

Sets

- Sets
- Representation of Sets
- Power Set
- Venn Diagram
- Operations on Sets
- Laws of Algebra of Sets
- Cardinal Number of a Finite and Infinite Set

1.1 Sets

In Mathematical language all living and non-living things in universe are known as objects. The collection of well defined distinct objects is known as a set.

Well defined means in a given set, it must be possible to decide whether or not the object belongs to the set and by distinct means object should not be repeated. The object in the set is called its **member** or **element**. A set is represented by { }.

Generally, sets are denoted by capital letters A, B, C, \dots and its elements are denoted by small letters a, b, c, \dots .

Let A is a non-empty set. If x is an element of A , then we write ' $x \in A$ ' and read as ' x is an element of A ' or ' x belongs to A '. If x is not an element of A , then we write ' $x \notin A$ ' and read as ' x is not an element of A ' or ' x does not belong to A '.

e.g., A = Set of all vowels in English alphabets.

In this set a, e, i, o and u are members.

The theory of sets was developed by German Mathematician Georg Cantor (1845-1918). The concept of sets is widely used in the foundation of relations, functions, logic, probability theory, etc. According to Cantor 'A set is any collection into a whole of definite and distinct objects of our intuition or thought'.

Sample Problem 1 Which of the following is a correct set?

- The collection of all the months of a year beginning with the letter J.
- The collection of ten most talented writers of India.
- A team of eleven best cricket batsmen of the world.
- A collection of most dangerous animals of the world.

Interpret (a)

- We are sure that members of this collection are January, June and July.
So, this collection is well defined and hence, it is a set.
- A writer may be most talented for one person and may not be for other. Therefore, it cannot be said accountely that which writers will be there in the relation.
So, this collection is not well defined. Hence, it is not a set.

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- (c) A batsman may be best for one person and may not be so for other. Therefore, it cannot be said accountably that which batsman will be there in our relation.
So, this collection is not well defined. Hence, it is not a set.
- (d) The term most dangerous is not a clear term. An animal may be most dangerous to one person and may not be for the other.
So, it is not well defined, hence it is not a set.

1.2 Representation of Sets

We can use the following two methods to represent a set.

- (i) **Listing Method** In this method, elements are listed and put within a braces { } and separated by commas.

This method is also known as **Tabular method** or **Roster method**.

e.g., $A = \text{Set of all prime numbers less than 11}$
 $= \{2, 3, 5, 7\}$

- (ii) **Set Builder Method** In this method, instead of listing all elements of a set, we list the property or properties satisfied by the elements of set and write it as

$$A = \{x : P(x)\} \text{ or } \{x \mid P(x)\}$$

It is read as "A is the set of all elements x such that x has the property $P(x)$." The symbol ":" or "|" stands for such that.

This method is also known as **Rule method** or **Property method**.

e.g., $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
 $= \{x : x \in N \text{ and } x \leq 8\}$

Note

- The order of elements in a set has no importance e.g., $\{1, 2, 3\}$ and $\{3, 1, 2\}$ are same sets.
- The repetition of elements in a set does not effect the set, e.g., $\{1, 2, 3\}$ and $\{1, 1, 2, 3\}$ both are same sets.

Notation of Some Standard Sets

- (i) Set of all natural numbers, $N = \{1, 2, 3, \dots\}$
 (ii) Set of all whole numbers, $W = \{0, 1, 2, 3, \dots\}$
 (iii) (a) Set of all integers, I or $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$
 (b) Set of all positive or negative integers,
 $I^+ = \{1, 2, 3, \dots, \infty\}$
 or $I^- = \{-1, -2, -3, \dots, \infty\}$
 (c) Set of all even (E) or odd (O) integers,
 $E = \{\dots, -4, -2, 0, 2, 4, \dots\}$
 or $O = \{\dots, -3, -1, 0, 1, 3, \dots\}$
 (iv) (a) Set of all rational numbers,
 $Q = \{p/q, \text{ where } p \text{ and } q \text{ are integers and } q \neq 0\}$
 (b) Set of all irrational numbers,
 $IR = \{\text{which cannot be } p \text{ and } I \in I, q \neq 0\}$
 (c) Set of all real numbers,
 $R = \{x : -\infty < x < \infty\}$
 (v) Set of all complex numbers,
 $C = \{a + ib, a, b \in R \text{ and } i = \sqrt{-1}\}$

Sample Problem 2 The builder form of following set is
 $A = \{3, 6, 9, 12\}$, $B = \{1, 4, 9, \dots, 100\}$ [NCERT]

- (a) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 5\}$,
 $B = \{x : x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$
 (b) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$,
 $B = \{x : x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$
 (c) $A = \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$,
 $B = \{x : x = n^2, n \in N \text{ and } 1 < n < 10\}$
 (d) None of the above

Interpret (b) Given, $A = \{3, 6, 9, 12\}$ and

$$= \{x : x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$$

and $B = \{1, 4, 9, \dots, 100\} = \{x : x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$

Different Types of Sets

(i) Empty (Void/Null) Set

A set which has no element, is called an empty set. It is denoted by ϕ or $\{\}$.

e.g., $A = \text{Set of all odd numbers divisible by 2}$
 and $B = \{x : x \in N \text{ and } 5 < x < 6\}$

Such sets which have atleast one element, are called non-void set.

Note If ϕ represents a null set, then ϕ is never written with in braces i.e., $\{\phi\}$ is not the null set.

(ii) Singleton Set

A set which have only one element, is called a singleton set.

e.g., $A = \{x : x \in N \text{ and } 3 < x < 5\}$
 and $B = \{5\}$

(iii) Finite and Infinite Sets

A set in which the process of counting of elements surely comes to an end, is called a finite set. In other words 'A set having finite number of elements is called a finite set'.

Otherwise it is called infinite set i.e., if the process of counting of elements does not come to an end in a set, then set is called an infinite set.

e.g., $A = \{x : x \in N \text{ and } x < 5\}$

and $B =$ Set of all points on a plane

In above two sets A and B , set A is finite while set B is infinite. Since, in a plane any number of points are possible.

(iv) Equivalent Sets

Two finite sets A and B are said to be equivalent, if they have the same number of elements.

e.g., If $A = \{1, 2, 3\}$ and $B = \{3, 7, 9\}$

Number of elements in $A = 3$

and number of elements in $B = 3$

$\therefore A$ and B are equivalent sets.

(v) Equal Sets

If A and B are two non-empty sets and each element of set A is an element of set B and each element of set B is an element of set A , then sets A and B are called equal sets.

Symbolically, if $x \in A \Rightarrow x \in B$

and $x \in B \Rightarrow x \in A$

e.g., $A = \{1, 2, 3\}$ and $B = \{x : x \in N, x \leq 3\}$

Here, each element of A is an element of B , also each element of B is an element of A , then both sets are called equal sets.

Note Equal sets are equivalent sets while its converse need not to be true.

(vi) Subset and Superset

Let A and B be two non-empty sets. If each element of set A is an element of set B , then set A is known as subset of set B . If set A is a subset of set B , then set B is called the superset of A .

Also, if A is a subset of B , then it is denoted as $A \subseteq B$ and read as 'A is a subset of B'.

Thus, if $x \in A \Rightarrow x \in B$,

then $A \subseteq B$

If $x \in A \Rightarrow x \notin B$,

then $A \not\subseteq B$

and read as 'A is not a subset of B'

e.g., If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$

Here, each element of A is an element of B . Thus, $A \subseteq B$ i.e., A is a subset of B and B is a superset of A .

Note

- Null set is a subset of each set.
- Each set is a subset of itself.
- If A has n elements, then number of subsets of set A is 2^n .

(vii) Proper Subset

If each element of A is in set B but set B has atleast one element which is not in A , then set A is known as proper subset of set B . If A is a proper subset of B , then it is written as ' $A \subset B$ ' and read as A is a proper subset of B .

e.g., If $N = \{1, 2, 3, 4, \dots\}$

and $I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

then $N \subset I$

Note If A has n elements, then number of proper subsets is $2^n - 1$.

(viii) Comparability of Sets

Two sets A and B are said to be comparable, if either $A \subset B$ or $B \subset A$ or $A = B$, otherwise, A and B are said to be incomparable.

e.g., Suppose $A = \{1, 2, 3\}$, $B = \{1, 2, 4, 6\}$ and $C = \{1, 2, 4\}$

Since, $A \not\subset B$ or $B \not\subset A$ or $A \neq B$

$\therefore A$ and B are incomparable.

But $C \subset B$

$\therefore B$ and C are comparable sets.

(ix) Universal Set

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set and it is denoted by S or U .

This set can be chosen arbitrarily for any discussion of given sets but after choosing it is fixed.

e.g., Suppose $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{7, 8, 9\}$

$\therefore U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is universal set for all three sets.

Sample Problem 3 Consider the following sets

$A =$ The set of lines which are parallel to the X-axis.

$B =$ The set of letters in the English alphabet.

and $C =$ The set of animals living on the earth. [NCERT]

Which of these is finite or infinite set?

(a) Finite set $\rightarrow A, B$, Infinite set $\rightarrow C$

(b) Finite set $\rightarrow B, C$, Infinite set $\rightarrow A$

(c) Finite set $\rightarrow A, C$, Infinite set $\rightarrow B$

(d) None of the above

Interpret (b)

$A =$ Infinite lines can be drawn parallel to X-axis

$B =$ There are finite 26 English alphabets

$C =$ There are finite number of animals living on earth

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Sample Problem 4 Two finite sets have m and n elements, respectively. The total number of subsets of the first set is 56 more than the total number of subsets of second set. What are the values of m and n , respectively?

- (a) 7, 6 (b) 6, 3 (c) 5, 1 (d) 8, 7

Interpret (b) Since, total possible subsets of sets A and B are 2^m and 2^n , respectively.

According to given condition,

$$2^m - 2^n = 56$$

$$\Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times (2^3 - 1)$$

On comparing both sides, we get

$$2^n = 2^3 \text{ and } 2^{m-n} = 2^3$$

$$\Rightarrow n = 3 \text{ and } m - n = 3$$

$$\Rightarrow m = 6 \text{ and } n = 3$$

1.3 Power Set

Let A be a non-empty set, then collection of all possible subsets of set A is known as power set. It is denoted by $P(A)$.

e.g., Suppose $A = \{1, 2, 3\}$

$$\therefore P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}.$$

- (a) $A \notin P(A)$ (b) $\{A\} \in P(A)$

Properties of Power Set

- (i) Each element of a power set is a set.
- (ii) If $A \subseteq B$, then $P(A) \subseteq P(B)$
- (iii) Power set of any set is always non-empty.
- (iv) If set A has n elements, then $P(A)$ has 2^n elements.
- (v) $P(A) \cap P(B) = P(A \cap B)$
- (vi) $P(A) \cup P(B) \subseteq P(A \cup B)$
- (vii) $P(A \cup B) \neq P(A) \cup P(B)$

Sample Problem 5 If set $A = \{1, 3, 5\}$, then number of elements in $P\{P(A)\}$ is

- (a) 8 (b) 256 (c) 248 (d) 250

Interpret (b) Given, $A = \{1, 3, 5\}$

$$\therefore n\{P(A)\} = 2^3 = 8$$

$$\therefore n\{P\{P(A)\}\} = 2^8 = 256$$

Sample Problem 6 Consider $A = \{1, 2\}$, $B = \{2, 3\}$. Then which of the following option is correct?

- (a) $P(A \cup B) \neq P(A) \cup P(B)$ (b) $P(A \cup B) = P(A) \cup P(B)$
 (c) $P(A \cup B) = P(A) \cap P(B)$ (d) None of these

Interpret (a) Here $P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$,

$$P(B) = \{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$$

$$AB = \{1, 2, 3\}$$

$$\therefore P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$$

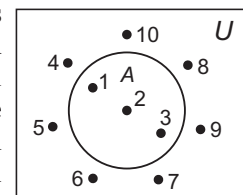
$$P(A) \cup P(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}\}$$

$$\therefore P(A \cup B) \neq P(A) \cup P(B)$$

1.4 Venn Diagram

A Swiss Mathematician (1707-1783) Euler gave an idea to represent a set by the points in a closed curve. Later on British Mathematician John Venn (1834-1923) brought this idea to practice. So, the diagrams drawn to represent sets are called Venn Euler diagram or simply Venn diagram.

In Venn diagram, the universal set is represented by a rectangular region and a set is represented by circle or a closed geometrical figure inside the universal set. Also, an element of a set A is represented by a point within the circle of set A .



e.g., If $U = \{1, 2, 3, 4, \dots, 10\}$ and $A = \{1, 2, 3\}$

Then, its Venn diagram is as shown in the figure.

1.5 Operations on Sets

Now, we introduce some operations on sets to construct new sets from the given ones.

(i) Union of Two Sets

Let A and B be two sets, then union of A and B is a set of all those elements which are in A or in B or in both A and B . It is denoted by $A \cup B$ and read as 'A union B'.

Symbolically, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly,

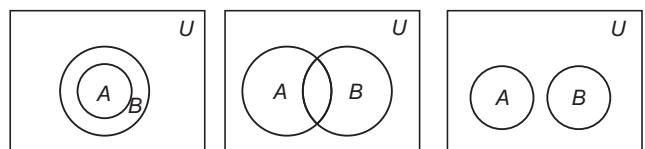
$$x \in A \cup B$$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\text{If } x \notin A \cup B$$

$$\Rightarrow x \notin A \text{ and } x \notin B$$

The venn diagram of $A \cup B$ is as shown in the figure and the shaded portion represents $A \cup B$.



$A \cup B$
(when $A \subseteq B$)

$A \cup B$ when neither
 $A \subseteq B$ nor $B \subseteq A$

$A \cup B$ when A and B
are disjoint sets

e.g., If

$$A = \{1, 2, 3, 4\}$$

and

$$B = \{4, 8, 5, 6\}$$

$$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 8\}.$$

General Form

The union of a finite number of sets A_1, A_2, \dots, A_n is represented by

$$A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n \text{ or } \bigcup_{i=1}^n A_i$$

Symbolically, $\bigcup_{i=1}^n A_i = \{x : x \in A_i \text{ for atleast one } i\}$

(ii) Intersection of Two Sets

If A and B are two sets, then intersection of A and B is a set of all those elements which are in both A and B . The intersection of A and B is denoted by $A \cap B$ and read as "A intersection B".

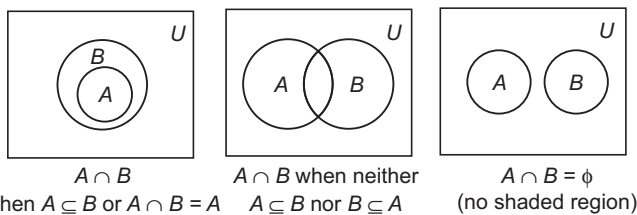
Symbolically,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

If $x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

and if $x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B$

The Venn diagram of $A \cap B$ is as shown in the figure and the shaded region represents $A \cap B$.



e.g., If $A = \{1, 2, 3, 4\}$ and $B = \{4, 3, 5, 6\}$

$\therefore A \cap B = \{3, 4\}$

General Form

The intersection of a finite number of sets

$A_1, A_2, A_3, \dots, A_n$ is represented by

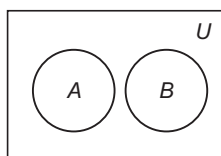
$$A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n \text{ or } \bigcap_{i=1}^n A_i$$

Symbolically, $\bigcap_{i=1}^n A_i = \{x : x \in A_i \text{ for all } i\}$

(iii) Disjoint of Two Sets

Two sets A and B are known as disjoint sets, if $A \cap B = \phi$ i.e., if A and B have no common element.

The Venn diagram of disjoint sets as shown in the figure



$$A \cap B$$

e.g., If $A = \{1, 2, 3\}$

and $B = \{4, 5, 6\}$,

then $A \cap B = \{ \} = \phi$

$\therefore A$ and B are disjoint sets.

(iv) Difference of Two Sets

If A and B are two non-empty sets, then difference of A and B is a set of all those elements which are in A but not in B . It is denoted as $A - B$. If difference of two sets is $B - A$, then it is a set of those elements which are in B but not in A .

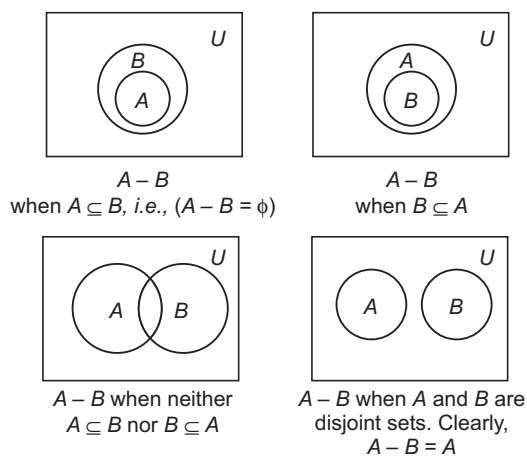
Hence, $A - B = \{x : x \in A \text{ and } x \notin B\}$

and $B - A = \{x : x \in B \text{ and } x \notin A\}$

If $x \in A - B \Rightarrow x \in A \text{ but } x \notin B$

and if $x \in B - A \Rightarrow x \in B \text{ but } x \notin A$

The Venn diagram of $A - B$ and $B - A$ are as shown in the figure and shaded region represents $A - B$ and $B - A$.



e.g., If $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7, 8\}$

$\therefore A - B = \{1, 2, 3\}$ and $B - A = \{5, 6, 7, 8\}$

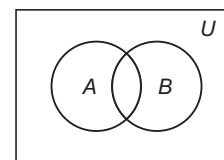
Note

- $A - B \neq B - A$
- $A - B \subseteq A$ and $B - A \subseteq B$
- $A - \phi = A$ and $A - A = \phi$
- The sets $A - B$ and $B - A$ are disjoint sets.

(v) Symmetric Difference of Two Sets

If A and B are two sets, then set $(A - B) \cup (B - A)$ is known as symmetric difference of sets A and B and is denoted by $A \Delta B$.

The Venn diagram of $A \Delta B$ is as shown in the figure and shaded region represents $A \Delta B$.



e.g., $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$,

then $A \Delta B = (A - B) \cup (B - A)$
 $= \{1, 2\} \cup \{4, 5, 6\} = \{1, 2, 4, 5, 6\}$

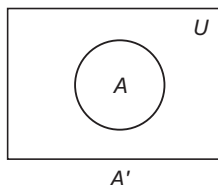
Note

- Symmetric difference can also be written as $A \Delta B = (A \cup B) - (A \cap B)$
- $A \Delta B = B \Delta A$ (commutative)

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(vi) Complement of a Set

The complement of a set A is the set of all those elements which are in universal set but not in A . It is denoted by A' or A^c .

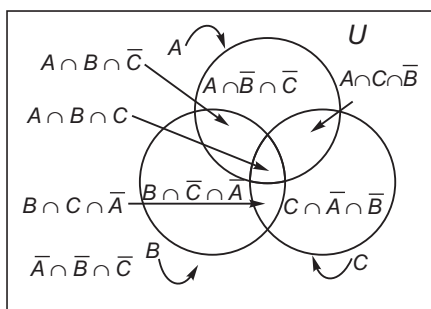


If U is a universal set and $A \subset U$, then $A' = U - A = \{x : x \in U \text{ but } x \notin A\}$
i.e., $x \in A \Rightarrow x \notin A'$

The Venn diagram of complement of a set A is as shown in the figure and shaded portion represents A' .

e.g., If $U = \{1, 2, 3, 4, 5, \dots\}$
and $A = \{2, 4, 6, 8, \dots\}$
 $\therefore A' = U - A = \{1, 3, 5, 7, \dots\}$

By Venn diagram, the operation between three sets can be represented given below.



Note

- $\phi = U'$
- $A \cup A' = U$
- $\phi' = U$
- $A \cap A' = \phi$
- $(A')' = A$

Sample Problem 7 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$, then $(A \cup B)'$, $(A' \cap B')$, $(A \Delta B)$ is equal to

- (a) $\{1, 9\}, \{2, 8\}, \{3, 4, 5, 6, 7, 8\}$
- (b) $\{1, 9\}, \{1, 9\} \{3, 4, 5, 6, 7, 8\}$
- (c) $\{1, 9\}, \{1, 9\} \{5, 6, 7, 8\}$
- (d) None of the above

Interpret (b) Given sets are

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

$$A = \{2, 4, 6, 8\} \text{ and } B = \{2, 3, 5, 7\}$$

Now, $A' = U - A$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$$

$$= \{1, 3, 5, 7, 9\}$$

and $B' = U - B$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9\}$$

$$(i) A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$$

$$= \{2, 3, 4, 5, 6, 7, 8\}$$

$$\therefore (A \cup B)' = U - A \cup B$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5, 6, 7, 8\}$$

$$= \{1, 9\}$$

$$(ii) (A' \cap B') = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$$

$$= \{1, 9\}$$

$$(iii) \text{ Now, } A - B = \{2, 4, 6, 8\} - \{2, 3, 5, 7\}$$

$$= \{4, 6, 8\}$$

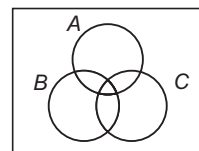
$$\text{ and } B - A = \{2, 3, 5, 7\} - \{2, 4, 6, 8\}$$

$$= \{3, 5, 7\}$$

$$\therefore A \Delta B = (A - B) \cup (B - A)$$

$$= \{4, 6, 8\} \cup \{3, 5, 7\} = \{3, 4, 5, 6, 7, 8\}$$

Sample Problem 8 The shaded region in the given figure is



$$(a) B \cap (A \cup C)$$

$$(c) B \cap (A - C)$$

$$(b) B \cup (A \cap C)$$

$$(d) B - (A \cup C)$$

Interpret (d) It is clear from the figure that set $A \cup C$ is not shading and set B is shading other than $A \cup C$.

i.e.,

$$B - (A \cup C)$$

Sample Problem 9 If $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$,

$A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$ what is $(A \cap B)'$ equal to ?

$$(a) \{1, 3\}$$

$$(c) \{0, 1, 3\}$$

$$(b) \{1, 2, 3\}$$

$$(d) \{0, 1, 2, 3\}$$

Interpret (c) $\therefore U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$$= \{0, 1, 2, 3\}$$

$$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$$

and

$$B = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$$

\therefore

$$A \cap B = \{2\}$$

Hence,

$$(A \cap B)' = U - (A \cap B)$$

$$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$$

1.6 Laws of Algebra of Sets

If A, B and C are three non-empty sets, then

(i) Idempotent law

$$(a) A \cup A = A$$

$$(b) A \cap A = A$$

(ii) Identity law

$$(a) A \cup \phi = A$$

$$(b) A \cap U = A$$

- (iii) Commutative law
 - (a) $A \cup B = B \cup A$
 - (b) $A \cap B = B \cap A$
- (iv) Associative law
 - (a) $(A \cup B) \cup C = A \cup (B \cup C)$
 - (b) $(A \cap (B \cap C)) = (A \cap B) \cap C$
- (v) Distributive law
 - (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- (vi) De-morgan's law
 - (a) $(A \cup B)' = A' \cap B'$
 - (b) $(A \cap B)' = A' \cup B'$
 - (c) $A - (B \cup C) = (A - B) \cap (A - C)$
 - (d) $A - (B \cap C) = (A - B) \cup (A - C)$

Sample Problem 10 If A and B are two sets, then $A \cap (A \cup B)'$ is equal to

- (a) A
- (b) B
- (c) ϕ
- (d) None of these

Interpret (c) $A \cap (A \cup B)' = A \cap (A' \cap B')$ (\because by de-Morgan's law)
 $= (A \cap A') \cap B'$ (\because by associative law)
 $= \phi \cap B'$ ($\because A \cap A' = \phi$)
 $= \phi$

Sample Problem 11 If A and B are non-empty sets, then $(A \cap B) \cup (A - B)$ is equal to

- (a) B
- (b) A
- (c) A'
- (d) B'

Interpret (b) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$
 $[\because (A - B) = (A \cap B')]$
 $= A \cap (B \cup B')$ (\because by distributive law)
 $= A \cap U$ ($\because B \cup B' = U$)
 $= A$

1.7 Cardinal Number of a Finite and Infinite Set

The number of distinct elements in a finite set A is called cardinal number and it is denoted by $n(A)$. And if it is not finite set, then it is called infinite set.

e.g., If $A = \{-3, -1, 8, 10, 13, 17\}$,
 then $n(A) = 6$

Properties

If A, B and C are finite sets and U be the finite universal set, then

- (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- (ii) If A and B are disjoint sets then, $n(A \cup B) = n(A) + n(B)$
- (iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

- (iv) $n(A - B) = n(A) - n(A \cap B)$
- (v) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$
- (vi) $n(A') = n(U) - n(A)$
- (vii) $n(A' \cup B') = n(U) - n(A \cap B)$
- (viii) $n(A' \cap B') = n(U) - n(A \cup B)$
- (ix) $n(A \cap B') = n(A) - n(A \cap B)$

Method to Find Common Roots

Sometimes the number of common elements cannot found easily by solving the given sets. That type of problems can be solved by drawing a curves.

The intersection point of a curve is equal to the number of common elements in a set.

e.g., Consider the sets

$$A = \{(x, y) | y = \frac{1}{x}, 0 \neq x \in R\}$$

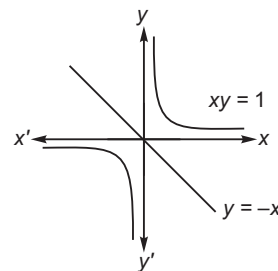
and $B = \{(x, y) | y = -x, x \in R\}$,
 then determine $n(A \cap B)$.

Here, we see that, $\forall x$, we get infinite values of y . Hence, we find infinite sets A and B . And it is difficult to find the common elements between A and B .

Now, firstly we make the graph of given sets.

$A =$ The set of all points on a curve $xy = 1$,
 $[\because xy = c$ is a rectangular hyperbola curve]

and $B =$ The set of all points on a curve $y = -x$.
 $[\because y = -x$ is a straight which passes through origin]



Since, there is no intersection point on a curve. So, there is no common elements between two sets.

Sample Problem 12 In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all of three newspapers, then the number of families which buy A only, is

- (a) 4400
- (b) 3300
- (c) 2000
- (d) 500

Interpret (b) $n(A) = 40\%$ of 10000 = 4000, $n(B) = 2000$,
 $n(C) = 1000$, $n(A \cap B) = 500$, $n(B \cap C) = 300$, $n(C \cap A) = 400$,
 $n(A \cap B \cap C) = 200$

$$\begin{aligned} \therefore n(A \cap \bar{B} \cap \bar{C}) &= n\{A \cap (B \cup C)'\} = n(A) - n\{A \cap (B \cup C)\} \\ &= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) \\ &= 4000 - 500 - 400 + 200 = 3300 \end{aligned}$$

WORKED OUT

Examples

Example 1 If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$, then $(A \cup D) \cap (B \cup C)$ is equal to

- (a) $\{5, 7, 9, 11, 15\}$ (b) $\{7, 9, 11, 15\}$
 (c) $\{7, 9, 11, 13, 15\}$ (d) None of these

Solution (b) Given, $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$

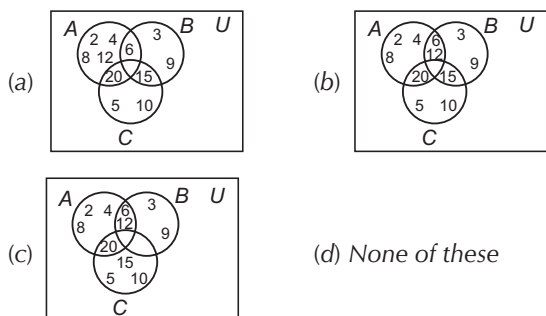
$$\begin{aligned} \text{Now, } A \cup D &= \{3, 5, 7, 9, 11\} \cup \{15, 17\} \\ &= \{3, 5, 7, 9, 11, 15, 17\} \\ \text{and } B \cup C &= \{7, 9, 11, 13\} \cup \{11, 13, 15\} = \{7, 9, 11, 13, 15\} \\ \therefore (A \cup D) \cap (B \cup C) &= \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\} \\ &= \{7, 9, 11, 15\} \end{aligned}$$

Example 2 The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

- (a) $B \cap C'$ (b) $A \cap C$
 (c) $B' \cap C'$ (d) None of these

Solution (a) $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$
 $= (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$
 $= (\phi \cup B \cup C) \cap C'$
 $= (B \cup C) \cap C'$
 $= (B \cap C') \cup \phi = B \cap C'$

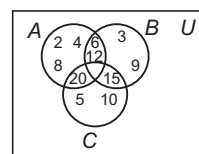
Example 3 Let A, B and C are subsets of universal set U . If $A = \{2, 4, 6, 8, 12, 20\}$, $B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers. Then, the correct Venn diagram is [NCERT]



Solution (b) Given, $A = \{2, 4, 6, 8, 12, 20\}$
 $B = \{3, 6, 9, 12, 15\}$

$$C = \{5, 10, 15, 20\}$$

Now, $A \cap B = \{2, 4, 6, 8, 12, 20\} \cap \{3, 6, 9, 12, 15\} = \{6, 12\}$
 $B \cap C = \{3, 6, 9, 12, 15\} \cap \{5, 10, 15, 20\} = \{15\}$
 $C \cap A = \{5, 10, 15, 20\} \cap \{2, 4, 6, 8, 12, 20\} = \{20\}$
 and $A \cap B \cap C = \phi$



Example 4 In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee. 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee? [NCERT]

- (a) 310 (b) 320 (c) 327 (d) 325

Solution (d) Let C and T denote the students taking coffee and tea, respectively.

$$\begin{aligned} \text{Here, } n(T) &= 150, n(C) = 225, n(C \cap T) = 100 \\ \text{Using the identity } n(C \cup T) &= n(T) + n(C) - n(C \cap T), \text{ we have} \\ n(C \cup T) &= 150 + 225 - 100 = 375 - 100 \\ \Rightarrow n(C \cup T) &= 275 \end{aligned}$$

Given, total number of students = 600 = $n(U)$

We are to find the number of students taking neither tea nor coffee i.e., $n(C \cup T)'$.

$$\therefore n(C \cup T)' = n(U) - n(C \cup T) = 600 - 275 = 325$$

Example 5 If there are three athletic teams in a school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 15 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. The total number of members is

- (a) 42 (b) 43
 (c) 45 (d) None of these

Solution (b) $\because n(B) = 21, n(H) = 26, n(F) = 29, n(H \cap B) = 14,$
 $n(H \cap F) = 15, n(F \cap B) = 12, n(B \cap H \cap F) = 8$
 $\therefore n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(B \cap H)$
 $\quad \quad \quad - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$
 $= 21 + 26 + 29 - 14 - 15 - 12 + 8 = 43$