Mathematics (Standard)- Theory

Time allowed: 3 hours **Maximum marks : 80**

General instruction

(i) This question paper comprise four sections – A, B, C and D this question paper carries 40 questions . All question are compulsory.

(ii) Section A: Q. No.1 to 20 comprises of 20 questions of one marks each.

(iii) Section B :Q. No.21 to 26 comprises of 6 questions of two marks each.

(iv) Section C :Q. No.27 to 34 comprises of 8 questions of three marks each.

(v) Section D :Q. No.35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper . However, an internal choice has been provided in 2 questions of one marks each, 2 questions of two marks each, 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choice in such questions.

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculators is not permitted.

Section A

Q. Nos. 1 to 10 are multiple choice type questions of 1 mark each. Select the correct option.

1. The HCF and the LCM of 12, 21,15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Answer: (c)

$12 = 2^2 \times 3$

$21 = 7 \times 3$ H.C.F. = 3

$15 = 5 \times 3$ L.C.M. = $2^4 \times 7 \times 5 = \times 3 = 420$

2. The value of x for which 2x, (x + 10) and (3x + 2) are the three consecutive terms of an AP, is 4

(a) 6 (b) –6

(c) 18 (d) –18

Answer: (a)

$2b = a + c$

$2(x + 10) = 2x + 3x + 2$

$18 = 3x \Rightarrow x = 6$

3. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

(a) −2 (b) ≠2

(c) 3 (d) 2

Answer: (b)

$x + y - y = 0$

$2x + 1c - 3 = 0$

$$\frac{1}{2} = \frac{1}{1c} \Rightarrow \frac{-4}{-3}$$

4. The first term of an AP is p and the common difference is q, then its 10^{th} term is

(a) $q + 9p$ (b) $p - 9q$

(c) $p + 9q$ (d) $2p + 9q$

Answer: (c)

10^{th} term $= p + (10 - 1)q = p + 9q$

5. The quadratic polynomial, the sum of whose zeroes is −5 and their product is 6, is

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Answer: (a)

$\alpha + \beta = -5$, $\alpha\beta = 6$

$x^2 + 5x + 6 = 0$

6. The distance between the points $(a \cos\theta + b \sin\theta, 0)$ and $(0, a \sin\theta - b \cos\theta)$, is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

Answer: (c)

$$d = \sqrt{(a\cos 0 + b\sin\theta)^2 + (a\sin 0 - b\cos\theta)} = \sqrt{a^2 + b^2}$$

7. The total number of factors of a prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Answer: (c)

Correct option is (c)

8. If the point P(k, 0) divides the line segment joining the points A(2, −2) and B(−7, 4) in the ratio 1 : 2, then the value of k is

(a) 1 (b) 2

(c) −2 (d) −1

Answer: (d)

Diagram

$$K = \frac{1\times 7 + 2\times 2}{1+2} = \frac{-3}{3} = -1$$

9. The value of p, for which the points A(3,1), B(5, p) and C(7, —5) are collinear, is

(a) −2 (b) 2

(c) −1 (d) 1

Answer: (b)

A(3, 1), B(5, P) & C(7, −5)

10. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) −10

(c) −7 (d) −2

Answer: (b)

$x^2 + 3x + K = 0$

$\alpha = 2, \beta$

$2 \times \beta = -3$

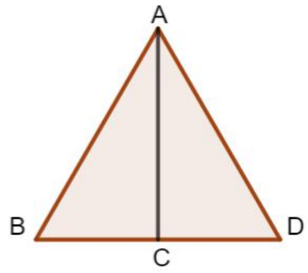
$2 + \beta = -3$

$K = -1 \Leftarrow \frac{K}{2} = -3$

In Q. No, s. 11 to 15, fill in the blanks. Each question is of 1 m.

11. ABC is an equilateral triangle of side 2a, then length of one of its altitude is____.

Answer: $\sqrt{3}a$



We have,
AB = 2a
BC = ½ × 2a = a
In ΔACB, By Pythagoras theorem
$AB^2 = AC^2 + BC^2$
$\Rightarrow (2a)^2 = AC^2 + a^2$
$\Rightarrow 4a^2 = AC^2 + a^2$
$\Rightarrow AC^2 = 3a^2$
$\Rightarrow AC = \sqrt{3}\, a$

12. In Fig. 1, ΔABC is circumscribing a circle, the length of BC is____cm.

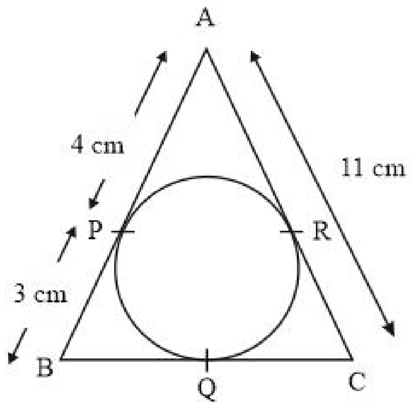


Fig. 1

Answer: 10 cm
AP = AR = 4 cm
CR = AC – AR = 11 – 4 = 7 cm
CR = QC = 7 cm
BQ = PB = 3 cm
BC = BQ + PB
3 + 7 = 10

13. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$ ______.

OR

The Value of $(1+\tan^2\theta)\,(1-\sin\theta)\,(1+\sin\theta) =$ _____.

Answer: 1

$$\sin^2\theta + \frac{1}{\sec^2\theta} = 1$$

∵ $\sin^2\theta + \cos^2\theta = 1$

OR

$(\sec^2\theta)(1 - \sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

14. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right) - 2\cos 60° =$ ______.

Answer: 1

$$\frac{\sin^2 35°}{\cos^2 55°} + \frac{\cos^2 43°}{\sin^2 47°} - 2\times\frac{1}{2} = 1 + 1 - 1 = 1$$

15. ABC and BDE are two equilateral triangles such the D is the mid-point of BC. Ratio of the areas of triangles ABC and ADE is_____.

Answer: 4

$$\frac{\text{area } \Delta ABC}{\text{ar}(\Delta BDE)} = \frac{\frac{S_3}{4}a^2}{\frac{S_3}{4}\left(\frac{A}{2}\right)^2} = 4$$

Q. Nos. 16 to 20 short answer type questions of 1 mark each.

16. A die is thrown once. What is probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Answer: $\frac{1}{3}$

S = {1, 2, 3, 4, 5, 6}

P(Number of less than 3)= $\frac{2}{6} = \frac{1}{3}$

OR

93

Probability of losing = 1 – Probability of winning

$= 1 - 0.07 = 1 - \frac{7}{100} = \frac{93}{100} = .93$

17. If the mean of the first n natural number is 15, then find n.

Answer: 29

$$\frac{1+2+-x}{x}=15 \Rightarrow \frac{x(x+1)}{x^2}=15$$

x + 1 = 30

x = 29

18. Two ones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Answer: 3

$$V_1=\frac{1}{3}\pi(3r)^2h$$

$$V_2=\frac{1}{3}\pi r^2 3h=3$$

19. Find the angle of elevation of the sun at that moment?

The ratio of the length of a vertical rod and the length of its shadow is $1:\sqrt{3}$.

Answer: 30°

$$\tan\theta=\frac{1}{\sqrt{3}}$$

θ = 30°

20. A die is thrown once. What is the probability of getting an even prime number?

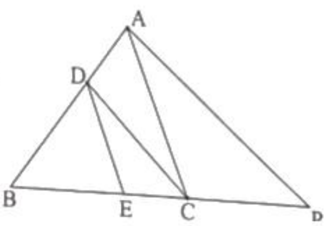
Answer: $\frac{1}{6}$

S = {1, 2, 3, 4, 5, 6}, E = {2}

Probability = $\frac{1}{6}$

21. In Fig. 2 DE || AC and DC || AP. Prove that $\frac{BE}{EC}=\frac{BC}{CP}$

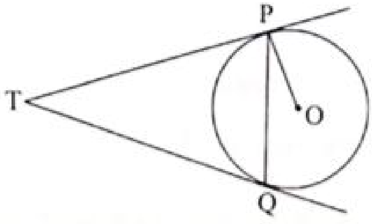
Fig. 2



OR

In Fig, 3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.

Fig. 3



Solution:

∵ DE || AC, By basic proportionality theorem

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$ [1]

Also, DC || AP

$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP}$ [2]

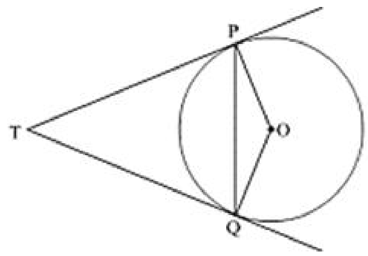
From [1] and [2], we get

$\Rightarrow \frac{BE}{EC} = \frac{BC}{CP}$

Hence, proved!

OR

Let TP and TQ are two tangents of a circle at points P and Q respectively with center O.



To prove: ∠PTQ = 2∠OPQ

Let ∠PTQ = θ

As lengths of tangents drawn from an external point to the circle are equal, therefore TP = TQ.

∴ ΔPQT is an isosceles triangle.

∴ ∠TPQ = ∠TQP = ½(180° - θ) = 90° – ½θ

Also, tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ ∠OPQ = ∠OPT – ∠TPQ = 90° – (90°– ½θ)

= ½θ

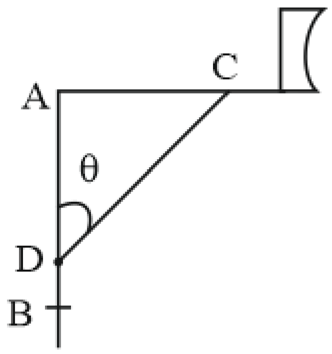
= ½∠PTQ

Thus, ∠PTQ = 2∠OPQ.

Hence, proved.

22. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in fig. 4. If AC = 1.5 m long and CD = 3 m, find (i) tan θ (ii) sec θ + cosec θ.

Fig. 4



Solution:

We have,

$$\sin\theta = \frac{Perpendicular}{Hypotenuse} = \frac{AC}{CD}$$

⇒ sin θ = 1.5/3 = ½

⇒ θ = 30°

(i) $\tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$

(ii) $\sec\theta + \text{cosec}\,\theta = \sec 30° + cosec\ 30° = \frac{2}{\sqrt{3}} + 2$

$$= 2\left(\frac{1}{\sqrt{3}} + 1\right) = \frac{2(1+\sqrt{3})}{\sqrt{3}}$$

23. If a number x is chosen at random from the numbers -3, -2, -1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Solution:

Total number of outcomes = 7

The numbers for which $x^2 \leq 4$ are -1, -2, 0, 1, 2

Favorable outcomes = 5

$$P(x^2 \leq 4) = \frac{5}{7}$$

24. Find the mean of the following distribution:

Class:	3-5	5-7	7-9	9-11	11-13
Frequency:	5	10	10	7	8

OR

Find the mode of the following data:

Class:	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency:	6	8	10	12	6	5	3

Solution:

Class	**Frequency (f_i)**	**Class Mark (x_i)**	**$f_i x_i$**
3-5	5	4	20
5-7	10	6	60
7-9	10	8	80
9-11	7	10	70
11-13	8	12	96
	$\sum f_i = 40$		$\sum f_i x_i = 323$

Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{323}{40} = 8.075$

OR

Class	**Frequency**
0-20	6
20-40	8
40-60	10
60-80	12
80-100	6
100-120	5
120-140	3

Model class = 60-80
Lower limit of model class, l = 60
Frequency of model class, $f_1 = 12$
Frequency of class preceding model class, $f_0 = 10$
Frequency of class following model class, $f_2 = 6$
Height of model class, h = 10
We know,

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{24 - 10 - 6} \times 10$$

= 60 + 0.4
= 60.4

25. Find the sum of first 20 terms of the following AP :
1,4,7,10,_____ .

Solution:

We have,
First term, a = 1
Common difference, d = 3

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Sum of 20 terms

$$S_{20} = 10\big(2(1) + 19(3)\big)$$

= 10(2 + 57)
= 590

26. The perimeter of a sector of a circle of radius 5.2 cm is 16.4cm. Find the area of the sector.

Solution:

Perimeter of sector $= 2r + \frac{\theta}{180}\pi r$

We have to find $\frac{\theta}{360}\pi r^2$

$$\Rightarrow 16.4 = 2(5.2) + \frac{\theta}{180}\pi r$$

$$\Rightarrow 16.4 - 10.4 = \frac{\theta}{180}\pi r$$

$$\Rightarrow 6 = \frac{\theta}{180}\pi r$$

Multiplying both side by (r/2), we have

$$\Rightarrow \frac{6r}{2} = \frac{\theta}{360}\pi r^2$$

$$\Rightarrow Area\ of\ sector = \frac{6(5.2)}{2} = 15.6\ \text{cm}^2$$

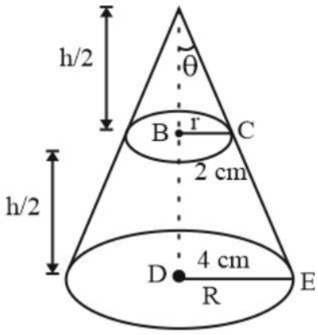
Section-C

Q. Nos. 27 to 34 carry 3 marks each.

27. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-points of its height and parallel to its base. Compare the volume of the two parts.

Solution:

volume of the cone = $1/3\pi r^2 h$, r = radius, h = Height)



When cone is divided in two parts , upper past ids cone and bottom part is frustum .

In ΔABC, tanθ = BC/AB

In ΔADE

$$\tan\theta = \frac{DE}{AD}$$

$$\Rightarrow \frac{BC}{AB} = \frac{DE}{AD}$$

$$\Rightarrow \frac{BC}{\frac{\cancel{h}}{2}} = \frac{4}{\cancel{h}}$$

$$\Rightarrow BC = \frac{4}{2} = 2cm$$

$$\frac{\text{Volume of upper part}}{\text{Volume of lower part}} = \frac{\frac{1}{3}\pi r^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi\left(\frac{h}{2}\right)\left(r^2 + R^2 + rR\right)}$$

$$= \frac{r^2}{r^2 + R^2 + rR} = \frac{2^2}{2^2 + 4^2 + 2(4)}$$

$$= \frac{4}{4 + 16 + 8}$$

$$= \frac{1}{7}$$

28. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

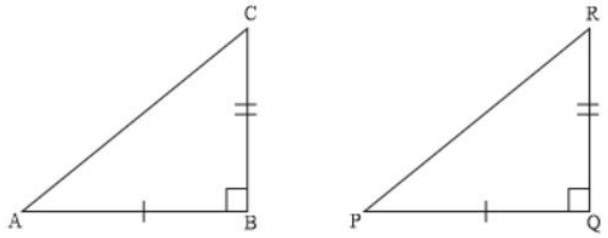
Solution:

Consider ΔABC such that $AC^2 = AB^2 + BC^2$

To prove : ∠B = 90°

Construction : We construct another ΔPQR right angled at Q such that PQ = AB and QR = BC

In ΔPQR , by Pythagoras theorem, we have



$PR^2 = PQ^2 + QR^2$

$\Rightarrow PR^2 = AB^2 + BC^2$

$\Rightarrow PR^2 = AC^2 \quad \left[\because AC^2 = AB^2 + BC^2\right]$

$\Rightarrow PR = AC$

In Δ ABC & ΔPQR

AB = PQ [by construction]

BC = QR [by construction]

AC = PR [Proved above]

$\Delta ABC \cong \Delta PQR$

∠B = ∠Q = 90° [by CPCT]

Hence proved

29. Find the area of triangle PQR formed by the points P(-5,7), Q(-4,-5) and R(4,5)

OR

If the point C(-1,2) divides internally the line segment joining A(2,5) and B(x, y) in the ratio 3: 4, find the coordinates of B.

Solution:

Area of triangle

$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$\frac{1}{2}\left|-5(-5-5) + (-4)(5-7) + 4(7+5)\right|$

$\frac{1}{2}\left|+50 + 8 + 48\right|$

$= 25 + 4 + 24$

$= 53 \text{sq.units}$

OR

$\left(x_1 y\right) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$(-1, 2) = \frac{3(x) + 4(2)}{3+4} \quad , \quad \frac{3y + 4(5)}{3+4}$

$\frac{3x+8}{7}=-1$

$\Rightarrow 3x+8=-7$

$\Rightarrow 3x=-15$

$\Rightarrow x=-5$

$\frac{3y+20}{7}=2$

$\Rightarrow 3y+20=14$

$\Rightarrow 3y=-6$

$\Rightarrow y=-2$

(-5,-2)

30. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x)=a x^2+bx + c$, $a \neq 0$, $c \neq 0$

OR

Divide the polynomial $f(x)=3 x^2-x^3-3 x+5$ by the polynomial $g(x)=x-1-x^2$ and verify the division algorithm.

Solution: $ax^2 + bx + c$, let its zeroes be α and β we have to find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

Now , to form equation we need to find $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$ and $\left(\frac{1}{\alpha\beta}\right)$

∴ Required quadratic equation will be

$$x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\frac{1}{\alpha\beta} \qquad \text{(i)}$$

Now,

$\alpha+\beta=\frac{-b}{a}, \alpha\beta=\frac{c}{a}$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta} \Rightarrow \frac{1}{\alpha\beta}=\frac{a}{c}$

$=\frac{\frac{-b}{a}}{\frac{c}{a}}=\frac{-b}{c}$

$\therefore \text{required equation} = x^2+\frac{b}{c}x+\frac{a}{c}$

$=\frac{1}{c}\left(cx^2+bx+a\right)$

$$g(x) = x - 1 - x^2$$
$$= -x^2 + x - 1$$
$$f(x) = 3x^2 - x^3 - 3x + 5$$
$$= -x^3 + 3x^2 - 3x + 5$$

$$-x^2 + x - 1 \overline{) -x^3 + 3x^2 - 3x + 5}$$
$$-x^3 + x^2 - x$$
$$(+) \quad (-) \quad (+)$$
$$\overline{\qquad\qquad}$$
$$2x^2 - 2x + 5$$
$$2x^2 - 2x + 5$$
$$(-) \quad (+) \quad (-)$$
$$\overline{\qquad\qquad}$$
$$3$$

According to division algorithm

F(x) = g(x). q(x) + r (x)

q(x) = $-x^2 + x - 1$

q(x) = x-2

r(x) = 3

RHS = $(-x^2+x-1)(x-2)+3$

= $-x^3+2x^2+x^2-2x-x+2+3$

= $-x^3 + 3x^2 - 3x + 5$

= LHS

Hence verified

31. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by 2 y-x=8,5 y-x=14 and y-2 x=1

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Solution:

2y -x=8

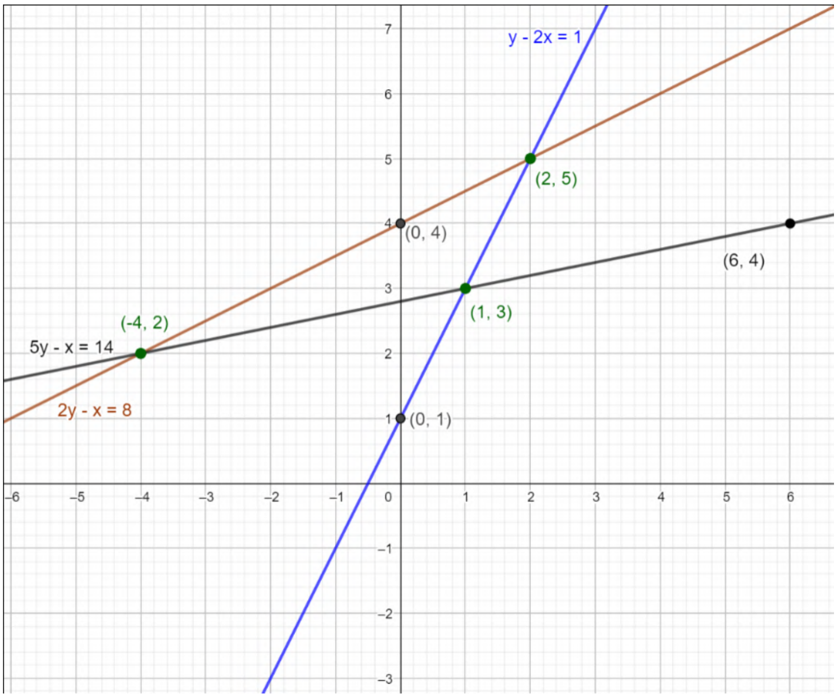
x	0	2
y	4	5

5y-x =14

x	1	6
y	3	4

y-2x = 1

x	0	1
y	1	3



Vertices of triangle are (2,5) , (1,3) and (-4,2)

OR

$x = 4$ is a zero of $(x) = x^3 - 3x^2 - 10x + 24$

$(x-4)$ is a factor of $x^3 - 3x^2 - 10x + 24$

Now

$x^3 - 3x^2 - 10x + 24$

$= x^3 - 4x^2 + x^2 - 4x - 6x + 24$

$= x^2(x-4) + x(x-4) - 6(x-4)$

$= (x-4)(x^2 + x - 6)$

$= (x-4)(x^2 + 3x - 2x - 6)$

$= (x-4)\{x(x+3) - 2(x+3)\}$

$= (x-4)(x-2)(x+3)$

$\therefore$ Zero are x = 4, x = 2 and x = -3

32. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Solution:

Let the speed of train be 'x' km/h.

Time taken to cover 480 km $= \frac{480}{x}$ [$\therefore$ time = $\frac{\text{distance}}{\text{Speed}}$]

Given,

Time taken with speed (x – 8) = Time taken originally + 3 hours

$$\frac{480}{x-8} = \frac{480}{x} + 3$$

$$\Rightarrow 480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 3$$

$$\Rightarrow 160\left(\frac{x-x+8}{x(x-8)}\right) = 3$$

$$\Rightarrow 160 \times 8 = x^2 - 8x$$

$$\Rightarrow x^2 - 8x - 1280 = 0$$

$$\Rightarrow x^2 - 40x + 32x - 1280 = 0$$

$$\Rightarrow x(x-40) + 32(x-40) = 0$$

$$\Rightarrow (x-40)(x+32) = 0$$

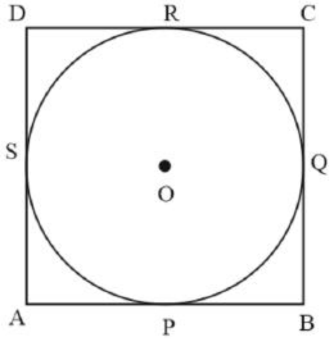
$\Rightarrow x = 40$ or x = – 32 [Negative speed not possible]

$\therefore$ Speed = 40 km/h.

33. Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Consider a parallel gram ABCD circumscribing a circle with center O.



To Prove: ABCD is a rhombus or AB = BC = CD = AD

We know, tangents from an external point to a circle are equal

AP = AS

BP = BQ

DR = DS

CR + CQ

Adding above equations, we get

AP + BP + CR + DR = AS + DS + BQ + CQ

AB + CD = AD + BC

Now, In a parallelogram AB = CD and BC = AD

$\Rightarrow AB + AB = BC + BC$

$\Rightarrow AB = BC$

$\Rightarrow AB = BC = CD = AD$

Hence, Proved!

34. Prove that : $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$.

Solution: $2\left(\sin^6\theta + \cos^6\theta\right) - 3\left(\sin^4\theta + \cos^4\theta\right) + 1 = 0$

We have,

$\sin^4\theta + \cos^4\theta = \left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

$\sin^6\theta + \cos^6\theta = \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta\right)$

$= \left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right)$

$= \left(1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right)$

$= \left(1 - 3\sin^2\theta\cos^2\theta\right)$

$\therefore$ LHS = $2\left(1 - 3\sin^2\theta\cos^2\theta\right) - 3\left(1 - 2\sin^2\theta\cos^2\theta\right) + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

= 0

=RHS.

Section D

Q. Nos. 35 to 40 carry 4 marks each.

35. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village :

Production yield/hect.	40-45	45-50	50-55	55-60	60-65	65-70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

OR

The median of the following data is 525. Find the values of x and y, if total

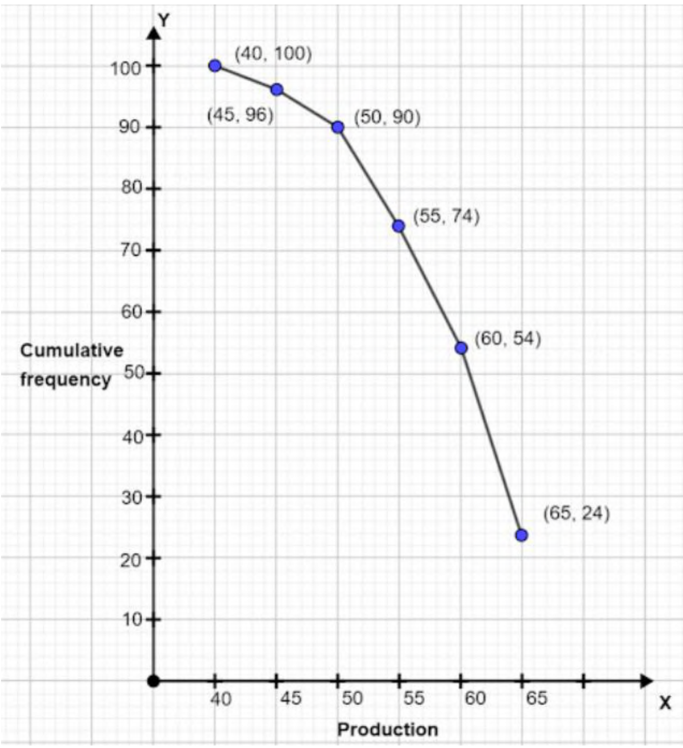
Class:	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Frequency:	2	5	x	12	17	20	y	9	7	4

Solution:

Production	**No. of Farms**
40 - 45	4
45 – 50	6
50 – 55	16
55 – 60	20
60 – 65	30
65 - 70	24

Production yield (More than type)	C.F
More than 40	100
More than 45	100 – 4 = 96
More than 50	96 – 6 = 90
More than 55	90 – 16 = 74
More than 60	74 – 20 = 54
More than 65	54 – 30 = 24

Plot points (40, 100) (45, 96) (50, 90) (55, 74) (60,54) (65, 24) on a graph



OR

Class-interval	Frequency	Cumulative Frequency
0 —100	2	2
100 — 200	5	2 + 5=7
200 —300	x	7 + x
300 — 400	12	7 + x + 12=19 + x
400 — 500	17	19 + x + 17=36 + x (F)
500 — 600	20(f)	36 + x + 20=56 + x
600 — 700	y	56 + x + y
700 —800	9	56 + x + y + 9=65 + x + y
800 — 900	7	65 + x + y + 7=72 + x + y
900 —1000	4	72 + x + y + 4=76 + x + y

Given Median =525

Then, median Class = 500-600
the lower limit (l) = 500
cumulative frequency of the class preceding 500-600(cf) = 36 + x
frequency of the median class 500-600 = 20,
class size (h) = 100
Total frequencies (n) = 100
So, 76 + x + y = 100
⇒ x + y = 100 − 76
⇒ x + y = 24 …(i)
and $\frac{n}{2} = \frac{100}{2} = 50$

Using the formula, $\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$,we have

$525 = 500 + \frac{50-(36+x)}{20} \times 100$

$\Rightarrow 525 - 500 = \frac{14-x}{20} \times 100$

⇒ 25 = (14 − x) × 5
⇒5 = 14 − x
⇒ x = 9
Putting the value of x in eq. (i), we get
⇒ 9 + y = 24
⇒ y = 24 − 9
⇒ y = 15

36. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take √3 = 1.73)

Solution:

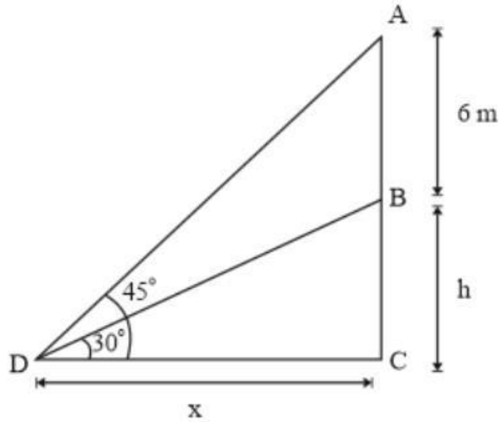
AB → Flag staff
BC → Tower of height (h)
DC → km (say)
In Δ BDC

$$\tan 30^o = \frac{BC}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$x = h\sqrt{3}$ --(i)

In ΔADC



$\tan 45^{O} = \frac{AC}{DC}$

$1 = \frac{AC}{DC}$

$1 = \frac{AB + BC}{DC}$

$1 = \frac{6 + h}{x}$

X = 6 + h --(ii)

From Eq. (i) and (ii)

$h\sqrt{3} = 6 + h$

$h(\sqrt{3} - 1) = 6$

$h = \frac{6}{\sqrt{3} - 1} \text{ or } \frac{6}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{6(\sqrt{3} + 1)}{2}$

$h = 3(\sqrt{3} + 1)$

37. Show that the square of any positive integer cannot be of the form (5q+2) or (5 q+3) for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3 .

Solution:

By Euclid's division algorithm, By Euclid's Lemma, $b = a \times q + r, 0 \le r < a$

Here, b is any positive integer.

Let a be an arbitrary positive integer. Then corresponding to the positive integers a and 5, there exist non-negative integers m and r such that

$a = 5m + r$, where $0 \leq r < 5$

Squaring both the sides using $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow a^2 = (25m^2 + r^2 + 10mr)$

$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2$

Where $0 \leq r < 5$

Case I When $r = 0$ we get

$a^2 = 5(5m^2)$

$\Rightarrow a^2 = 5q$

where $q = 5m^2$ is an integer.

Case II When $r = 1$ we get

$a^2 = 5(5m^2 + 2m) + 1$

$\Rightarrow a^2 = 5q + 1$

where $q = (5m^2 + 2m)$ is an integer.

Case III When $r = 2$ we get

$\Rightarrow a^2 = 5(5m^2 + 4m) + 4$

$\Rightarrow a^2 = 5q + 4$

Where $q = (5m^2 + 4m)$ is an integer.

Case IV When $r = 3$ we get

$\Rightarrow a^2 = 5(5m^2 + 6m) + 9 = 5\ (5m^2 + 6m) + 5 + 4$

$\Rightarrow a^2 = 5(5m^2 + 6m + 1) + 4 = 5q + 4$

where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V when $r = 4$ we get

$\Rightarrow a^2 = 5(5m^2 + 8m) + 16 = 5\ (5m^2 + 8m) + 15 + 1$

$\Rightarrow a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$

where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.

OR

By Euclid's division algorithm, By Euclid's Lemma, $b = a \times q + r$, $0 \leq r < a$

Here, b is any positive integer.

Let a be an arbitrary positive integer. Then corresponding to the positive integers a and 5, there exist non-negative integers m and r such that

$a = 5m + r$, where $0 \leq r < 5$

Squaring both the sides using $(a + b)^2 = a^2 + 2ab + b^2$

$\Rightarrow a^2 = (25m^2 + r^2 + 10mr)$

$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2$

Where $0 \le r < 5$

Case I When r = 0 we get

$a^2 = 5(5m^2)$

$\Rightarrow a^2 = 5q$

where $q = 5m^2$ is an integer.

Case II When r = 1 we get

$a^2 = 5(5m^2 + 2m) + 1$

$\Rightarrow a^2 = 5q + 1$

where $q = (5m^2 + 2m)$ is an integer.

Case III When r = 2 we get

$\Rightarrow a^2 = 5(5m^2 + 4m) + 4$

$\Rightarrow a^2 = 5q + 4$

Where $q = (5m^2 + 4m)$ is an integer.

Case IV When r = 3 we get

$\Rightarrow a^2 = 5(5m^2 + 6m) + 9 = 5\ (5m^2 + 6m) + 5 + 4$

$\Rightarrow a^2 = 5(5m^2 + 6m + 1) + 4 = 5q + 4$

where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V when r = 4 we get

$\Rightarrow a^2 = 5(5m^2 + 8m) + 16 = 5\ (5m^2 + 8m) + 15 + 1$

$\Rightarrow a^2 = 5(5m^2 + 8m + 3) + 1 = 5q + 1$

where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

38. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7: 15 . Find the numbers.

OR

Solve: 1+4+7+10+……+x =287

Solution:

Let the four conductive terms of AP be

(a - 3d), (a - d) , (a + d) , (a + 3d)

Case I : a − 3d + a − d + a + d + a + 3d = 32

4a = 32

a = 8

CASE II :

$$\frac{(a-3d)(a+3d)}{(a-d)}=\frac{7}{15}$$

$$\frac{(8-3d)(8+3d)}{(8-d)(8+d)}=\frac{7}{15}$$

$$\frac{64-9d^2}{64-d^2}=\frac{7}{15}$$

$$15(64-9d^2)=7(64-d^2)$$

$$960-135d^2=448\,7d^2$$

$$960-448=135d^2-7d^2$$

$$512=128d^2$$

$$\Rightarrow d^2=\frac{512}{128}=4$$

$$d=\pm 2$$

When d = 2 & a = 8

a – 3d = 8 – 3(2) = 2

a – d = 8 – 2 = 6

a + d = 8 + 2 =10

a + 3d = 8 + 3(2) = 14

when d = -2 & a = 8

a - 3d = 8 – 3(-2) = 14

a - d = 8 – (-2) = 10

a + d = 8 + (-2) = 6

a + 3d = 8 + 3(-2) = 2

OR

Let the number of terms be ‘n’

Given, sum = 287

$\Rightarrow \frac{n}{2}(2a+(n-1)d)=287$

$\Rightarrow \frac{n}{2}(2+(n-1)3)=287$

⇒ n(2 + 3n – 3) = 574

⇒ n(3n – 1) = 574

⇒ $3n^2 - n - 574 = 0$

⇒ $3n^2 - 42n + 41n - 574 = 0$

⇒ 3n(n – 14) + 41(n – 14) = 0

⇒ (3n + 41)(n – 14) = 0

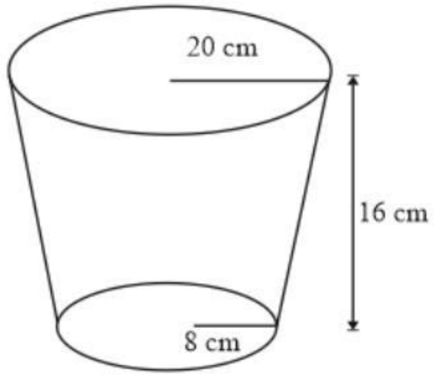
⇒ n = 14 or -41/3 [Not possible]

Now, 14^{th} term = a + 13d = 1 + 13(3) = 1 + 39 = 40

39. A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre. (Use π =3.14)

Solution:

r_1 = 8cm , r_2 = 200 cm, h = 16cm



Volume of bucket = $\frac{1}{3}\pi(r_1^2 + r_2^2 + r_1 r_2)h$

$\frac{1}{3} \times 3.14 \times \left[(8)^2 + (20)^2 + (8 \times 20)\right]16$

$\frac{1}{3} \times 3.14\left[64 \times 400 \times 160\right] \times 16$

= 10,449.92 cm^3

= 10,449.92 × 10^{-6} × m^3 $\left\{\because 1m^3 = 10^6 cm^3\right\}$

= 10,449.92 × 10^{-6} × 10^3 L $\left\{\because 1m^3 = 10^3 L\right\}$

= 10, 449.92 × 10^{-3} L

= 10. 449 L

Now , Cost of filling the milk = Rs 40 /L

∴ Cost of filling 10.449 L of milk = 10.449 × 40

= Rs 417.96

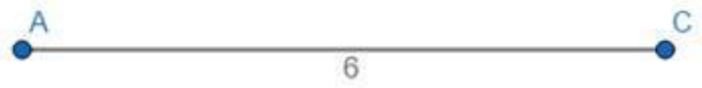
= Rs 418 (approx.)

40. Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are 2/3 times the corresponding sides of the first triangle.

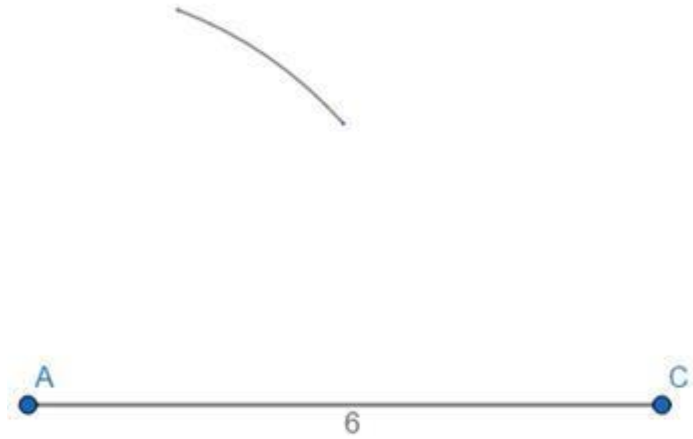
Solution:

Let the triangle with sides 4 cm,5 cm and 6 cm be ΔABC

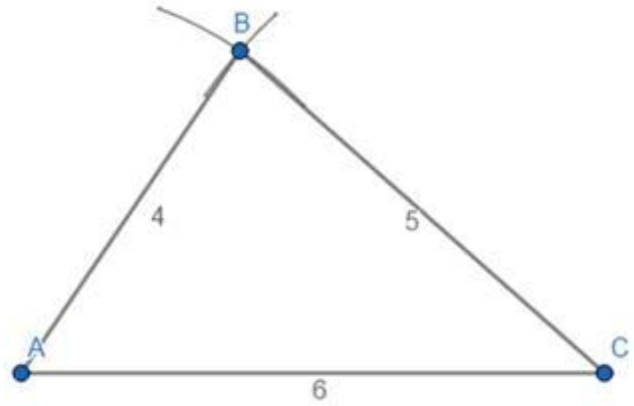
Step1: construct segment AC of 6 cm



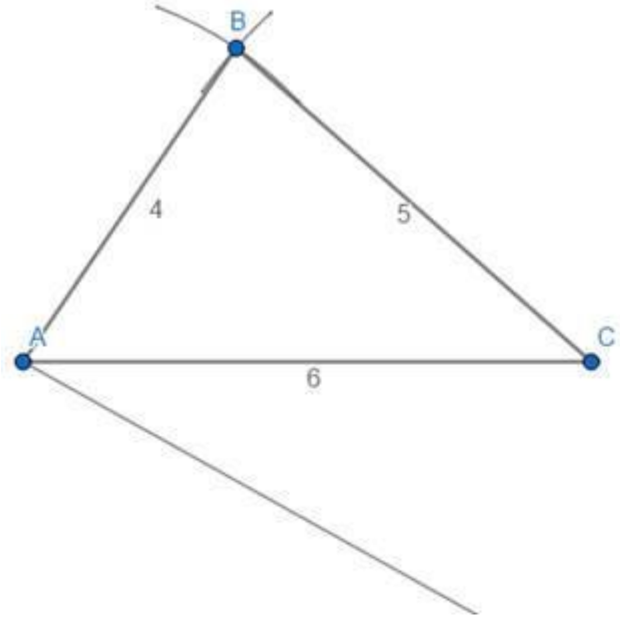
Step2: take distance 4 cm in compass keep the needle of the compass on point A and mark an arc above AC



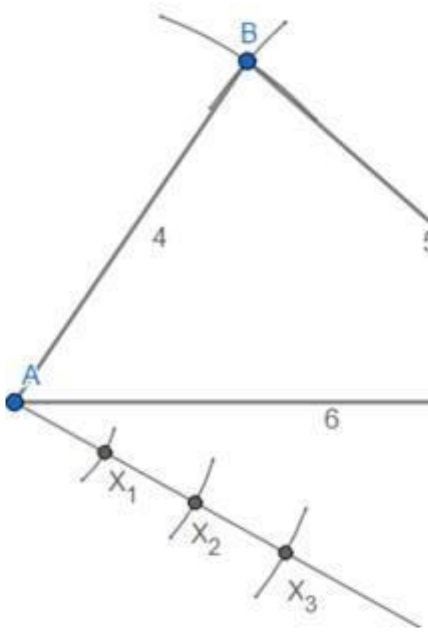
Step3: take distance 5 cm in compass keep the needle of the compass on point C and mark an arc intersecting the arc drawn in step2. Mark intersection point as B join AB and AC



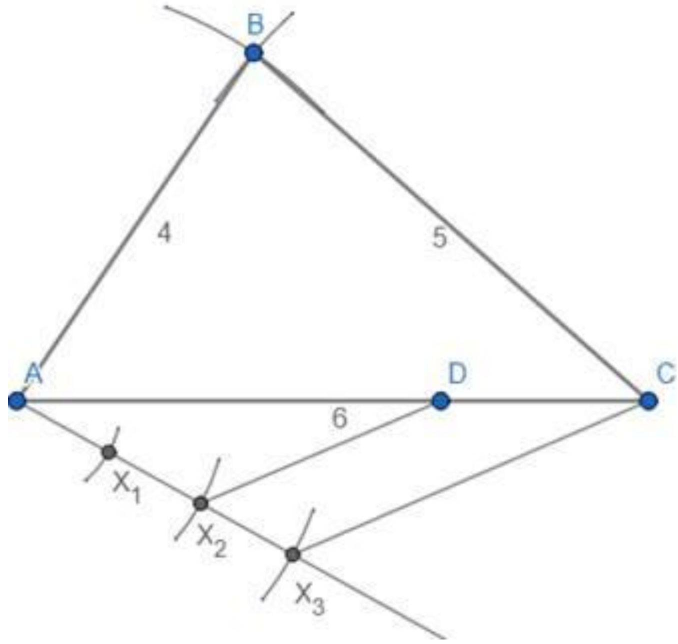
Step4: draw a ray from point A below AC at any angle



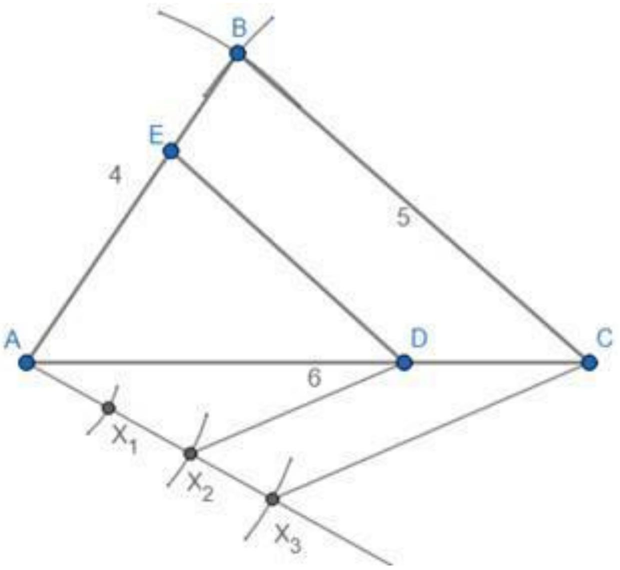
Step5: take any distance in compass and keeping the needle of the compass on point A cut an arc on ray constructed in step4 and name that point X_1. Keeping the distance in compass same keep the needle of the compass on point X_1 and cut an arc on the same ray and mark that point as X_2. Draw 3 such parts (greater of 2 and 3 in 2/3), i.e. by repeating this process mark points upto X_3



Step6: join X_3 and C and from X_2 (because X_2 is the second point 2 being smaller in 2/3) construct line parallel to X_3C and mark the intersection point with AC as D



Step7: construct line parallel to BC from point D and mark the intersection point with AB as E thus ΔADE ~ ΔACB is ready



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