Example 3. The sum of one-fourth of a number and 3 is 15. Find the number.

Solution. To find The number **Given** Sum of one-fourth of a number and 3 is 15

Assume: Let the required number be x.

One-fourth of the number $= \frac{1}{4}x$

Sum of one-fourth of the number $\frac{1}{4}x$ and 3 is

$$\frac{1}{4}x + 3$$

Equal quantities are:
Sum of one-fourth of the number and 3 and 15
Equate to form an equation

$$\Rightarrow \quad \frac{1}{4}x + 3 = 15$$

Solve the equation

$$\text{or} \quad \frac{1}{4}x = 12$$

$$\text{or} \quad x = 48$$

Thus, the required number is 48

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Example 4. The denominator of a fraction is 3 more than the numerator. If 5 is added to both parts, the resulting fraction is equivalent to $\frac{4}{5}$. Find the fraction.

Solution.

Unknown Fraction **Known** Denominator is 3 more than the numerator

Assume: Let the numerator of the fraction be x.
$\Rightarrow$ Denominator of the fraction be $x + 3$

So the required fraction is $\frac{x}{x+3}$

5 is added to both parts, so the fraction becomes

$$\frac{x+5}{x+3+5} = \frac{x+5}{x+8}$$

Equal quantities are

Resulting fraction $= \frac{4}{5}$

Equate to form an equation

$$\Rightarrow \quad \frac{x+5}{x+8} = \frac{4}{5}$$

Solve by cross-multiplication

$$\Rightarrow \quad 5(x + 5) = 4(x + 8)$$
$$\Rightarrow \quad 5x + 25 = 4x + 32$$
$$\Rightarrow \quad 5x - 4x = 32 - 25$$
$$\text{or} \quad x = 7$$

Thus, the required fraction is

$$\frac{x}{x+3} = \frac{7}{7+3} = \frac{7}{10}$$

Example 5. The sum of the digits of a two-digit number is 7. The number obtained by interchanging the digits exceeds the original number by 27. Find the number.

Solution.

To find A two-digit number or find ones and tens digits as once the digits are known, number can be determined. **Given** Sum of the digits is 7

Assume

Let ones digit of the number be x then its tens digit is $7 - x$ (as sum of the digits is 7).

$\therefore$ Required number
$= 10 \times$ (tens digit) $+ 1 \times$ ones digit
$= 10 \cdot (7 - x) + 1 \cdot x$
$= 70 - 10x + x$
$= 70 - 9x$

Number obtained by interchanging the digits *i.e.*, when ones digit beocmes $7 - x$ and tens digit becomes x is $10x + 7 - x$

It is given that the number obtained by interchanging the digits exceeds original number by 27.

Form an equation

New number = original number + 27

$$\Rightarrow \quad 10x + 7 - x = 10(7 - x) + x + 27$$

Solve equation

$$\text{or} \quad 9x + 7 = 70 - 9x + 27$$
$$\text{or} \quad 9x + 9x = 97 - 7$$
$$\text{or} \quad 18x = 90$$
$$\text{or} \quad x = 5$$

Thus, ones digit of the number is 5
and tens digit of the number is $7 - 5 = 2$
$\therefore$ The required number is
$10 \times 2 + 1 \times 5$
$= 20 + 5 = 25.$

Example 6. Sarita's mother is four times as old as Sarita. Twenty years later, she will be just twice as old as Sarita will be then. Find the present age of Sarita.

Solution. Let Sarita's present age be x years

Then her mother's present age is $4x$ years

20 years later. Sarita's age = $(x + 20)$ years

Her mother's age will be $(4x + 20)$ years

Given. 20 years later mother's age will be $2 \times$ Sarita's age then

$$\Rightarrow \quad 4x + 20 = 2(x + 20)$$
$$4x + 20 = 2x + 40$$
$$\text{or} \quad 4x - 2x = 40 - 20$$
$$\text{or} \quad 2x = 20 = 10$$

Thus, Sarita's present age is 10 years.

Value Based Question. Example 8. Indu's flower garden is a square. If she enlarges it by increasing the width 1 metre and the length by 3 metres, the area will be 19 sq. metres more than the original area. What is the length of a side now? What can you say about Indu?

Solution. Let side of the garden be x m

Area of the garden = $(\text{side})^2 = x^2$ sq. m

After enlargement,

Length of the garden = $(x + 3)$ metres

and breadth of the garden = $(x + 1)$ metres

Therefore, its area will be L × B

$$= (x + 3)(x + 1)$$
$$= (x^2 + 4x + 3) \text{ sq. m}$$

Area after enlargement = original area + 19

$$\Rightarrow \quad x^2 + 4x + 3 = x^2 + 19$$
$$\text{or} \quad x^2 + 4x + 3 - x^2 = 19$$
$$\text{or} \quad 4x + 3 = 19$$
$$\text{or} \quad 4x = 19 - 3$$
$$\text{or} \quad x = \frac{16}{4} = 4 \text{ m}$$

Length of the side now is 4 m.

Example 7. The present ages of Kuber and Vaibhav are in the ratio 4 : 5. Four years later their ages will be in the ratio 5 : 6. Find their present ages.

Solution. Let the ratio constant be x

Then Kuber's present age is $4x$ years

Vaibhav's present age is $5x$ years

Four years later, Kuber's age will be $(4x + 4)$ years

and Vaibhav's age will be $(5x + 4)$ years

Given. For years later ratio of their ages will be 5 : 6

$$\text{Therefore,} \quad \frac{4x+4}{5x+4} = \frac{5}{6}$$
$$\text{or} \quad 24x + 24 = 25x + 20$$
$$\text{or} \quad 25x - 24x = 24 - 20$$
$$\text{or} \quad x = 4$$

Thus, Kuber's present age

$$= 4x = 4 \times 4 = 16 \text{ years}$$

and Vaibhav's present age

$$= 5x = 5 \times 4 = 20 \text{ years}$$

Example 9. The length of a rectangle is 8 cm more than its width. If the perimeter of the rectangle is 44 cm, find the length and breadth of the rectangle.

Solution. Let width of the rectangle be x cm then, its length is $(x + 8)$ cm.

$$\text{Perimeter} = 44 \text{ cm}$$
$$2(L + B) = 44$$
$$\text{or} \quad L + B = 22$$
$$\text{or} \quad x + 8 + x = 22$$
$$\text{or} \quad 2x = 22 - 8$$
$$\text{or} \quad 2x = 14$$
$$\text{or} \quad x = 7 \text{ cm}$$

Breadth is $x = 7$ cm

and length is $x + 8 = (7 + 8) = 15$ cm

Example 10. The altitude of a triangle is two-third the length of its corresponding base. If the altitude is increased by 4 cm and the base decreased by 2 cm, the area of the triangle remains the same. Find the base and the altitude of the triangle.

Solution. Let base of the triangle be x cm

So, length of its altitude will be $\frac{2}{3}x$ cm

Area of a triangle

$$= \frac{1}{2} \times x \times \frac{2}{3}x \text{ sq. cm} \quad = \frac{x^2}{3} \text{ sq. cm}$$

Altitude of the triangle after increasing by 4 cm

$$= \left(\frac{2}{3}x + 4\right) \text{ cm}$$

Base after decreasing by 2 cm = $(x - 2)$ cm

New area = $\frac{1}{2}(x - 2)\left(\frac{2}{3}x + 4\right)$

Given that both areas are equal.

$$\frac{x^2}{3} = \frac{1}{2}(x-2)\left(\frac{2}{3}x + 4\right)$$
$$\text{or} \quad \frac{x^2}{3} = \frac{1}{2}\left[\frac{2}{3}x^2 - \frac{4}{3}x + 4x - 8\right]$$
$$\text{or} \quad 2x^2 = \cancel{3}\left[\frac{2x^2 - 4x + 12x - 24}{\cancel{3}}\right]$$
$$\text{or} \quad 2x^2 = 2x^2 + 8x - 24$$
$$\text{or} \quad x^2 = x^2 + 4x - 12$$
$$\text{or} \quad x^2 + 4x - 12 - x^2 = 0$$
$$\text{or} \quad 4x - 12 = 0 \quad x = \frac{12}{4} \quad \text{or} \quad x = 3$$
$$\text{or} \quad 4x = 12$$

Thus, base of the altitude = $x = 3$ cm and its altitude

$$= \frac{2}{3}x = \frac{2}{3} \times 3 \text{ cm} = 2 \text{ cm}.$$

Value Based Question. Example 11. Two cars leave Delhi at the same time, travelling in opposite direction. If the average speed of one car is 5 km/hr more than that of the other and they are 425 km apart at the end of 5 hours, what is the average speed of each car? Speed limit is 60 km/hr. Are the speeds within limits? Why should we follow speed limit?

Solution. Let the two cars be car A and car B.

Let speed of the car A be x km/hr

and speed of the car B be $(x + 5)$ km/hr

Distance covered by car A in 5 hours

$= x \times 5$ km $= 5x$ km

Distance covered by car B in 5 hours

$= (x + 5)5$ km

$= (5x + 25)$ km

Cars are 425 km apart after 5 hours

$\therefore$ Distance covered by car A + distance covered by car B = 425 km

$\Rightarrow \quad 5x + 25 + 5x = 425$

or $\quad 10x + 25 = 425$

or $\quad 10x = 425 - 25$

or $\quad 10x = 400$

or $\quad x = 40$

Thus speed of car A $= x = 40$ km/hr

and the speed of car B

$= (x + 5)$ km/hr $= 45$ km/hr

Yes, speeds are within limits. We should follow speed limits for safe driving *i.e.*, to avoid accidents.

Example 12. A steamer, going downstream in a river, covers the distance between two towns in 20 hours. Coming back upstream, it covers this distance in 25 hours. The speed of water is 4 km/hr. Find the distance between the two towns.

Solution: Distance can be found if speed and time are known so first we will find the speed.

Let speed of the steamer in the still water be x km/hr.

Speed of water is 4 km/hr

Speed of the steamer going downstream

= Speed of the steamer in still water + speed of water (why?)

$= (x + 4)$ km/hr

$\therefore$ Distance covered by the steamer going downstream in 20 hours

$= s \times t = (x + 4)20$ km

Speed of the steamer going upstream

= Speed of the steamer in still water – speed of water (why?)

$= (x - 4)$ km/hr

$\therefore$ Distance covered by the steamer going upstream in 25 hours

$= (x - 4)25$ km

$= (25x - 100)$ km

But distance covered upstream

$\Rightarrow \quad (x + 4)20 = (x - 4)25$

or $\quad 20x + 80 = 25x - 100$

$\quad 25x - 20x = (80 + 100)$ km/hr

$\quad 5x = 180$ km/hr

or $\quad x = 36$ km/hr

$\therefore$ Distance covered $= (x + 4)20$

$= (36 + 4) \times 20 = 40 \times 20$

$= 800$ km

Example 13. A car travelling at 60 km/hr left Vrindavan at 7.30 p.m. One hour later another car travelling at 80 km/hr started over the same road to overtake the first. How long must then second car travel?

Solution. Let the second car overtakes the first, x hours after it started.

First car travelled for $(x + 1)$ hours and distance covered by both cars is the same.

Distance covered by first car in $(x + 1)$ hr

$= s \times t$

$= 60(x + 1)$ km

and distance covered by second car in x hrs

$= 80x$ km

But $\quad 60(x + 1) = 80x$

$\Rightarrow \quad 6(x + 1) = 8x$

or $\quad 6x + 6 = 8x$

or $\quad 8x - 6x = 6$

or $\quad 2x = 6 \quad$ or $\quad x = 3$

Hence, the second car travelled for 3 hours.

Example 14. A painter can paint a building in 4 days and his apprentice can do it in 6 days. How long will it take them to paint this building if they work together on it except for 1 day when painter is ill and the apprentice works alone?

Solution. Suppose the painter works for x days then his apprentice works for $(x + 1)$ days (why?)

Work done by painter in 1 day $= \frac{1}{4}$

and work done by apprentice in 1 day $= \frac{1}{6}$

Work done by painter in x days

$$= \frac{1}{4} \times x = \frac{x}{4}$$

Work done by his apprentice in $(x + 1)$ days

$$= \frac{x+1}{6}$$

Work done by them together

$$= \frac{x}{4} + \frac{x+1}{6}$$

$$= \frac{3x + 2(x+1)}{12} = \frac{3x + 2x + 2}{12}$$

$$= \frac{5x+2}{12}$$

Since the whole work which is completed *i.e.*, painting the building can be taken as 1, therefore.

$$\frac{5x+2}{12} = 1$$

or $5x + 2 = 12$

or $5x = 10$

or $x = \frac{10}{5} = 2$

or $x = 2$

$\therefore$ Painter paints for x days *i.e.*, 2 days and his apprentice for $(2 + 1) = 3$ days

Thus, time taken is 3 days.

Example 15. In an auditorium, 300 tickets were sold. The total sale of tickets was ₹ 1250. If the tickets were of two denominations of ₹ 2.50 and ₹ 5.00, how many of each denomination were sold?

Solution. Let the number of tickets of the denomination ₹ 2.50 be x

Therefore, the number of tickets of the denomination ₹ 5.00 will be $(300 - x)$

Amount spent on ₹ 2.50 tickets

$$= ₹\ 2.50 \times x = ₹\ 2.50x$$

Amount spent on ₹ 5 tickets

$$= ₹\ 5 \times (300 - x)$$

Total amount spent = ₹ 1250

$\Rightarrow$ $2.50x + 5(300 - x) = 1250$

or $2.50x + 1500 - 5x = 1250$

or $1500 - 1250 = 5x - 2.50x$

or $250 = 2.50x$

or $x = \frac{250}{2.50} = \frac{\cancel{250} \times 100}{\cancel{250}}$

$= 100$

Therefore, number of tickets of denomination ₹ 2.50 = x = 100 and number of tickets of denomination ₹ 5 will be

$(300 - x) = (300 - 100) = 200$