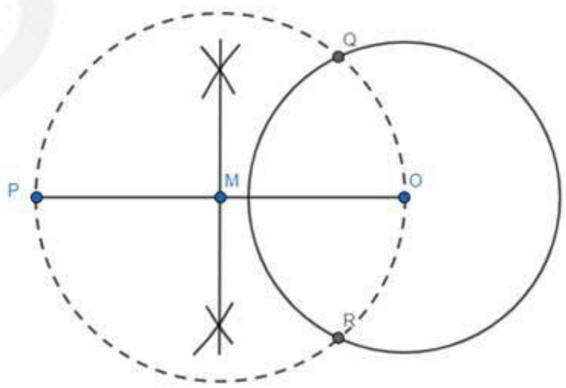
3. Draw a perpendicular bisector of OP. Let M be the mid-point of OP.



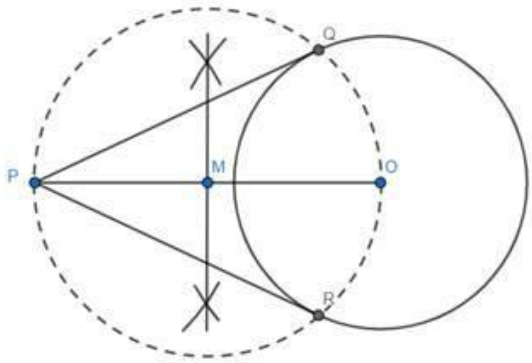
4. Taking M as center and OM as radius, draw a circle.



5. Let this circle intersect the given circle at the points Q and R.



6. Join PQ and PR.



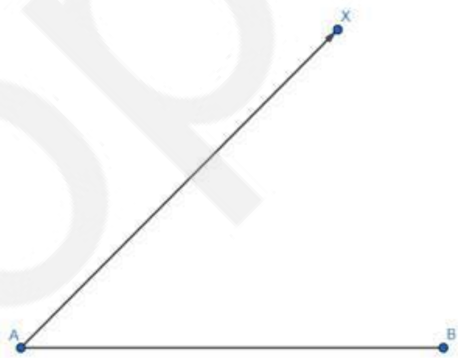
Here, PQ and PR are the required tangents.

OR

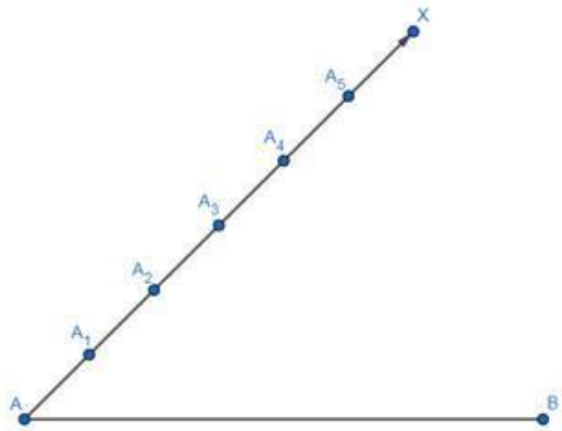
<u>Steps of Construction:</u>

1. Draw a line segment AB of length 8 cm.

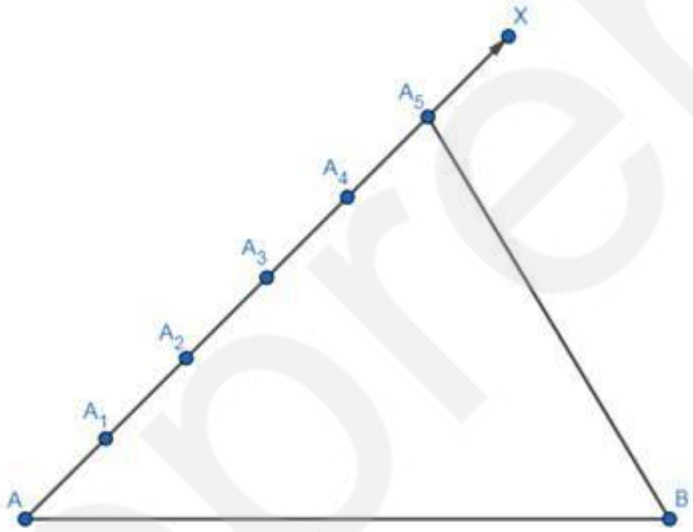
2. Draw any ray AX, making an acute angle with AB.



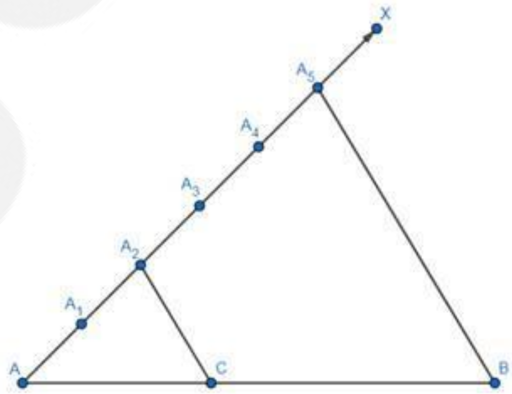
3. Mark 5 (2 + 3) points A_1, A_2, .., A_5 on AX such that $AA_1 = A_1A_2 = .. = A_4A_5$ by drawing equal arcs.



4. Join BA_5.



5. Since we want the ratio 2:3, through A_2, draw A_2C parallel to A_5B such that C lies on AB.



6. Thus AC:CB = 2:3

29. A solid metallic cuboid of dimension 24 cm × 11 cm × 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Answer: Let 'n' cones are formed

⇒ Volume of cuboid = n × volume of one cone

$$\Rightarrow 24 \times 11 \times 7 = n \times \frac{1}{3} \times \pi \times (3.5)^2 \times 6$$

$$\Rightarrow 24 \times 11 \times 7 = n \times \frac{22}{7} \times 3.5 \times 3.5 \times 2$$

$$\Rightarrow n = \frac{24 \times 11 \times 7}{22 \times 3.5}$$

⇒ n = 24 cones are formed.

30. Prove that (1 + tan A – sec A) × (1 + tan A + sec A) = 2 tan A

OR

Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Answer: Taking LHS

(1 + tan A – sec A)(1 + tan A + sec A)

Using $(a - b)(a + b) = a^2 - b^2$

$= (1 + \tan A)^2 - \sec^2 A$

$= 1 + \tan^2 A + 2\tan A - \sec^2 A$

$= \sec^2 A + 2\tan A - \sec^2 A$

= 2 tan A

= RHS

OR

Taking LHS

$$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$

$$= \frac{\operatorname{cosec}\theta(\operatorname{cosec}\theta + 1) + \operatorname{cosec}\theta(\operatorname{cosec}\theta - 1)}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$$

$$= \frac{\operatorname{cosec}^2\theta + \operatorname{cosec}\theta + \operatorname{cosec}^2\theta - \operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$$

$$= \frac{2\,\text{cosec}^2\,\theta}{\cot^2\theta}$$

$$= \frac{2}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} = \frac{2}{\cos^2\theta}$$

$= 2\sec^2\theta$

= RHS

31. Given that √3 is an irrational number, show that (5+2√3) is an irrational number.

OR

An army contingent of 612 members is too much behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer: Let 5 + √3 be a rational number then it can be written in the form

$5 + \sqrt{3} = \frac{p}{q}$

Where p and q are coprime integers.

$$\Rightarrow \sqrt{3} = \frac{p}{q} - 5 = \frac{p - 5q}{q}$$

(p – 5q) and 'q' are integers, therefore RHS is a rational number

⇒ √3 is a rational number which is a contradiction.

∴ our assumption is wrong and 5 + 2√3 is an irrational number.

OR

Suppose, both groups are arranged in 'n' columns, for completely filling each column,
The maximum no of columns in which they can march is the highest common factor of their number of members. i.e. n = HCF(612, 48)

Now,

612 = 48 × 12 + 36

48 = 36 × 1 + 12

36 = 12 × 3 + 0

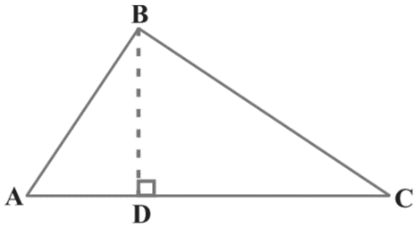
∴ n = HCF(612, 48) = 12

32. Prove that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Answer: Let us consider a right triangle ABC right-angled at B.

To Prove: $AB^2 + BC^2 = AC^2$

Construction: Draw BD ⊥ AC



In ΔADB and ΔABC, we have

∠BAD = ∠BAC [Common]

∠ABC = ∠ADB [Both 90°]

⇒ ΔADB ~ ΔABC [By AA similarity]

Sides of similar triangles are proportional, therefore

$$\frac{AD}{AB} = \frac{AB}{AC}$$

⇒ $AD.AC = AB^2$ [1]

In ΔBDC and ΔABC, we have

∠BCD = ∠BCA [Common]

∠ABC = ∠BDC [Both 90°]

⇒ ΔBDC ~ ΔABC [By AA similarity]

Sides of similar triangles are proportional, therefore

$$\frac{CD}{BC} = \frac{BC}{AC}$$

⇒ $CD.AC = BC^2$ [2]

Adding [1] and [2], we get

$AD.AC + CD.AC = AB^2 + BC^2$

⇒ $AC(AD + CD) = AB^2 + BC^2$

⇒ $AC.AC = AB^2 + BC^2$

⇒ $AC^2 = AB^2 + BC^2$

Hence, Proved!

Read the following passage carefully and then answer the equations given at the end.

33. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pot have been placed at a distance 1 m from each other along AD, as shown in Fig. 5. Niharika runs 1/4 th the distance AD on the 2^{nd} line and posts are green flag. Preet runs $1/5^{th}$ the distance AD on the eight line and posts a red flag.

(i) what is the distance between the two flags?

(ii) If Rashmi has two post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?

Fig. 5

Answer: Total distance = 100 m [100 flower points]

Niharika is on the second line and has run $\frac{1}{4} \times 100 = 25$ meters

⇒ coordinate of green flag = (2, 25)

Preet is on the eight line and has run $\frac{1}{5} \times 100 = 20$ meters

⇒ coordinate of green flag = (8, 20)

(i) Distance between them $= \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36+25} = \sqrt{61}$ units.

(ii) We have to find mid-point of line segment joining (2, 25) and (8, 20) i.e.

$$\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = (5, 22.5)$$

∴ she should post her flag at 22.5 m on the fifth lane.

34. Solve graphically: 2x +3y = 2, x − 2y = 8

Answer: Let's find two solutions for each equation

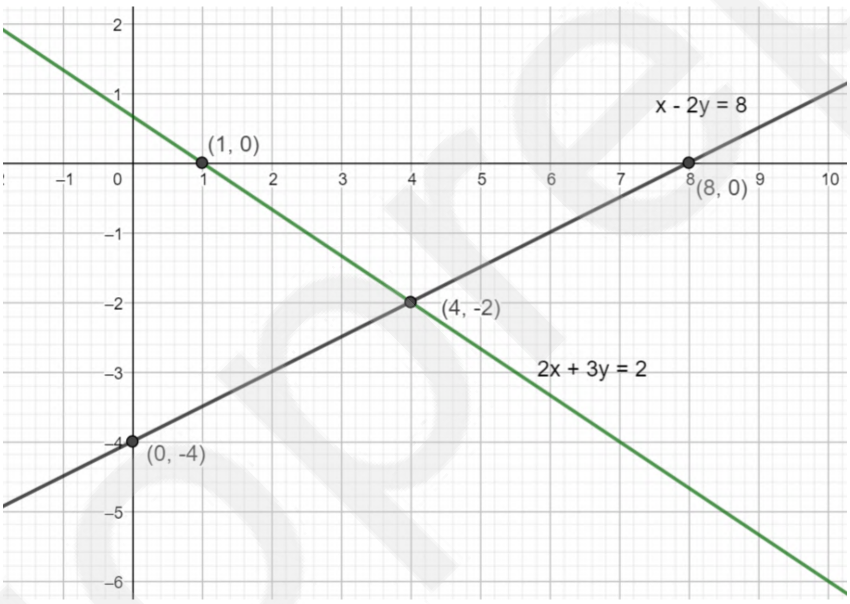
For equation x − 2y = 8

x	0	8
y	-4	0

For equation 2x + 3y = 2

x	1	4
y	0	-2

From graph, solution is (4, -2)



SECTION – D

Q. Nos. 35 to 40 carry 4 marks each.

35. A two-digit number is such that the product of its digit is 14. If 45 is added to the number; the digits interchange their places. Find the number.

Answer: Let the digit on units place be x & on that place be y.

∴ The original number = 10y + x and number with interchanged digits = 10x + y

A.T.Q

x.y=14..(i)

$(10y+x)+45=10x+y$

$\rightarrow 9x-9y=45$

Or $x-y=5$

$X=5+y$…(ii)

Substitute the value of x in (i)

$Y(5+y)=14$

$5y+y^2=14$

$Y^2+5y-14=0$

$Y^2+7y-2y-14=0$

$Y(y+7)-2(y+7)=0$

$(y+7)(y-2)=0$

$Y=-7$ and $y=2$

Neglecting $y = -7$

Now, put $y = 2$ in eq…(i)

$X(2) =14$

$X=7$

$\therefore$ The number $= 10y+x=10(2)+7=27$

36. If 4 times the 4th term of an AP is equal to 18 times the 18th term, then find the 22nd term.

OR

How many terms of the AP: 24, 21, 18, ….. must be taken so that their sum is 78?

Answer: Given $4a_4=18a_{18}$, $a_{22}=7$

$2(a+3d)=9(a+17d)$

$2a+6d=9a+153d$

$7a+197d=0$

$7(a+21d)=0$

$a+21d=0$

$\therefore a_{22}=0$

OR

Given: 24,21,18, ….s_n=78

A=24, d=21-24=-3

S_n=n/2 [2a+(n-1)d]

78=n/2 [2(24)+(n-1)(-3)]

156=n[48-3x+3]

156=n[51-3x]

156=51_n-3_n^2

$3x^2$-51n+156=0

X^2-17n+52=0

X^2-13n-4n+52=0

n(n-13)-4(n-13)=0

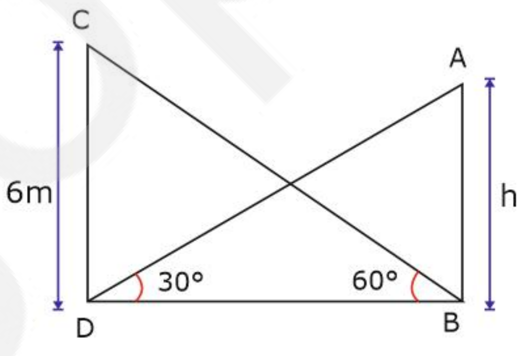
(n-13)(n-4)=0

n=13, n=4

37. The angle of elevation of the top a building from the foot of a tower is 30°. The angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Answer: Tower → CD

Building → AB



In ΔADB

$$\text{Tan}60° = \frac{AB}{DB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{DB} \Rightarrow DB = h\sqrt{3} .....(i)$$

In ΔCDB

$$\tan 60^\circ = \frac{CD}{DB}$$

$$\sqrt{3} = \frac{60}{DB} \Rightarrow DB = \frac{60}{\sqrt{3}} .....(ii)$$

From (i) & (ii)

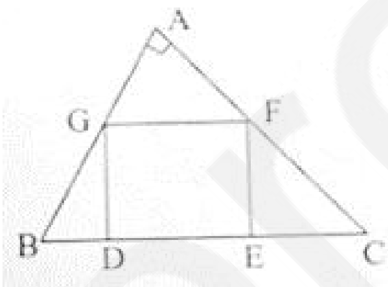
$$h\sqrt{3} = \frac{60}{\sqrt{3}}$$

$$h = \frac{60}{3} = 20$$

$$\boxed{h = 20m}$$

∴ Height of the building in 20m.

38. In DEFG is a square in a triangle ABC right angled at A.



Prove that

(i) ΔAGF ~ Δ DBG (ii) Δ AGF ~ Δ EFC

OR

In an obtuse Δ ABC (∠B is obtuse), AD is perpendicular to CB produced. Then prove that $AC^2 = AB^2 + BC^2 + 2BC \times BD$.

Answer: (i) Given: DEFG is a square and ∠BAC = 90°

To Prove: $DE^2 = BD \times EC$.

In Δ AGF and Δ DBG

∠GAF = ∠BDG [each 90°]

∠AGF = ∠DBG

[corresponding angles because GF|| BC and AB is the transversal]

∴Δ AFG ~ Δ DBG [by AA Similarity Criterion] ...(1)

(ii) In Δ AGF and Δ EFC

$\angle GAF = \angle CEF$ [each 90°]
$\angle AFG = \angle ECF$

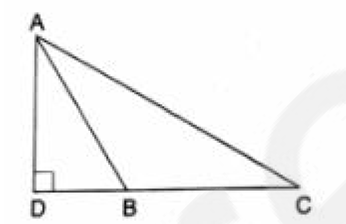
[corresponding angles because GF|| BC and AC is the transversal]

$\therefore \Delta$ AGF ~ Δ EFC [by AA Similarity Criterion] ...(2)

OR

Given: AD $\perp$ CB (produced)

To prove: $AC^2 = AB^2 + BC^2 + 2BC \bullet BD$



In Δ ADC, DC = DB + BC ....(i)
First, in Δ ADB,
Using Pythagoras theorem, we have
$AB^2 = AD^2 + DB^2 \Rightarrow AD^2 = AB^2 - DB^2$ ....(ii)
Now, applying Pythagoras theorem in Δ ADC, we have

$AC^2 = AD^2 + DC^2$
$= (AB^2 - DB^2) + DC^2$ [Using (ii)]
$= AB^2 - DB^2 + (DB + BC)^2$ [Using (i)]

Now, $\because (a + b)^2 = a^2 + b^2 + 2ab$

$\therefore AC^2 = AB^2 - DB^2 + DB^2 + BC^2 + 2DB \bullet BC$

$\Rightarrow AC^2 = AB^2 + BC^2 + 2BC \bullet BD$

39. An open metal bucket is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper ends are 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of Rs 40 per litre.

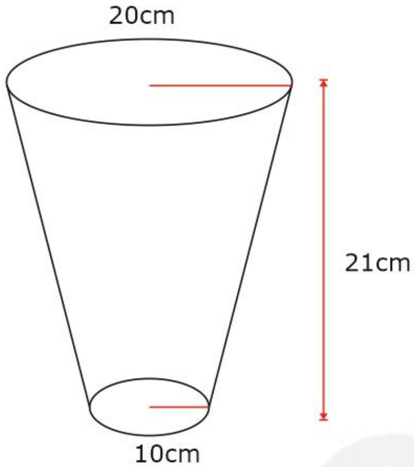
OR

A solid in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Answer: r_1=10cm

r_2=20cm

h=21cm



Vol. of socket =

$$\frac{1}{3}\pi(r_1^1 + r_2^2 + r_1 r_2)h$$

$$= \frac{1}{3} \times \frac{22}{7} \times [(10)^2 + (20)^2 + 10 \times 20] \times 21$$

$$= 22[100 + 400 + 200]$$

$$= 22 \times 700$$

$$= 15400 cm^2$$

$$= 15400 \times 10^{-6} \times 10^3 L = 15.4L$$

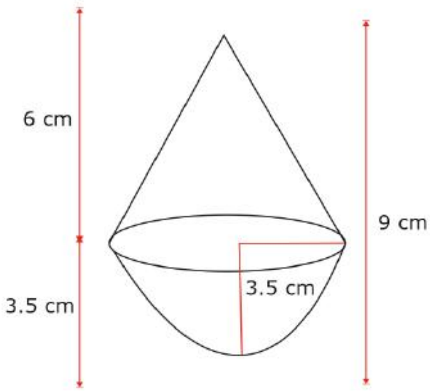
Cost of 1L of milk = ₹ 40

∴ Cost pr 15.4L of milk = 40× 15.4 =₹616

OR

R=3.5cm

H= 9.5-3.5=6cm



Vol. of the solid = vol. of hemisphere to vol. of cone.

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1}{3}\pi r^2[2r + h]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5[2(3.5) + 6]$$

$$= \frac{38.5}{3}[13]$$

$$= \frac{500.5}{3}$$

$$= 166.833 cm^3$$

40. Find the mean of the following data:

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	20	35	52	44	38	31

Answer:

Class	f_i	Xi	d:hi-a, a=50	$U_i = d_i/h$	f_iU_i
0-20	20	10	-40	-2	-90
20-40	35	30	-20	-1	-35
40-60	52	50	0	0	0
60-80	44	70	1	1	44
80-100	38	90	2	2	76
100-120	31	110	3	3	93
	∈fi=220				$\in f_iU_i$, vi=138

$$\bar{x} = a + h\left(\frac{\in fivi}{\in fi}\right)$$

$$\bar{x} = 50 + \left[\frac{138}{224}\right] \times 20$$

$$\bar{x} = 50 + \frac{69}{11}$$

$$\bar{x} = \frac{550 + 69}{11} = \frac{619}{11}$$

$$\boxed{\bar{x} = 56.27}$$

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