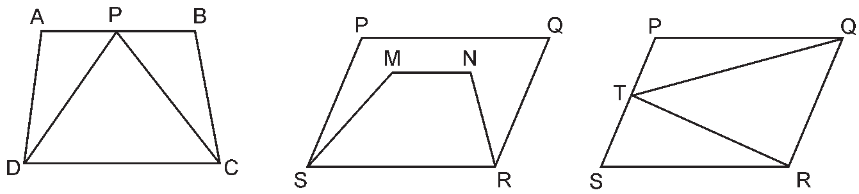
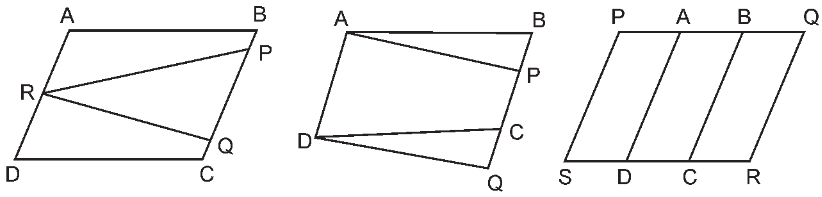
9 AREAS OF PARALLELOGRAMS AND TRIANGLES

EXERCISE 9.1

Q.1. *Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.*



(i) (ii) (iii)

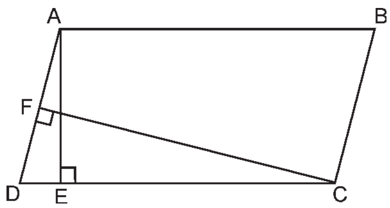


(iv) (v) (vi)

Sol. (i) Base DC, parallels DC and AB
(iii) Base QR, parallels QR and PS
(v) Base AD, parallels AD and BQ.

EXERCISE 9.2

Q.1. *In the figure, ABCD is a paralle-logram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.*



Sol. Area of parallelogram ABCD
$= AB \times AE$
$= 16 \times 8 \text{ cm}^2 = 128 \text{ cm}^2$
Also, area of parallelogram ABCD
$= AD \times FC = (AD \times 10) \text{ cm}^2$
$\therefore \quad AD \times 10 = 128$

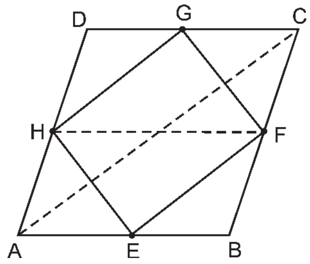
$$\Rightarrow \quad AD = \frac{128}{10} = \mathbf{12.8 \text{ cm Ans.}}$$

Q.2. *If E, F, G, and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).*

Sol. **Given :** A parallelogram ABCD · E, F, G, H are mid-points of sides AB, BC, CD, DA respectively

To Porve : ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Construction : Join AC and HF.



Proof : In ΔABC,
E is the mid-point of AB.
F is the mid-point of BC.

$\Rightarrow$ EF is parallel to AC and EF = $\frac{1}{2}$ AC ... (i)

Similarly, in ΔADC, we can show that

HG $\parallel$ AC and HG = $\frac{1}{2}$ AC ... (ii)

From (i) and (ii)
EF $\parallel$ HG and EF = HG
$\therefore$ EFGH is a parallelogram. [One pour of opposite sides is equal and parallel]

In quadrilateral ABFH, we have

HA = FB and HA $\parallel$ FB [AD = BC $\Rightarrow \frac{1}{2}$AD = $\frac{1}{2}$BC $\Rightarrow$ HA = FB]

$\therefore$ ABFH is a parallelogram. [One pair of opposite sides is equal and parallel]

Now, triangle HEF and parallelogram HABF are on the same base HF and between the same parallels HF and AB.

$\therefore$ Area of $\Delta HEF = \frac{1}{2}$ area of HABF ... (iii)

Similarly, area of $\Delta HGF = \frac{1}{2}$ area of HFCD ... (iv)

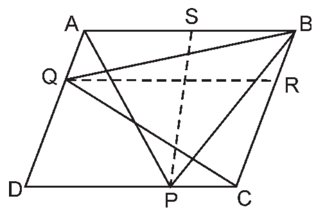
Adding (iii) and (iv),
Area of ΔHEF + area of ΔHGF

$= \frac{1}{2}$ (area of HABF + area of HFCD)

$\Rightarrow$ ar (EFGH) $= \frac{1}{2}$ ar (ABCD) **Proved.**

Q.3. *P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).*

Sol. **Given :** A parallelogram ABCD. P and Q are any points on DC and AD respectively.



To prove : ar (APB) = ar (BQC)

Construction : Draw PS $\parallel$ AD and QR $\parallel$ AB.

Proof : In parallelogram ABRQ, BQ is the diagonal.

$\therefore$ area of $\Delta BQR = \frac{1}{2}$ area of ABRQ ... (i)

In parallelogram CDQR, CQ is a diagonal.

$\therefore$ area of $\Delta RQC = \frac{1}{2}$ area of CDQR ... (ii)

Adding (i) and (ii), we have

area of ΔBQR + area of ΔRQC

$= \frac{1}{2}$ [area of ABRQ + area of CDQR]

$\Rightarrow$ area of $\Delta BQC = \frac{1}{2}$ area of ABCD ... (iii)

Again, in parallelogram DPSA, AP is a diagonal.

$\therefore$ area of $\Delta ASP = \frac{1}{2}$ area of DPSA ... (iv)

In parallelogram BCPS, PB is a diagonal.

$\therefore$ area of $\Delta BPS = \frac{1}{2}$ area of BCPS ... (v)

Adding (iv) and (v)

area of ΔASP + area of $\Delta BPS = \frac{1}{2}$(area of DPSA + area of BCPS)

$\Rightarrow$ area of $\Delta APB = \frac{1}{2}$ (area of ABCD) ... (vi)

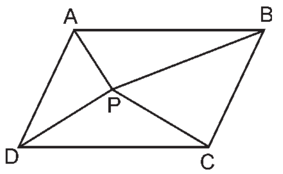
From (iii) and (vi), we have

area of ΔAPB = area of ΔBQC. **Proved.**

Q.4. *In the figure, P is a point in the interior of a parallelogram ABCD. Show that*

(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar(APB) + ar (PCD)



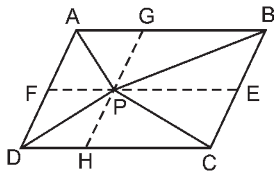
Sol. **Given :** A parallelogram ABCD. P is a point inside it.

To prove : (i) ar (APB) + ar(PCD)

$= \frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC)

= ar (APB) + ar (PCD)



Construction : Draw EF through P parallel to AB, and GH through P parallel to AD.

Proof : In parallelogram FPGA, AP is a diagonal,

$\therefore$ area of ΔAPG = area of ΔAPF ... (i)

In parallelogram BGPE, PB is a diagonal,

$\therefore$ area of ΔBPG = area of ΔEPB ... (ii)

In parallelogram DHPF, DP is a diagonal,

∴ area of ΔDPH = area of ΔDPF ... (iii)

In parallelogram HCEP, CP is a diagonal,

∴ area of ΔCPH = area of ΔCPE ... (iv)

Adding (i), (ii), (iii) and (iv)

area of ΔAPG + area of ΔBPG + area of ΔDPH + area of ΔCPH

= area of ΔAPF + area of ΔEPB + area of ΔDPF + area ΔCPE

⇒ [area of ΔAPG + area of ΔBPG] + [area of ΔDPH + area of ΔCPH]

= [area of ΔAPF + area of ΔDPF] + [area of ΔEPB + area of ΔCPE]

⇒ area of ΔAPB + area of ΔCPD = area of ΔAPD + area of ΔBPC ... (v)

But area of parallelogram ABCD

= area of ΔAPB + area of ΔCPD + area of ΔAPD + area of ΔBPC ... (vi)

From (v) and (vi)

area of ΔAPB + area of ΔPCD = $\frac{1}{2}$ area of ABCD

or, ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD) **Proved.**

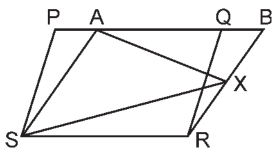
(ii) From (v),

⇒ ar (APD) + ar (PBC) = ar (APB) + ar (CPD) **Proved.**

Q.5. *In the figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that*

(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ *ar (PQRS)*



Sol. Given : PQRS and ABRS are parallelograms and X is any point on side BR.

To prove : (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

Proof : (i) In ΔASP and BRQ, we have

∠SPA = ∠RQB [Corresponding angles] ...(1)

∠PAS = ∠QBR [Corresponding angles] ...(2)

∴ ∠PSA = ∠QRB [Angle sum property of a triangle] ...(3)

Also, PS = QR [Opposite sides of the parallelogram PQRS] ...(4)

So, ΔASP ≅ ΔBRQ [ASA axiom, using (1), (3) and (4)]

Therefore, area of ΔPSA = area of ΔQRB

[Congruent figures have equal areas] ...(5)

Now, ar (PQRS) = ar (PSA) + ar (ASRQ]

= ar (QRB) + ar (ASRQ]

= ar (ABRS)

So, ar (PQRS) = ar (ABRS) **Proved.**

(ii) Now, ΔAXS and ||gm ABRS are on the same base AS and between same parallels AS and BR

$\therefore$ area of $\Delta AXS = \frac{1}{2}$ area of ABRS

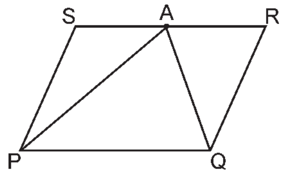
$\Rightarrow$ area of $\Delta AXS = \frac{1}{2}$ area of PQRS [$\because$ ar (PQRS) = ar (ABRS]

$\Rightarrow$ ar of (AXS) = $\frac{1}{2}$ ar of (PQRS) **Proved.**

Q.6. *A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?*

Sol. The field is divided in three triangles.

Since triangle APQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.



$\therefore$ ar (APQ) = $\frac{1}{2}$ ar (PQRS)

$\Rightarrow$ 2ar (APQ) = ar(PQRS)

But ar (PQRS) = ar(APQ) + ar (PSA) + ar (ARQ)

$\Rightarrow$ 2 ar (APQ) = ar(APQ) + ar(PSA) + ar (ARQ)

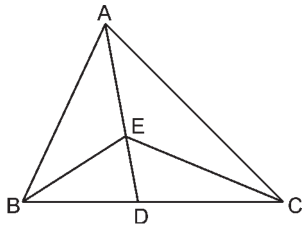
$\Rightarrow$ ar (APQ) = ar(PSA) + ar(ARQ)

Hence, area of ΔAPQ = area of ΔPSA + area of ΔARQ.

To sow wheat and pulses in equal portions of the field separately, farmer sow wheat in ΔAPQ and pulses in other two triangles or pulses in ΔAPQ and wheat in other two triangles. **Ans.**

EXERCISE 9.3

Q.1. *In the figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).*



Sol. **Given :** A triangle ABC, whose one median is AD. E is a point on AD.

To Prove : ar (ABE) = ar (ACE)

Proof : Area of ΔABD = Area of ΔACD ... (i)

[Median divides the triangle into two equal parts]

Again, in ΔEBC, ED is the median, therefore,

Area of ΔEBD = area of ΔECD .... (ii)

[Median divides the triangle into two equal parts]

Subtracting (ii) from (i), we have

area of ΔABD – area of ΔEBD = area of ΔACD – area of ΔECD

$\Rightarrow$ area of ΔABE = area of ΔACE

$\Rightarrow$ ar (ABE) = ar (ACE) **Proved.**

Q.2. *In a triangle ABC, E is the mid-point on median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).*

Sol. **Given :** A triangle ABC, in which E is the mid-point of median AD.

To Prove : ar(BED) = $\frac{1}{4}$ ar (ABC)

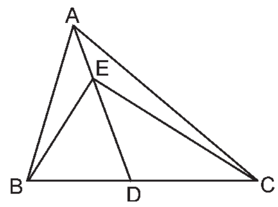
Proof : In ΔABC, AD is the median.

$\therefore$ area of ΔABD = area of ΔADC ... (i)

[Median divides the triangle into two equal parts]

Again, in ΔADB, BE is a median.

$\therefore$ area of ΔABE = area of ΔBDE ... (ii)



From (i), we have

area of $\Delta ABD = \frac{1}{2}$ area of ΔABC ... (iii)

From (ii), we have

area of $\Delta BED = \frac{1}{2}$ area of ΔABD ... (iv)

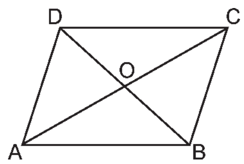
From (iii) and (iv), we have

area of $\Delta BED = \frac{1}{2} \times \frac{1}{2}$ area of ΔABC

$\Rightarrow$ area of $\Delta BED = \frac{1}{4}$ area of ΔABC

$\Rightarrow$ ar (BED) $= \frac{1}{4}$ ar(ABC) **Proved.**

Q.3. *Show that the diagonals of a parallelogram divide it into four triangles of equal area.*



Sol. Given : A parallelogram ABCD.

To Prove : area of ΔAOB = area of ΔBOC = area of ΔCOD = area of ΔAOD

Proof : AO = OC and BO = OD

[Diagonals of a parallelogram bisect each other]

In ΔABC, O is mid-point of AC, therefore, BO is a median.

$\therefore$ area of ΔAOB = area of ΔBOC ... (i)

[Median of a triangle divides it into two equal parts]

Similarly, in ΔCBD, O is mid-point of DB, therefore, OC is a median.

$\therefore$ area of ΔBOC = area of ΔDOC ... (ii)

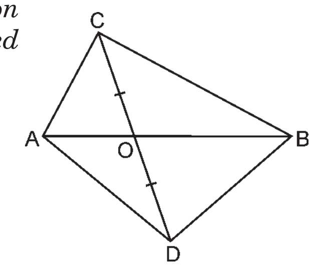
Similarly, in ΔADC, O is mid-point of AC, therefore, DO is a median.

$\therefore$ area of ΔCOD = area of ΔDOA ... (iii)

From (i), (ii) and (iii), we have

area of ΔAOB = area of ΔBOC = area of ΔDOC = area of ΔAOD **Proved.**

Q.4. *In the figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bsisected by AB at O, show that ar (ABC) = ar (ABD).*



Sol. Given : ABC and ABD are two triangles on the same base AB and line segment CD is bisected by AB at O.

To Prove : ar (ABC) = ar(ABD)

Proof : In ΔACD, we have

CO = OD [Given]

$\therefore$ AO is a median.

$\therefore$ area of ΔAOC = area of ΔAOD ... (i)

[Median of a triangle divides it into two equal parts]

Similarly, in ΔBCD, OB is median

∴ area of ΔBOC = area of ΔBOD ... (ii)

Adding (i) and (ii), we get

area of ΔAOC + area of ΔBOC = area of ΔAOD + area of ΔBOD

⇒ area of ΔABC = area of ΔABD

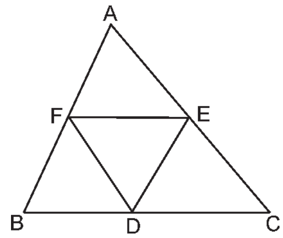
⇒ ar(ABC) = ar (ABD) **Proved.**

Q.5. *D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that*

(i) BDEF is a parallelogram. *(ii) ar(DEF) =* $\frac{1}{4}$ *ar(ABC)*

(iii) ar (BDEF) = $\frac{1}{2}$ *ar (ABC)*

Sol. Given : D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC.



To Prove : (i) BDEF is a parallelogram.

(ii) ar(DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar(BDEF) = $\frac{1}{2}$ ar (ABC)

Proof : (i) In ΔABC, E is the mid-point of AC and F is the mid-point of AB.

∴ EF || BC or EF || BD

Similarly, DE || BF.

∴ BDEF is a parallelogram ... (1)

(ii) Since DF is a diagonal of parallelogram BDEF.

Therefore, area of ΔBDF = area of ΔDEF ... (2)

Similarly, area of ΔAFE = area of ΔDEF ... (3)

and area of ΔCDE = area of ΔDEF ... (4)

From (2), (3) and (4), we have

area of ΔBDF = area of ΔAFE = area of ΔCDE = area of ΔDEF ... (5)

Again ΔABC is divided into four non-overlapping triangles BDF, AFE, CDE and DEF.

∴ area of ΔABC = area of ΔBDF + area of ΔAFE + area of ΔCDE + area of ΔDEF

= 4 area of ΔDEF ... (6) [Using (5)]

⇒ area of ΔDEF = $\frac{1}{4}$ area of ΔABC

⇒ ar (DEF) = $\frac{1}{4}$ ar (ABC) **Proved.**

(iii) Now, area of parallelogram BDEF = area of ΔBDF + area of ΔDEF

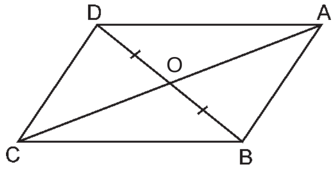
= 2 area of ΔDEF

= $2\cdot\frac{1}{4}$ area of ΔABC

= $\frac{1}{2}$ area of ΔABC

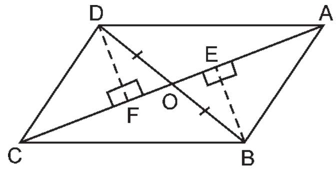
Hence, ar (BDEF) = $\frac{1}{2}$ ar (ABC) **Proved.**

Q.6. *In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that :*
(i) ar (DOC) = ar(AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CD or ABCD is a parallelogram.



Sol. Given : Diagonal AC and BD of quadrilateral ABCD intersect at O such that OB = OD and AB = CD.

To Prove : (i) ar (DOC) = ar (AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram.



Construction : Draw perpendiculars DF and BE on AC.

Proof : (i) area of ΔDCO = $\frac{1}{2}$ CO × DF ... (1)

area of ΔABO = $\frac{1}{2}$ AO × BE ... (2)

In ΔBEO and ΔDFO, we have
BO = DO [Given]
∠BOE = ∠DOF [Vertically opposite angles]
∠BEO = ∠DFO [Each = 90°]
⇒ ΔBOE ≅ ΔBOF [SAS congruence]
⇒ BE = DF [CPCI] ... (3)
OE = OF [CPCT] ... (4)

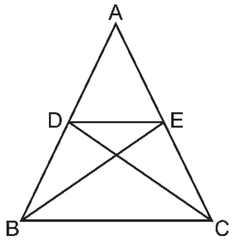
In ΔABE and ΔCDF, we have,
AB = CD [Given]
BE = DF [Proved above]
∠AEB = ∠CFD [Each = 90°]
∴ ΔABE ≅ CDF [RHS congruence]
⇒ AE = CF [CPCT] ... (5)

From (4) and (5), we have
OE + AE = OF + CF
⇒ AO = CO ... (6)
Hence, ar (DOC) = ar (AOB). [From (1), (2), (3) and (6)] **Proved.**

(ii) In quadrilateral ABCD, AC and BD are its diagonals, which intersect at 0.
Also, BO = OD [Given]
AO = OC [Proved above]
⇒ ABCD is a parallelogram [Diagonals of a quadrilateral bisect each other]
⇒ BC||AD.
So, ar(DCB) = ar(DCB) **Proved.**

(iii) In (ii), we have proved that ABCD is a parallelogram.
Hence, ABCD is a parallelograms **Proved.**

Q.7. *D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.*



Sol. **Given :** D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC)

To Prove : DE || BC

Proof : ar (DBC) = ar (EBC) [Given]

Also, triangles DBC and EBC are on the same base BC.

∴ they are between the same parallels

i.e., DE || BC **Proved.**

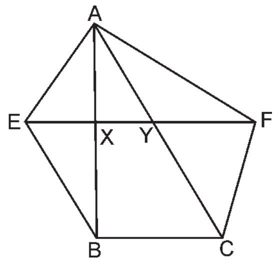
[∵ triangles on the same base and between the same parallels are equal in area]

Q.8. *XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that*

ar (ABE) = ar (ACF)

Sol. **Given :** XY is a line parallel to side BC of a ΔABC.

BE || AC and CF || AB



To Prove : ar (ABE) = ar (ACF)

Proof : ΔABE and parallelogram BCYE are on the same base BC and between the same parallels BE and AC.

$\therefore$ ar (ABE) = $\frac{1}{2}$ ar (BCYE) ... (i)

Similarly,

ar (ACF) = $\frac{1}{2}$ ar (BCFX) ... (ii)

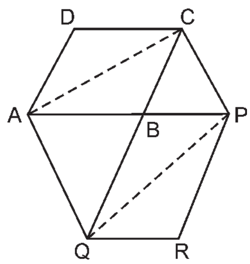
But parallelogram BCYE and BCFX are on the same base BC and between the same parallels BC and EF.

∴ ar (BCYE) = ar (BCFX) ... (iii)

From (i), (ii) and (iii), we get

ar (ABE) = ar (ACF) **Proved.**

Q.9. *The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure,). Show that ar (ABCD) = ar (PBQR).*



Sol. **Given :** ABCD is a parallelogram.

CP || AQ, BP || QR, BQ || PR

To Prove : ar (ABCD) = ar (PBQR)

Construction : Join AC and PQ.

Proof : AC is a diagonal of parallelogram ABCD.

∴ area of ΔABC = $\frac{1}{2}$ area of ABCD ... (i)

[A diagonal divides the parallelogram into two parts of equal area]

Similarly, area of $\Delta PBQ = \frac{1}{2}$ area of PBQR ... (ii)

Now, triangles AQC and AQP are on the same base AQ and between the same parallels AQ and CP.

$\therefore$ area of ΔAQC = area of ΔAQP ... (iii)

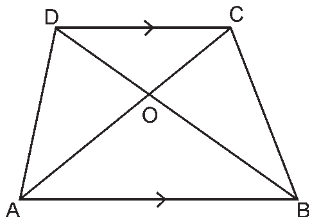
Subtracting area of ΔAQB from both sides of (iii),

area of ΔAQC – area of ΔAQB = area of ΔAQP – area of ΔAQB

$\Rightarrow$ area of ΔABC = area of ΔPBQ ... (iv)

$\Rightarrow \frac{1}{2}$ area of ABCD = $\frac{1}{2}$ area of PBQR [From (i) and (ii)]

$\Rightarrow$ area of ABCD = area of PBQR **Proved.**



Q.10. *Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).*

Sol. **Given :** Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

To Prove : ar (AOD) = ar (BOC)

Proof : Triangles ABC and BAD are on the same base AB and between the same parallels AB and DC.

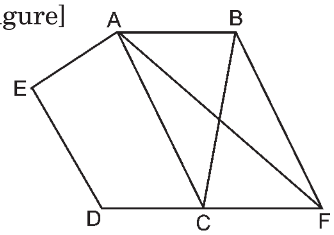
$\therefore$ area of ΔABC = area of ΔBAD

$\Rightarrow$ area of ΔABC – area of ΔAOB = area of ΔABD – area of ΔAOB

[subtracting area of ΔAOB from both sides]

$\Rightarrow$ area of ΔBOC = area of ΔAOD [From figure]

Hence, ar (BOC) = ar (AOD) **Proved.**



Q.11. *In the Figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that*

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Sol. **Given :** ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

To Prove : (i) ar(ACB) = ar (ACF)

(ii) ar(AEDF) = ar (ABCDE)

Proof : (i) ΔACB and ΔACF lie on the same base AC and between the same parallels AC and BF.

Therefore, ar (ACB) = ar (ACF) **Proved.**

(ii) So, ar (ACB) + ar (ACDE) = ar (ACF) + ar (ACDE)

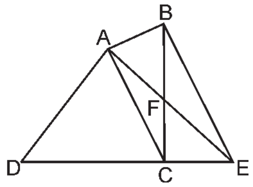
[Adding same areas on both sides]

$\Rightarrow$ ar (ABCDE) = ar(AEDF) **Proved.**

Q.12. *A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.*

Sol. ABCD is the plot of land in the shape of a quadrilateral. From B draw BE | | AC to meet DC produced at E.

To Prove : ar (ABCD) = ar (ADE)



Proof : ΔBAC and ΔEAC lie on the same base AC and between the same parallels AC and BE.

Therefore, ar (BAC) = ar (EAC)

So, ar (BAC) + ar (ADC) = ar (EAC) + ar (ADC)

[Adding same area on both sides]

or, ar (ABCD) = ar (ADE)

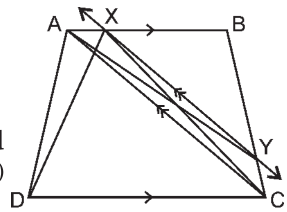
Hence, the gram Panchayat took over ABD and gave ΔCEF.

Q.13. *ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).*

Sol. **Given :** ABCD is a trapezium with AB || DC.

AC || XY.

To Prove : ar(ΔADX) = ar(ΔACY).



Construction : Join XC

Proof : Since AB || DC ∴ AX || DC

⇒ ar (ADX) = ar (AXC) ... (i)

(Having same base AX and between same parallels)

Since AC || XY

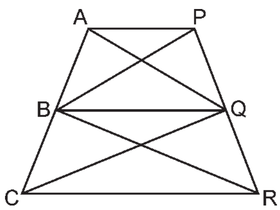
⇒ ar (AXC) = ar (ACY) ... (ii)

(Having same base AC and between same parallels)

⇒ ar (ADX) = ar (ACY) [From (i), (ii)] **Proved.**

Q.14. *In the figure, AP || BQ || CR. Prove that ar(AQC) = ar (PBR).*

Sol. **Given :** In figure, AP || BQ || CR.



To Prove : ar (AQC) = ar (PBR)

Proof : Triangles ABQ and PBQ are on the same base BQ and between the same parallels AP and BQ.

∴ ar(ABQ) = ar(PBQ) ... (1)

[Triangles on the same base and between the same parallels are equal in area]

Similarly triangle BQC and BQR on the same base BQ and between the same parallels BQ and CR

∴ ar(BQC) = ar (BQR) ... (2) [Same reason]

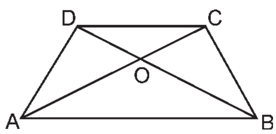
Adding (1) and (2), we get

ar (ABQ) + ar (BQC) = ar (PBQ) + ar (BQR)

⇒ ar(AQC) = ar (PBR). **Proved.**

Q.15. *Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.*

Sol. **Given :** Diagonals AC and BD of a quadrilateral ABCD intersect at O, such that ar (AOD) = ar (BOC)



To Prove : ABCD is a trapezium.

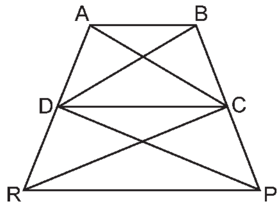
Proof : ar (ΔAOD) = ar(ΔBOC)

⇒ ar(AOD) + ar(BOA) = ar (BOC) + ar (BOA)

⇒ ar(ABD) = ar(ABC)

But, triangle ABD and ABC are on the same base AB and have equal area.
∴ they are between the same parallels,
i.e., AB || DC
Hence, ABCD is a trapezium. [∵ A pair of opposite sides is parallel]
Proved.

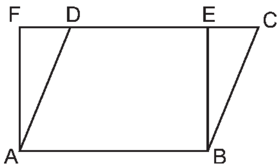
Q.16. *In the figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.*



Sol. Given : ar(DRC) = ar (DPC) and ar (BDP) = ar (ARC)
To Prove : ABCD and DCPR are trapeziums.
Proof : ar (BDP) = ar (ARC)
⇒ ar (DPC) + ar (BCD) = ar (DRC + ar (ACD)
⇒ ar (BCD) = ar(ACD) [∵ ar (DRC) = ar (DPC)]
But, triangles BCD and ACD are on the same base CD.
∴ they are between the same parallels,
i.e., AB || DC
Hence, ABCD is a trapezium. ... (i) **Proved.**
Also, ar (DRC) = ar (DPC) [Given]
Since, triangles DRC and DPC are on the same base CD.
∴ they are between the same parallels,
i.e., DC || RP
Hence, DCPR is a trapezium ... (ii) **Proved.**

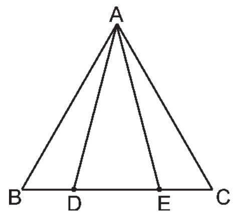
EXERCISE 9.4 (Optional)

Q.1. *Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.*



Sol. Given : A parallelogram ABCD and a rectangle ABEF having same base and equal area.
To Prove : 2(AB + BC) > 2(AB + BE)
Proof : Since the parallelogram and the rectangle have same base and equal area, therefore, their attitudes are equal.
Now perimeter of parallelogram ABCD.
= 2 (AB + BC) ... (i)
and perimeter of rectangle ABEF
= 2 (AB + BE) ... (ii)
In ΔBEC, ∠BEC = 90°
∴ ∠BCE is an acute angle.
∴ BE < BC ... (iii) [Side opposite to smaller angle is smaller]
∴ From (i), (ii) and (iii) we have
2(AB + BC) > 2(AB + BE) **Proved.**

Q.2. *In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).*



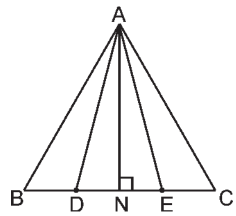
Can you now answer the question that you have left the 'Introduction' of this chapter, whether the field of Budhia *has been actually divided into three parts of equal area?*

[Remark : Note that by taking BD = DE = CE, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n *equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into n triangles of equal areas.]*

Sol. **Given :** A triangle ABC, in which D and E are the two points on BC such that BD = DE = EC

To Prove : ar (ABD) = ar (ADE) = ar (AEC)

Construction : Draw AN ⊥ BC



Now, ar (ABD) $= \frac{1}{2} \times$ base × altitude (of ΔABD)

$= \frac{1}{2} \times BD \times AN$

$= \frac{1}{2} \times DE \times AN$ [As BD = DE]

$= \frac{1}{2} \times$ base × altitude (of ΔADE)

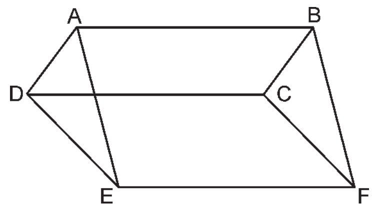
= ar (ADE)

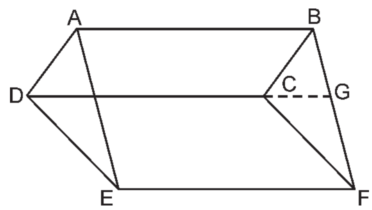
Similarly, we can prove that

ar (ADE) = ar (AEC)

Hence, ar (ABD) = ar (ADE) = ar (AEC) **Proved.**

Q.3. *In the figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar(BCF)*





Sol. **Given :** Three parallelograms ABCD, DCFE and ABFE.

To Prove : ar (ADE) = ar (BCF)

Construction : Produce DC to intersect BF at G.

Proof : ∠ADC = ∠BCG ... (i) [Corresponding angles]

∠EDC = ∠FCG ... (ii) [Corresponding angles]

⇒ ∠ADC + ∠EDC = ∠BCG + ∠FCG [By adding (i) and (ii)]

⇒ ∠ADE = ∠BCF ... (iii)

In ΔADE and ΔBCF, we have

AD = BC [Opposite sides of || gm ABCD]

DE = CF [Opposite sides of || gm DCEF]

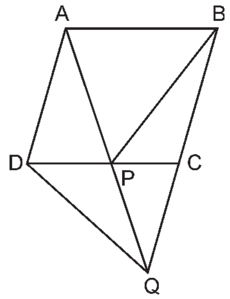
$\angle ADE = \angle BCF$ [From (iii)]

$\therefore \Delta ADE \cong \Delta BCF$ [SAS congruence]

$\Rightarrow$ ar (ADE) = ar (BCF)

[Congruent triangles are equal in area] **Proved.**

Q.4. *In the figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).*



Sol. Given : ABCD is a parallelogram, in which BC is produced to a point Q such that AD = CQ and AQ intersects DC at P.

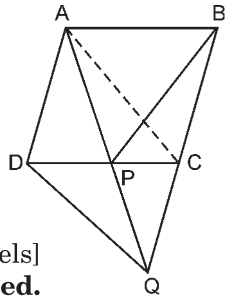
To Prove : ar(BPC) = ar (DPQ)

Construction : Join AC.

Proof : Since AD ∥ BC $\Rightarrow$ AD ∥ BQ

ar (ΔADC) = ar (ΔADQ) ... (i)

[Having same base AD and between same parallel]



$\Rightarrow$ ar (ΔADP) + ar (ΔAPC) = ar (ΔADP) + ar (ΔDPQ)

[From figure]

$\Rightarrow$ ar (ΔAPC) = ar (ΔDPQ)

Now, since AB ∥ DC $\Rightarrow$ AB ∥ PC

ar (ΔAPC) = ar (ΔBPC) ... (ii)

[Having same base PC and between same parallels]

$\Rightarrow$ ar (ΔBPC, = ar (ΔDPQ) [From (i) and (ii)] **Proved.**

Q.5. *In figugre, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. IF AE intersects BC at F, show that*

(i) ar (BDE) = $\frac{1}{4}$ *ar(ABC)*

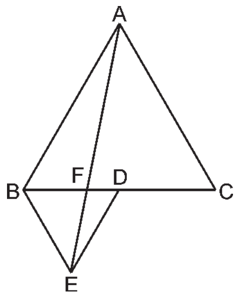
(ii) ar (BDE) = $\frac{1}{2}$ *ar(ABC)*

(iii) ar (ABC) = 2 ar(BEC)

(iv) ar (BEF = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ *ar (AFC)*



[Hint : Join EC and AD. Show that BE | | AC and DE | | AB, etc.]

Sol. Given : ABC and BDE are equilateral triangles, D is the mid-point of BC and AE intersects BC at F.

Construction : Join AD abd EC.

Proof : $\angle ACB = 60°$

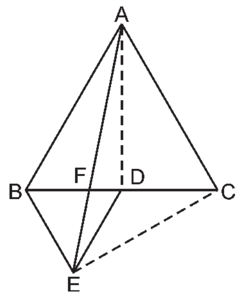
[Angle of an equilateral triangle] ..(1)

$\angle EBC = 60°$

$\Rightarrow \angle ACB = \angle EBC$ [Same reason]

$\Rightarrow$ AC | | BE [Alternate angle are equal]

Similarly, we can prove that AB | | DE ... (2)



(i) D is the mid-point of BC, so AD is a median of ΔABC

$\therefore$ ar (ABD) = $\frac{1}{2}$ ar (ABC) ... (3)

ar(DEB) = ar(DEA) [Triangles on the same base DE and between the same parallels DE and AB]

⇒ ar(DEB) = ar(ADF) + ar(DEF) ...(4)

Also, ar(DEB) = $\frac{1}{2}$ ar(BEC) [DE is a median]

⇒ = $\frac{1}{2}$ ar(BEA) [Triangles on the same base DE and between the same parallels BE and AC] ... (5)

⇒ 2 ar(DEB) = ar(BEA)

⇒ 2 ar(DEB) = ar(ABF) + ar(BEF) ... (6)

Adding (4) and (6), we get

3 ar(DEB) = ar(ADF) + ar(DEF) + ar(ABF) + ar(BEF)

⇒ 3 ar(DEB) = ar(ADF) + ar(ABF) + ar(DEF) + ar(BEF)

= ar(ABD) + ar(BDE)

⇒ 2 ar(DEB) = ar(ABD)

⇒ ar(DEB) = $\frac{1}{2}$ ar(ABC) [From (3)]

⇒ ar(DEB) = $\frac{1}{4}$ ar(ABC) **Proved.**

(ii) From (5) above, we have

ar(BDE) = $\frac{1}{2}$ ar(BAE) **Proved.**

(iii) ar(DEB) = $\frac{1}{2}$ ar(BEC) [DE is a median]

⇒ $\frac{1}{4}$ ar(ABC) = ar $\frac{1}{2}$ (BEC) [From part (i)]

⇒ ar(ABC) = 2 ar (BEC) **Proved.**

(iv) ar(DEB) = ar(BEA) [Triangles on the same base DE and between the same parallels DE AB] ... (7)

⇒ ar(DEB) – ar(DEF) = ar (DEA) – ar (DEF)

⇒ ar(BFE) = ar(AFD) **Proved.**

(v) *****

(vi) From (v), we have ar(FED) $= \frac{1}{2}$ ar(BFE)

$= \frac{1}{2}$ ar(AFD) [From part (iv)]

Now ar (AFC) = ar(AFD) + ar(ADC)

= ar(AFD) + $\frac{1}{2}$ ar(ABC) [BE is a median]

= ar(AFD) + 2ar(BDE) [From part (i)]

= ar(AFD) + 2ar(ADE)

= ar(AFD) + 2ar(AFD) + 2 ar(DEF)

= 3 ar(AFD) + ar(BFE) [From part (v)]

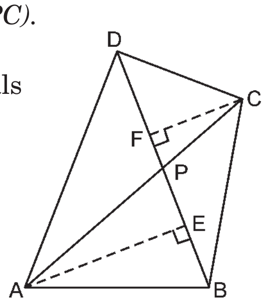
= 3 ar(AFD) + ar(AFD) [From part (iv)]

= 4 ar(AFD)

$\therefore \frac{1}{8}$ ar(AFC) $= \frac{1}{2}$ ar (AFD)

= ar (FED) (From above] **Proved.**

Q.6. *Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(APB) × ar(CPD) = ar (APD) × ar (BPC). Hint : From A and C, draw perpendiculars to BD.]*



Sol. Given : AB CD is a quadrilateral whose diagonals intersect each other at P.

Construction : Draw AE ⊥ BD and CF ⊥ BD.

Proof : ar (APB) $= \frac{1}{2} \times PB \times AE$ ... (i)

ar (CPD) $= \frac{1}{2} \times DP \times CF$ ... (ii)

Now, ar (BPC) $= \frac{1}{2} \times BP \times CF$ ... (iii)

ar (APD) $= \frac{1}{2} \times DP \times AE$ ... (iv)

From (i) and (ii),

ar (APB) × ar (CPD) $= \frac{1}{4} \times PB \times DP \times AE \times CF$ ... (v)

From (iii) and (iv), we have

ar (BPC) × ar (APD) $= \frac{1}{4} \times BP \times DP \times CF \times AE$ ... (vi)

$\therefore$ ar (APB) × ar (CPD) = ar (BPC) × ar (APD)

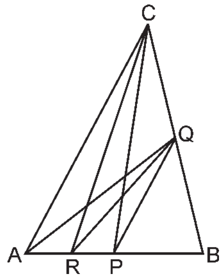
[From (v) and (vi)] **proved**

Q.7. *P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that*

(i) ar (PQR) $= \frac{1}{2}$ ar(ARC) *(ii) ar (RQC) $= \frac{3}{8}$ ar (ABC)*

(iii) ar(PBQ) = ar(ARC)

Sol. **Given :** A triangle ABC, P and Q are mid-points of AB and BC, R is the mid point of AP.



Proof : CP is a median of ΔABC

$\Rightarrow$ ar (APC) = ar (PBC) = ar $\frac{1}{2}$ (ABC)

median divides a triangle into two triangles of equal area] ... (1)

CR is a median of ΔAPC

$\therefore$ ar(ARC) = ar(PRC) = $\frac{1}{2}$ ar(APC) ...(2)

QR is a median of ΔAPQ.

$\therefore$ ar(ARQ) = ar(PRQ) = $\frac{1}{2}$ ar(APQ) ...(3)

PQ is a median of ΔPBC

$\therefore$ ar(PQC) = ar(PQB) = $\frac{1}{2}$ ar(PBC) ...(4)

PQ is a median of ΔRBC

ar(RQC) = ar(PQC) = $\frac{1}{2}$ ar(RBC) ...(5)

(i) ar(PQA) = ar (PQC) [Triangles on the same base PQ and between the same parallels PQ and AC]

$\Rightarrow$ ar(ARQ) + ar(PQR) = $\frac{1}{2}$ ar (PBC) [From (4)]

$\Rightarrow$ ar(PRQ) + ar(PRQ) = $\frac{1}{2}$ ar (APC) [From (3) and (1)]

$\Rightarrow$ 2 ar(PRQ) = ar(ARC) [From (2)]

$\Rightarrow$ ar(PRQ) = $\frac{1}{2}$ ar(ARC) **Proved.**

(ii) From (5), we have

ar(RQC) = $\frac{1}{2}$ ar (RBC)

= $\frac{1}{2}$ ar(PBC) + $\frac{1}{2}$ ar(PRC)

= $\frac{1}{4}$ ar(ABC) + $\frac{1}{4}$ ar(APC) [From (1) and (2)]

= $\frac{1}{4}$ ar(ABC) + $\frac{1}{8}$ ar(ABC) [From (1)]

= ar(RQC) = $\frac{3}{8}$ ar(ABC) **Proved.**

(iii) ar(PBQ) = $\frac{1}{2}$ ar(PBC) [From (4)]

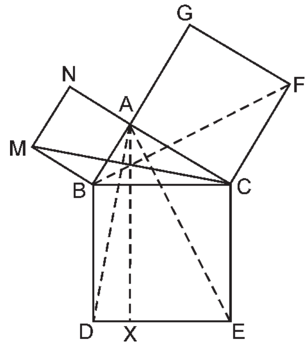
= $\frac{1}{4}$ ar(ABC) [From (1)] ... (6)

ar(ARC) = $\frac{1}{2}$ ar(APC) [From (2)]

$= \frac{1}{4}$ ar(ABC) [From (1)] ... (7)

From (6) and (7) we have ar(PBQ) = ar (ARC) **Proved.**

Q.8. *In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that :*



(i) ΔMBC ≅ ΔABD

(ii) ar(BYXD) = 2 ar (MBC)

(iii) ar(BYXD) = ar (ABMN)

(iv) ΔFCB ≅ ΔACE

(v) ar(CYXE) = 2 ar (FCB)

(vi) ar(CYXE) = ar (ACFG)

(vii) ar(BCED)) = ar (ADMN + ar (ACFG)

Note : *Result (vii) is the famous Theorem of Pythagoras. You shall learn a simpler proof of this theorem in Class X.*

Sol. (i) In DMBC and ΔABD, we have

MB = AB [Sides of a square]

BC = BD [Sides of a square]

∠MBC = ∠ABD [∠MBC = 90° + ∠ABC, and ∠ABC = 90° + ∠ABC]

∴ ΔMBC ≅ ΔABD [SAS congruent]

(ii) ar(ΔMBC) ≅ ar(ABD) [Congruent triangles have equal area]

$\Rightarrow \frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY [BC = BD]

$\therefore \text{ar(MBC)} = \frac{1}{2} \times BD \times BY$

⇒ Height of ΔMBC = BY [BC = BD]

$\therefore \text{ar(MBC)} = \frac{1}{2} \times BC \times BY$

⇒ 2 ar(MBC) = BC × BY ... (1)

Also, ar(BY × D) = BD × BY

= BC × BY [BC = BD] ... (2)

From (1) and (2), we have ar(BY × D) = ar (MBC) **Proved.**

(iii) ar(BY × D) = 2·ar(MBC) [From part (ii)]

$= 2 \times \frac{1}{2} \times MB \times$ height of MBC corresponding to BC

= MB × AB [MB | | NC and AB ⊥ MB]

= AB × AB [∵ AB = MB]

= AB^2

⇒ ar(BY × D) = ar (ABMN) **Proved.**

(iv) In ΔFCB and ΔACR, we have

FC = AC [Sides of a square]

BC = CE [Sides of a square]

$\angle FCB \cong \Delta ACE$ [SAS congruence] **Proved.**

(v) $\frac{1}{2} \times BC \times \text{height} = \frac{1}{2} \times CE \times CY$

$\Rightarrow$ Height of ΔFCB = CY [BC = CE]

$\therefore$ ar[FCB] $= \frac{1}{2} \times BC \times CY$

$\Rightarrow$ 2ar[FCB] = BC × CY ... (3)

Also, ar (CYXE) = CE × CY

= BC × CY ... (4)

From (3) and (4), we have

ar(CYXE) = 2 ar (FCB) **Proved.**

(vi) ar(CYXE) $= 2 \times \frac{1}{2} \times FC \times$ height of ΔFCB corresponding to FC

= FC × AC [FC || GB and AC ⊥ FC]

= AC × AC [AC = FC]

= AC^2

$\Rightarrow$ 2ar(CYXE) = ar(ACFG) **Proved.**

(vii) From (iii) and (vi), we have

ar(BYXD) + ar(CYXE) = ar(ABMN) + ar(ACFG)

$\Rightarrow$ ar(BCED) + ar(ABMN) + ar(ACFG) **Proved.**