10th Chapter Number System CBSE Test Paper – 06

Q.1. Based on Euclid's algorithm: a = b q + r; Using Euclid's algorithm: Find the HCF of 825 and 175.

Ans: Since 825>175, we use division lemma to 825 and 175 to get 825 = 175 x 4 + 125.

Since r ≠ 0, we apply division lemma to 175 and 125 to get 175 = 125 x 1 + 50

Again applying division lemma to 125 and 50 we get, 125 = 50 x 2 + 25.

Once again applying division lemma to 50 and 25 we get. 50 = 25 x 2 + 0.

Since remainder has now become 0, this implies that HCF of 825 and 125 is 25.

Q.2. Based on Showing that every positive integer is either of the given forms: Prove that every odd positive integer is either of the form 4q + 1 or 4q + 3 for some integer q.

Ans: Let a be any odd positive integer (first line of problem) and let b = 4. Using division Lemma we can write a = bq + r, for some integer q, where 0≤r<4. So a can be 4q, 4q + 1, 4q + 2 or 4q + 3. But since a is odd, a cannot be 4q or 4q + 2. Therefore any odd integer is of the form 4q + 1 or 4q + 3.

Q. Find H C F (26, 91) if LCM(26, 91) is 182

Sol: We know that LCM x HCF = Product of numbers.

or 182 x HCF = 26 x 91 or HCF = 26 x 91 = 13 Hence HCF (26, 91) = 13.

Q. Prove that √5 is irrational.

Solution: let us assume on the contrary that √5 is rational. That is we can find co-primes a and b b (≠0) such that √5 = a/b. Or √5b = a.

Squaring both sides we get $5b^2 = a^2$.

This means 5 divides a^2. Hence it follows that 5 divides a.

So we can write a = 5c for some integer c.

Putting this value of a we get, $5b^2 = (5c)^2$ Or $5b^2 = 25c^2$ Or $b^2 = 5b^2$.

It follows that 5 divides b^2. Hence 5 divides b.

Now a and b have at least 5 as a common factor.

But this contradicts the fact that a and b are co-primes.

This contradiction has arisen because of our incorrect assumption that √5 is rational. Hence it follows that √5 is irrational.

Q. Prove that product of three consecutive positive integers is divisible by 6?

Ans: Let three consecutive positive integers be, n, n + 1 and n + 2.

Whenever a number is divided by 3, the remainder obtained is either 0 or 1 or 2.

∴ n = 3p or 3p + 1 or 3p + 2, where p is some integer.

If n = 3p, then n is divisible by 3.

If n = 3p + 1, then n + 2 = 3p + 1 + 2 = 3p + 3 = 3(p + 1) is divisible by 3.

If n = 3p + 2, then n + 1 = 3p + 2 + 1 = 3p + 3 = 3(p + 1) is divisible by 3.

So, we can say that one of the numbers among n, n + 1 and n + 2 is always divisible by 3.

⇒ n (n + 1) (n + 2) is divisible by 3.

Similarly, whenever a number is divided 2, the remainder obtained is 0 or 1.

∴ n = 2q or 2q + 1, where q is some integer.

If n = 2q, then n and n + 2 = 2q + 2 = 2(q + 1) are divisible by 2.

If n = 2q + 1, then n + 1 = 2q + 1 + 1 = 2q + 2 = 2 (q + 1) is divisible by 2.

So, we can say that one of the numbers among n, n + 1 and n + 2 is always divisible by 2.

⇒ n (n + 1) (n + 2) is divisible by 2.

Since, n (n + 1) (n + 2) is divisible by 2 and 3. ∴ n (n + 1) (n + 2) is divisible by 6.

Q. Express HCF of F 65 and 117 in the form of 65 m +117 n

Ans: 117 = 65 x 1 + 52 ; 65 = 52 x 1 + 13 ; 52 = 4 x 13 + 0

In this step the remainder is zero. Thus, the divisor i.e. 13 in this step is the H.C.F. of the given numbers

The H.C.F. of 65 and 117 is 13

Now, 13 = 65 – 52 x 1 ⇒ 52 = 117 – 65 x 1 ⇒ 13 = 65 – (117 – 65 x 1) x 1 ⇒13 = 65 x 2 – 117 ⇒13 = 65 x 2 + 117 x (–1)

the H.C.F. of 65 and 117 is of the form 65m + 117 n, where m = 2 and n = –1

Q. Prove that: the cube of any positive integer is either of the form 2m or 2m+ 1, for some integer.

Ans: Let a be any positive integer. So, a is either an even positive integer or an odd positive integer.

∴ a = 2n or a = 2n + 1 $a^3 = (2n)^3 = 8n$ 3 $= 2(4n^3) = 2m$, where $m = 4n^3$

Or $a^3 = (2n + 1)^3 = 8n^3 + 12n^2 + 6n + 1 = 2(4n^3 + 6n^2 + 3n) + 1 = 2m + 1$, where $m = 4n^3 + 6n$ 2 $+ 3n$

So, the cube of any positive integer is either of the form 2m or 2m+ 1, where m is integer.

Q. Q. HCF of 657 and 963 is expressible in the form of 657x+963y x(-15)

Ans: Since 963 > 657, we apply the division lemma to 963 and 657 to obtain HCF.

963 = 657 x 1 + 306 ; 657 = 306 x 2 + 45 ; 306 = 45 x 6 + 36 ; 45 = 36 x 1 + 9 ; 36 = 9 x 4 + 0

In this step the remainder is zero. Thus, the divisor i.e. 9 So, The H.C.F. of 657 and 963 is 9.

Now, We can write 9 = 45 – 36 x 1 [from last steps of HCF]; 36 = 306 – 45 x 6 ;

9 = 45 – (306 – 45 x 6) x 1 = 45 x 7 – 306 x 1 ; 9 = (657 – 306 x 2) x 7 – 306 x 1 = 657 x 7 – 306 x 15

9 = 657 x 7 – (963 – 657 x 1) x 15 = 657 x 22 – 963 x 15

⇒ HCF of 657 and 963 = 657 x 22 – 963 x 15

∴ HCF, 9 can be expressed as linear combination of 657 and 963 as 9 = 657x + 963y, where x and y are not unique. Hence In linear combination, x = 22 and y = –15

Q. A rectangular courtyard measures 18m 72cm long and 13m 20 cm broad. It is to be paved with square tiles of same size. Find the least possible no. of such tiles.

Ans: Now, HCF of 1872 and 1320 = 24 Therefore, the side of the required square tile = 24 cm.

Thus, no. of such square tile required to pave the courtyard

= [Area if courtyard/area of 1 tiles]= [1872x 1320]/[24x24] = 4290

Hence, least possible no. of such tiles = 4290

Q. show that any positive odd integer is of the form 6q+1 ,or 6q+3 ,or 6q+5 where q is some integer.

Ans: Let 'a' be any positive odd integer. and b=6.

Let 'q' be quotient and 'r' be remainder.

a=6q+r where 0≤r<6 or a=6q+0 or a=6q+1 or a=6q+2 or a=6q+3 or a=6q+4 or a=6q+5

=> r=0,1,2,3,4,5 But Now, here odd integer are 6q+1, 6q+3, and 6q+5

Hence proved that any odd integer is of the form 6q+1, 6q+3 and 6q+5

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Q. Show that 5n can not end with the digit 2 for any natural number n.

Ans: Here Given Number=5n

Now Prime factors of this number=(5)n

Now, For the number to end with 2, it should be a factor of 2 which is not so in this case.

So it cannot end with the digit as 2.

Q. Show that the cube of any positive integer is either of the form 2m OR 2m+1 for some integer

Ans: Let a be any positive integer. So, a is either an even positive integer or an odd positive integer.

∴ a = 2n or a = 2n + 1

a 3 = (2n)3 = 8n 3 = 2(4n 3) = 2m, where m = 4n 3

Or a 3 = (2n + 1)3 = 8n 3 + 12n 2 + 6n + 1 = 2(4n 3 + 6n 2 + 3n) + 1 = 2m + 1, where m = 4n 3 + 6n 2 + 3n

So, the cube of any positive integer is either of the form 2m or 2m+ 1, where m is integer.

Q. Show that any positive odd integer is of the form 6q+1 ,or 6q+3 ,or 6q+5 where q is some integer.

Ans: Let 'a' be any positive odd integer. and b=6.

Let 'q' be quotient and 'r' be remainder.

a=6q+r where 0≤r<6 or a=6q+0 or a=6q+1 or a=6q+2 or a=6q+3 or a=6q+4 or a=6q+5

So r =0, 1, 2,3,4,5 For odd r =1, 3, 5

Then, odd integer will be 6q+1, 6q+3, and 6q+5

Hence proved that any odd integer is of the form 6q+1, 6q+3 and 6q+5

Q. show that n^2-1 is divisible by 8 if n is odd positive integer

Ans: Any odd positive integer is of the form 4*m* + 1 or 4*m* + 3 for some integer *m*.

When *n* = 4*m* + 1,

$$n^2-1=(4m+1)^2-1=16m^2+8m+1-1=16m^2+8m=8m(2m+1) \quad \Rightarrow n^2-1 \text{ is divisible by } 8$$

When $n = 4m + 3$ $\quad n^2-1=(4m+3)^2-1=16m^2+24m+9-1=16m^2+24m+8=8(2m^2+3m+1)$ $\Rightarrow n^2 - 1$ is divisible by 8. Hence, $n^2 - 1$ is divisible by 8 if *n* is an odd positive integer.