Mid-Term Examination, September, 2018

TECHNO MISSION SCHOOL

Samastipur-848101

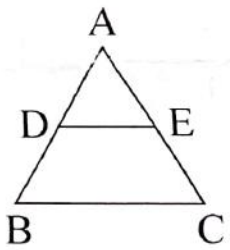
Class- Std X | Time- 3 Hours
Subject- Mathematics | F.M.- 80

General Instructions :

i. **All questions are compulsory.**
ii. **The questions paper consists of 30 questions divided into four sections A, B, C and D**
iii. **Use of calculator is not permitted.**

Section - A (6 marks)

1. State the fundamental theorem of Arithmetic.
2. Express $\sin 67^0 + \cos 75^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.
3. In the adjoing figure, DE || BC and AD = 1 cm, BD = 2 cm. What is the ratio of ΔABC? the area of to the area of ΔADE?



4. Write the number of solutions of the following pair of linear equations: $x + 2y - 8 = 0$, $2x + 4y = 16$
5. If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a.
6. If $\tan A = \frac{3}{4}$ and $A + B = 90^0$, then what is the value of $\cot B = ?$

Section - B (6 x 2 = 12 marks)

7.

P.T.O.

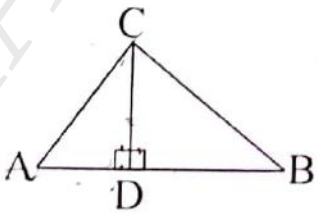
8. Check whether 6^n can end with the digit 0 for any natural number n.

9. Find the zeroes of the quafratic polynomial $4u^2 + 8u$ and verify the relationship between the zeroes and the co-efficients.

9. 5 years hence, the age of Jacob will be three times that of his son. 5 years ago, Jacob's age ws seven times that of his son. What are their present ages?

10. In the given fig. $\angle ACB = 90^\circ$ and $CD \perp AB$.

prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$



11. If A, B and C are interior angles of a ΔABC, then show that

$$\sec\left(\frac{A+B}{2}\right) = \operatorname{cosec}\frac{C}{2}$$

12. If $\sqrt{3}\tan\theta = 3\sin\theta$,, then find the value of $\sin^2\theta - \cos^2\theta$

Section - C (10 x 30 = 30 marks)

13. Use Euclid's divison lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

14. Solve the following system of equtions for x and y:
$(a - b)x + (a + b)y = a^2 + 2ab - b^2$
$(a + b)(x + y) = a^2 + b^2$

15. Represent the following system of liniar equations graphically. From the graph, find the points where the lines interest y-axis
$3x + y - 6 = 0$; $2x - y - 5 = 0$

16. Obtain all zeroes of $(3x^4 - 15x^3 + 13x^2 + 25x - 30)$; if two of its

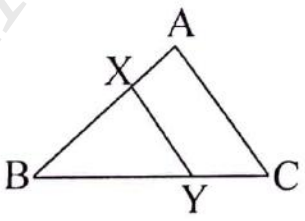
P.T.O.

zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

17. The diagonals of a quafrilateral ABCD intereset each other at the point O such that $\frac{AO}{OC} = \frac{BO}{OD}$. Show that ABCD is a traperium.

18. Prove that: $\frac{\tan\theta}{(1-\cot\theta)} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\theta$

19. In the given figure, the line segment xy is paralled to side AC of ΔABC and it divides the triangle into two parts of equal area. Find the ratio $\frac{AX}{AB}$



20. Find the mode of the following data.

Marks	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60	less than 70	less than 80
Students	3	8	24	36	49	69	75	80

21. In an acute angled ΔABC, if $\sin 2(A+B+C) = 1$, $\tan(B+C-A) = \sqrt{3}$, then find the values of A, B and C.

22. The sum of the zeroes of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero.

Prove that the product of zeroes of the equation is $\frac{-(a^2+b^2)}{2}$

Section - D **(8 x 4 = 32 marks)**

23. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides.

24. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another

P.T.O.

polynomial $x^2 - 2x + k$, the remainder comes out to be $(x + a)$, then find k and a.

25. In an equilateral ΔABC, D is a point on side BC, such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

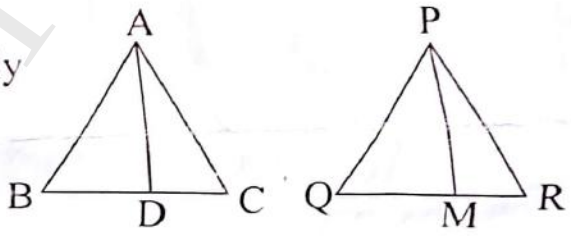
26. The table given below shows the frequency distribution of the scores obtained by 200 candidates in a BCA entrance examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of Candidates	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) less than type and (ii) more than type, Hence findthe median.

27. Sides AB and AC and median AD of a ΔABC are respectively proportional to sides PQ and PR and median PM of another ΔPQR.

Prove that $\Delta ABC \sim \Delta PQR$



28. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$ then prove that

$$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$$

29. A two digit number is obtained by either multiplying the sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number. How many such numbers are there.

P.T.O.

30. Evaluate the following:

(i) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$ $\frac{1}{\sqrt{2}}$

(ii) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \frac{x+b+x+a}{(x+a)(x+b)} = \frac{1}{c}$

$\frac{2x+a+b}{x^2+bx+ax+ab} = \frac{1}{c}$

$\Rightarrow x^2+bx+ax+ab = 2cx-ac-bc=0$

$\Rightarrow x^2+x(a+b-2c)+(ab-ac-bc)=0$

$\alpha+\beta = -\frac{a-b+2c=0}{1}$

$\therefore 2c = a+b$

$\alpha\beta = \frac{c}{a} = ab-ac-bc$

$= ab-a\left(\frac{a+b}{2}\right)-b\left(\frac{a+b}{2}\right)$

$= \frac{2ab-a^2-ab-ab-b^2}{2}$

$= -\frac{(a^2+b^2)}{2}$