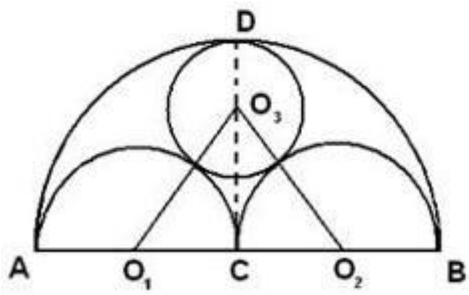
Class-10th (X) Mathematics Chapter: Tangents to Circles

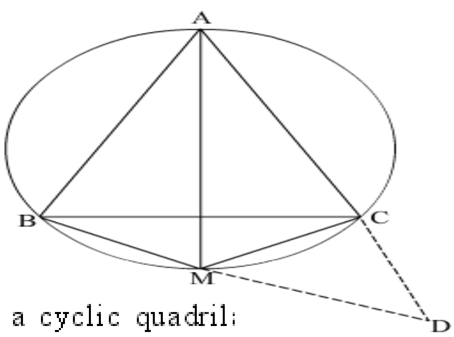
1. Q. AB is line segment of length 24 cm. C is its midpoint. On AB, AC and BC semicircles are described. Find the radius of the circle which touches all the three semicircles.

Solution: Let the required radius be r cm.
O_1O_3 = Radius of smaller semicircle + r = 24/4 + r = 6 + r
O_1C = Radius of smaller semicircle = 24/4 = 6 cm
In right triangle O_1O_3C:
$O_1O_3{}^2 = O_1C^2 + O_3C^2$
or $O_3C^2 = O_1O_3{}^2 – O_1C^2$
$= 36 + r^2 + 12r – 36$
$= r^2 + 12r$
or $O_3C = (r^2 + 12r)^{1/2}$
Also,
$DC = DO_3 + O_3C$
or $24/2 = 12 = r + (r^2 + 12r)^{1/2}$
or $12 – r = (r^2 + 12r)^{1/2}$
Squaring both sides
$144 + r^2 – 24r = r^2 + 12r$
or $36r = 144$
or $r = 144/36 = 4$
Hence, radius of the circle which touches all three semicircles is 4 cm.



2. Q. M is any point on the minor arc BC of circumcircle of an equilateral triangle ABC. Prove that AM = BM+ CM.

Solution:



Given, ΔABC is an equilateral triangle.
⇒ AB = BC = AC
Construction : Extend BM to point D. Such that DM = CM.
To Prove : AM = BM + CM
Proof : ΔABC is an equilateral triangle.
⇒ ∠A = ∠B = ∠C = 60° and AB = BC = AC
Now, ABCM is a cyclic quadrilateral.
$\Rightarrow \angle BAC + \angle BMC = 180°$ [opposite angles of a cyclic quadril
$\Rightarrow \angle BMC = 180° - 60° = 120°$
Now, $\angle BMC + \angle CMD = 180°$ [Linear pair]
$\Rightarrow \angle CMD = 180° - 120° = 60°$
$\Rightarrow \angle BAC = \angle CMD = 60°$
Also, ∠MDC = ∠MCD [Angles opposite to equal sides CM and DM are equal]
∴ Δ CMD is an equilateral triangle.
Now, In ΔAMC and BDC, we have

AC = BC

∠CAM = ∠CBD [Angles in the same segment of a circle are equal]

∠ABC = ∠AMC = ∠BDC = 60° [∠ABC and ∠AMC lies in the same segment of a circle and are equal to each other]

$\Rightarrow \Delta AMC \cong \Delta BDC \qquad [ASA]$

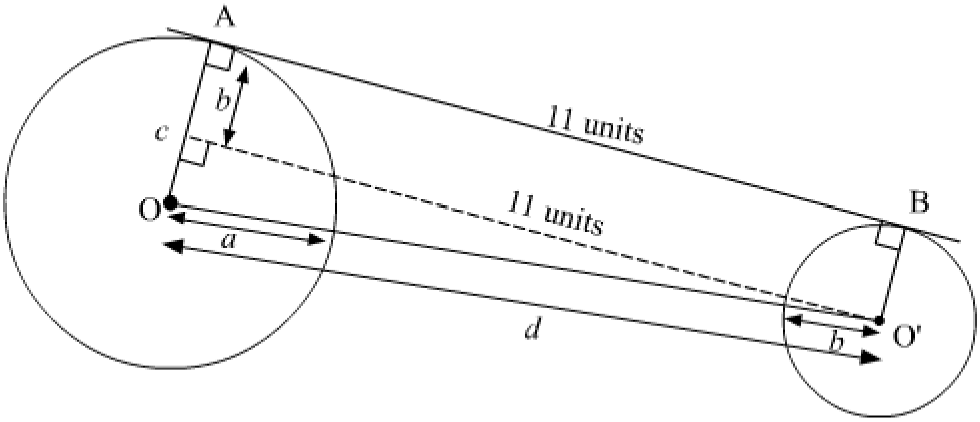
$\Rightarrow AM = BD \qquad [C.P.C.T]$

$\Rightarrow AM = BM + CM \qquad [As\ CM = DM]$

[Hence Proved]

3. Q. The length of a common internal tangent of two circles is 7 and a common external tangent is 11. Compute the product of the radii of two circles.

he length of common external tangent is 11 units. We can draw it as follows :



Let *d* be the distance of the centres of the circle. Let *a* be the radius of large circle and *b* be the radius of smaller circle.

From O', draw a line parallel to AB which meets OA and C.

$\therefore \angle O'CO = 90^\circ$ and $OC = a - b \qquad [\because O'C \parallel AB$ and $\angle CAB = 90^\circ]$

In Right angled ΔOCO', we have –

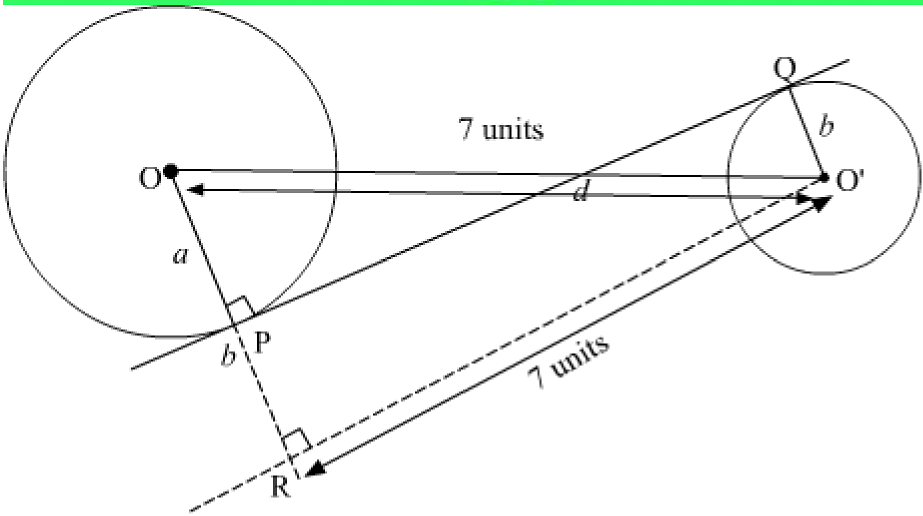
$(O'C)^2 + (OC)^2 = (OO')^2$

$\Rightarrow d^2 = (a-b)^2 + (11)^2$

$\Rightarrow 11^2 = d^2 - (a-b)^2$

$\Rightarrow 121 = d^2 - a^2 - b^2 + 2ab \quad \ldots \quad (1)$

Again, the length of common internal tangent to these two circles is 7 units. We can draw it as –



Here PQ = 7 units is the length of common internal tangent.
Draw a line O'R parallel to PQ so that it makes a right angled triangle OO'R with
OR = OP + PR = *a* + *b* units
O'R = PQ = 7 units
and OO' = *d* units
By Pythagoras theorem,

$$7^2 + (a+b)^2 = d^2$$

$$\Rightarrow 49 = d^2 - a^2 - b^2 - 2ab \qquad ..... \qquad (2)$$

subtracting (2) from (1) we get –

$$121 - 49 = 4ab$$

$$\Rightarrow 4ab = 72$$

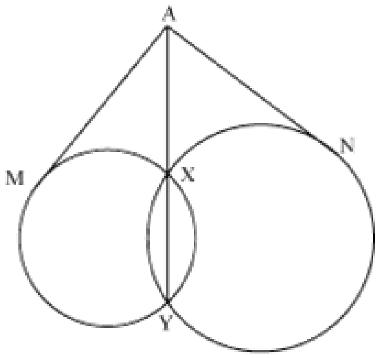
$$\Rightarrow ab = 18 \text{ square units}$$

Hence the product of radius of two circles is 18 square units.

4. Q. If from any point on the common chord of two intersecting circles, tangents be drawn to the circle, prove that they are equal.

Let PT be a tangent to the circle from an external point P and a secant to the circle through P intersects the circle at points A and B, then **PT^2 = PA × PB**

This property is used to solve the given question.



Let the two circles intersect at points X and Y. XY is the common chord.

Suppose A is a point on the common chord and AM and AN be the tangents drawn from A to the circle.

AM is the tangent and AXY is a secant.

$\therefore AM^2 = AX \times AY$ ...(1)

AN is the tangent and AXY is a secant.

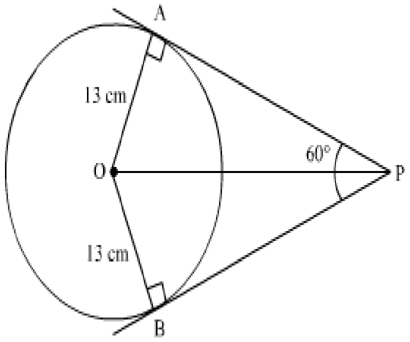
$\therefore AN^2 = AX \times AY$ ...(2)

From (1) and (2), we have

$AM^2 = AN^2$

$\therefore AM = AN$

5. Q. If two tangents inclined at an angle of 60 are drawn to circle of radius 13 cm , Find length of each tangent



Solution: Let PA and PB be two tangents to a circle with centre O and radius 13 cm.

We are given ∠APB = 60°

We know that two tangents drawn to a circle from an external point are equally inclined to the segment joining the centre to the point.

$\therefore \angle APO = \angle BPO = \frac{1}{2} \times \angle APB = \frac{1}{2} \times 60° = 30°$

Also, OA ⊥ AP and OB ⊥ BP (radius ⊥ tangent at point of contact)

In right ΔOAP,

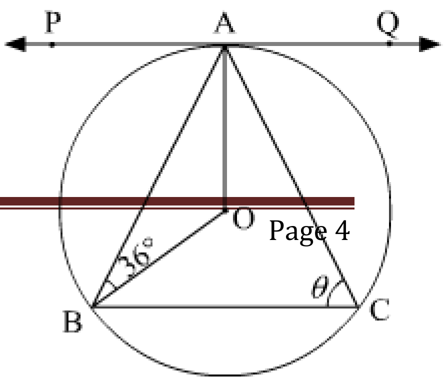
$\tan 30° = \frac{13}{PA}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{13}{PA}$

$\Rightarrow PA = 13\sqrt{3}$ cm

$\therefore PA = PB = 13\sqrt{3}$ cm (Lengths of tangents drawn from an external point to the circle are equal)

6. Q. PAQ is a tanget to a circle of centre O. a triangle is inscribed in circle ABC . if angle OBA is 36^0 0 and angle C is θ then Find measure of θ.



Solution:

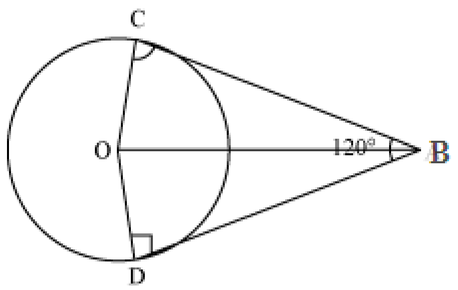
Given, PAQ is the tangent to the circle.∠OBA = 36° and ∠ACB = θ.

In ΔOAB,
OA = OB (Radius of the circle)
⇒ ∠OBA = ∠OAB (Equal sides have equal angles opposite to them)
⇒ ∠OAB = 36°
∠OAB + ∠AOB + ∠OBA = 180° (Angle sum property)
⇒ 36° + ∠AOB + 36° = 180°
⇒ ∠AOB = 180° – 72° = 108°
We know that, the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.
∴ ∠AOB = 2∠ACB
⇒ 2∠ACB = 108°
⇒ ∠ACB = 54°
Hence, the value of θ is 54°

7. Q. Two tangents BC and BD are drawn to a circle with centre 'O' such that angle DBC=120^0 .Prove that BO=2BC

It can be clearly show that OB bisects ∠DBC.

∴∠OBC = ∠OBD = 60
In ΔOBC,
∠OBC = 60, ∠OCB = 90
∠COB + ∠OBC +∠OCB = 180 [Angle sum property of triangle]
∠COB + 60 + 90 = 180
∠COB = 180 – 150 = 30



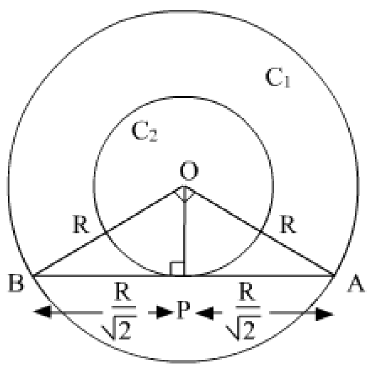
$$\sin(\angle COB) = \frac{BC}{BO}$$

$$\sin 30° = \frac{BC}{BO}$$

$$\frac{1}{2} = \frac{BC}{BO}$$

$$BO = 2BC$$

8. Q. two concentric circle has been drawn with centre o a right angled triangle inside the circle in such a way that hypotenuse touches the smaller circle as a tangent of of smaller circle and perpendicular is drawn as the radius of bigger circle and the base is also the radius of bigger circle find the radius of smaller circle



According to Question,
AB = hypotenuse of the right angled triangle
AB touches the smaller circle (C_2) as a tangent and OA = OB = R
(radius of bigger circle)
⇒ $(AB)^2 = R^2 + R^2 + = 2R^2$

$$\Rightarrow AB = \sqrt{2}R$$

As AB is a chord of circle C_1 and O is centre.
So OP ⊥ AB and P will bisect AB

$$\therefore BP = AP = \frac{\sqrt{2}R}{2} = \frac{R}{\sqrt{2}}$$

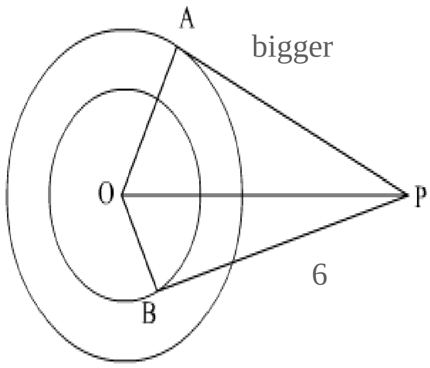
Now, OBD is a right angled triangle

$\therefore (OP)^2 = (BO)^2 - (BP)^2$

$$\Rightarrow (OP)^2 = R^2 - \frac{R^2}{2} = \frac{R^2}{2}$$

$$\Rightarrow OP = \frac{R}{\sqrt{2}}$$

Hence, Radius of smaller circle should by $1/\sqrt{2}$ times the radius of bigger circle.



9. Q.Two concentric circles with centre O are of radii 6 cm and 3 cm respectively.From an external point P, tangents PA and PB are drawn to these circles.If PA = 10 cm, find PB.

Given: Two concentric circles with radius O and radius O and radii 6 cm and 3 cm.

$\Rightarrow$ OA = 6 cm and OB = 3 cm

Also PA and PB are tangents to two circles such that PA = 10 cm

Now, the known that radius of a circle is perpendicular to the tangent at the point of center.

$\Rightarrow OA \perp PA$ and $OB \perp PB$

In right ΔOAP

$OP^2 = OA^2 + AP^2$ (Pythagoras theorem)

$\Rightarrow OP^2 = (6\text{ cm})^2 + (10\text{ cm})^2 = 36\text{ cm}^2 + 100\text{ cm}^2 = 136\text{ cm}^2$

In right ΔOBP

$OP^2 = OB^2 + PB^2$

$\Rightarrow PB^2 = OP^2 - OB^2 = 136\text{ cm}^2 - (3\text{ cm})^2 = 136\text{ cm}^2 - 9\text{ cm}^2 = 127\text{ cm}^2$

$PB = \sqrt{127}$ cm

10.Q. LT is a straight line of 6cm. S is the mid-point. Semi-circles are drawn on LT, TS and LS as diameter. Such circle is drawn which touches this three semi-circles. Prove that the radius of this circle is 1 cm.

Let the radius of the circle be r cm.

LS = ST = 3 cm.

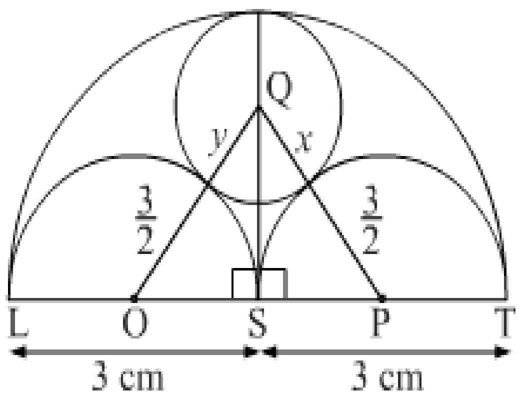
$SP = OS = \frac{3}{2}$ cm

QS = $(3 - r)$ cm.

$OQ = \left(r + \frac{3}{2}\right)$ cm

In right ΔOQS,

$OQ^2 = OS^2 + QS^2$



$$\therefore \left(r+\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2 + (3-r)^2$$

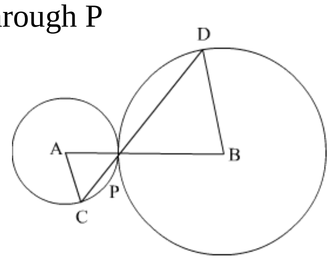
$$\Rightarrow r^2 + 3r + \frac{9}{4} = \frac{9}{4} + 9 + r^2 - 6r$$

$$\Rightarrow 9r = 9$$

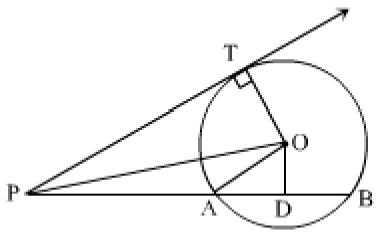
$$\Rightarrow r = 1$$

Thus, the radius of the circle is 1 cm.

11. Q. Two circles whose centres are A and B touches each other at point P. A line CD is drawn which passing through point P, which meets its circumference at C and D. Then prove that : AC is paralel to BD.



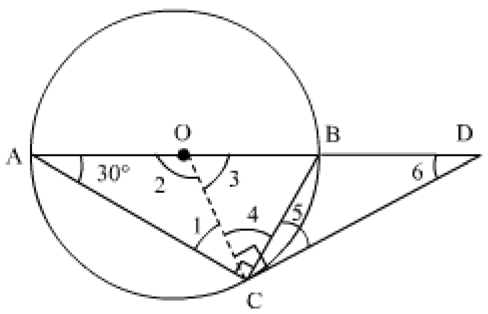
Given: Two circles with centre A and B touches at P and CD passing through P
In ΔACP
AC = AP (radius)
⇒ ∠APC = ∠ACP (Angles opposite to equal sides) ... (1)
In ΔBDP
BD = BP (radius)
⇒ ∠BPD = ∠BDP ... (2)
But
∠APC = ∠BPD (Vertically opposite angles) ... (3)
Now AC and DB are two lines and AB is the transversal such that
∠ACP = ∠BPD (from (1), (2) and (3))
Hence AC || BD



12.Q. If **PAB** is a **secant** to a circle intersecting the circle to **A** and **B** and **PT** is a **tangent**, then proove that :-
PA *PB = PT^2

Given: A secant PAB to a circle C(O, *r*) intersect it in A and B and PT is a tangent.
To prove: PA × PB = PT^2
Construction: Draw OD ⊥ AB. Join OP, OT and OA.
Proof:
Since, OD ⊥ AB
∴ AD = DB ...(1) (Perpendicular from the centre to the chord bisects the chord)
PA × PB = (PD – AD) (PD + BD)
= (PD – AD) (PD + AD) [Using (1)]
= $PD^2 - AD^2$
In right ΔOPD,
$OP^2 = OD^2 + PD^2$
⇒ $PD^2 = OP^2 - OD^2$
∴ PA × PB = $(OP^2 - OD^2) - AD^2 = OP^2 - (OD^2 + AD^2)$
In right ΔOAD,
$OA^2 = OD^2 + AD^2$
∴ PA × PB = $OP^2 - OA^2 = OP^2 - OT^2$ (∵ OA = OT)
In ΔOPT,
$OP^2 = PT^2 + OT^2$
⇒ $OP^2 - OT^2 = PT^2$
∴ PA × PB = PT^2

13. Q. AB is a diameter and AC is the cord of a circle such that angle BAC =30^0. if tangent at C intersects AB produced at D , prove that BC=BD



Given : A circle with AB as diameter having chord AC. ∠BAC = 30
Tangent at C meets AB produced at D. **To prove :** BC = BD **Construction :** Join OC
Proof : In Δ AOC,
OA = OC (radii of same circle)
⇒ ∠1 = ∠BAC (angles opposite to equal sides are equal) ⇒∠1 = 30
By angle sum property of Δ,
We have, ∠2 = 180 – (30 + 30) = 180 – 60= 120
Now, ∠2 + ∠3 = 180 (linear pair) ⇒ 120 + ∠3 = 180 ⇒ ∠3= 60
AB is diameter of the circle.
We know that angle in a semi circle is 90
⇒ ∠ACB = 90 ⇒ ∠1 + ∠4 = 90 ⇒ 30 + ∠4 = 90 ⇒ ∠4 = 60
Consider OC is radius and CD is tangent to circle at C.
We have OC ⊥ CD ⇒ ∠OCD = 90 ⇒ ∠4 + ∠5 (=∠BCD) = 90 ⇒ 60 + ∠5 = 90 ⇒ ∠5 = 30
In ΔOCD, by angle sum property of Δ

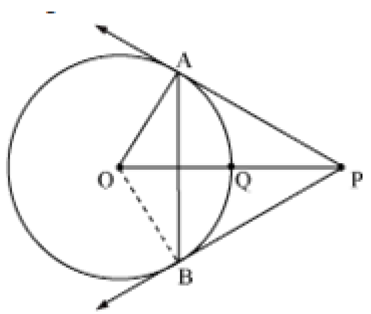
∠5 + ∠OCD + ∠6 = 180 ⇒ 60 + 90 + ∠6 = 18 ⇒ ∠6 + 15 = 180 ⇒ ∠6 = 30
In ΔBCD, ∠5 = ∠6 (= 30)
⇒ BC = CD (sides opposite to equal angles are equal)

14. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that the triangle APB is equilateral.

AP is the tangent to the circle.
∴ OA ⊥ AP (Radius is perpendicular to the tangent at the point of contact)
⇒ ∠ OAP = 90º
In Δ OAP,

$\sin \angle OPA = \frac{OA}{OP} = \frac{r}{2r}$ [OP = Diameter of the circle]

$\therefore \sin \angle OPA = \frac{1}{2} = \sin 30°$



⇒ ∠OPA = 30°
Similarly, it can be proved that ∠OPB = 30°.
Now, ∠APB = ∠OPA + ∠OPB = 30° + 30° = 60°
In ΔPAB,
PA = PB [lengths of tangents drawn from an external point to a circl
⇒∠PAB = ∠PBA ...(1) [Equal sides have equal angles opposite to
∠PAB + ∠PBA + ∠APB = 180° [Angle sum property]
⇒∠PAB + ∠PAB = 180° – 60° = 120° [Using (1)]
⇒2∠PAB = 120°
⇒∠PAB = 60° ...(2)
From (1) and (2)
∠PAB = ∠PBA = ∠APB = 60°
∴ ΔPAB is an equilateral triangle.