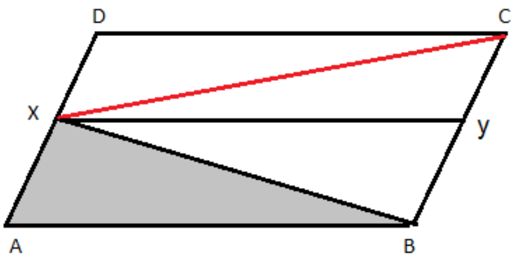
Class-IX Math SOLVED CBSE TEST PAPER

Chapter: Area of Parallelogram and Triangles

1. Q. The area of parallelogram ABCD is 40 cm2. If X be the midpoint of AD then find the area of triangle AXB

Solution



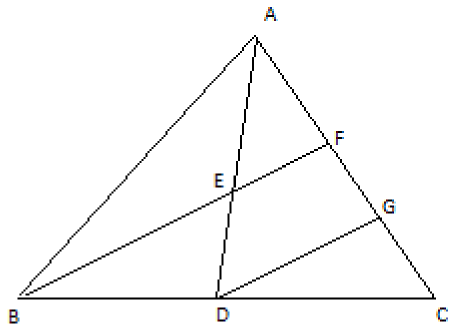
Draw XY II AB II CD

so, ABYX will be IIgm and Xb is its diagonal

Ar (AXB) = ½ ar(ABYX) = ½ x ½ ar(ABCD) = ¼ x 40 = 10

2. Q. AD is a median of triangle ABC and E is the midpoint of AD. BE produced meets AC in F . Prove that AF = 1/3 AC

Construction: draw DG parallel to BF.



In triangle ADG, EF parallel to base and E is the midpoint of AD.

Therefore AF = FG.(Midpoint theorem) ------------(i)

In triangle BCF, DG is parallel to BF and D is mid pint of BC

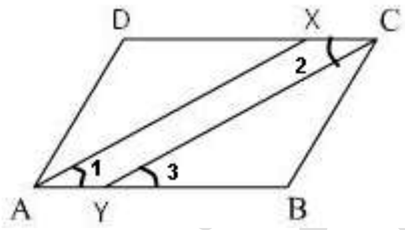
FG = GC (Midpoint theorem) -----------(ii)

From (i) and (ii)

Now, AF = FG = GC = 1/3 AC

3. Q. ABCD is a II gm and line segment AX and CY bisects angles A and C respectively where X is a point on AB. To prove AX II CY

Solution:



Given, ABCD is a parallelogram

∠A = ∠C (Opposite angles of the parallelogram are equal)

½ ∠A = ½ ∠C => ∠1 = ∠2 ------------------(1) [AX is bisector of ∠A and CY is bisector of ∠C]

Now, AB || CD and CY is the transversal.

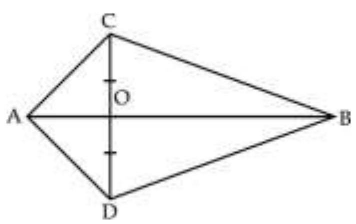
∴ ∠2 = ∠3 ...(2) [Alternate interior angles]

From (1) and (2), we get ∠1 = ∠3

Transversal AB intersects AX and CY at A and Y respectively such that ∠1 = ∠3 i.e., corresponding angles formed are equal. AX || CY

4. Q. ΔABC and ΔABD are two triangles on the same base AB. If line segment CD is bisected by AB at O. Show that ar(ABC) = ar(ABD)

Solution:



In ΔDAC, AO is the median, (given, CD is bisected by AB)

ar(DAO) = ar(CAO) -----------{i} (a median of a triangle divides it into 2 triangles of equal areas)

Similarly, BO is the median of ΔCBD

∴ ar(DBO) = ar(CBO) -----------{ii} (a median of a triangle divides it into 2 triangles of equal areas)

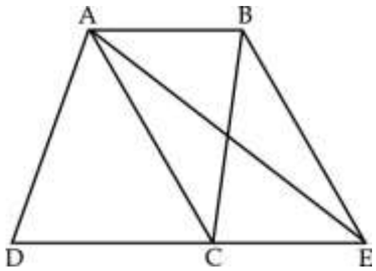
Adding {i} and {ii}, we get,

ar(DAO) + ar(DBO) = ar(CAO) + ar(CBO)

i.e., ar(ABD) = ar(ABC)

or, ar(ABC) = ar(ABD)

5. Q. In the figure, ABCD is a quadrilateral and BE||AC, also BE meets DC produced at E. Show that ar(ADE) = ar(ABCD)



Solution:

ABCD is a quadrilateral and BE||AC

ar (ABC) = ar (AEC)

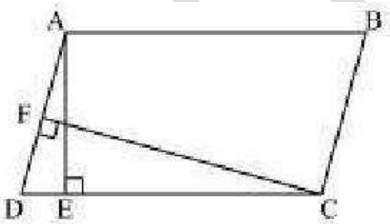
adding ar(ADC) in both sides

ar (ABC) + ar(ADC) = ar (AEC) + ar(ADC)

ar(ADE) = ar(ABCD)

6. Q. In the given figure, ABCD is parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

Answer:



In parallelogram ABCD, CD = AB = 16cm [Opposite sides of a parallelogram are equal]

We know that Area of a parallelogram = Base × Corresponding altitude

Area of parallelogram ABCD = CD × AE = AD × CF =>

16 cm × 8 cm = AD × 10 cm

Thus, the length of AD = (16 cm × 8 cm)/10 = 12.8 cm.

7. The diagonals of a quadrilateral are perpendicular to each other. Show that the quadrilateral formed by joining the mid points of its sides is a rectangle.

Solution:

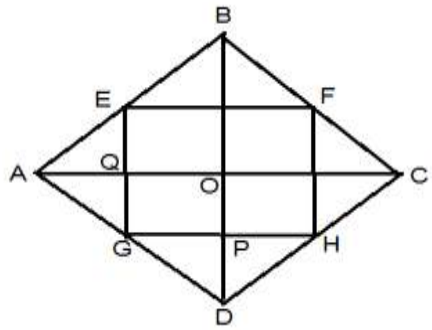
E and F are mid point of AB and BC

EF = ½ AC and EF II AC ----------(i)

G and H are midpoint of AD and CD

GH = ½ AC and GH II AC ---------(ii)

from (i) and (ii)



EF II GH and EF = GH

So EFGH is IIgm

Now,

Now, GH | | AC = > OQ II PG

=> <QO P + <OPG = 180^0

but AC ⊥ BD (Given) => <QO P = 90^0

=> 90^0 + <OPG = 180^0

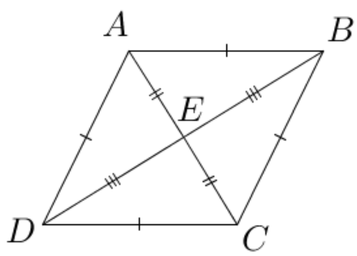
<OPG = 90^0

Here EFGH is IIgm having one angle 90^0

Thus, PQRS is a rectangle

8. Prove that "The diagonals of a rhombus are perpendicular to each other".

Solution:



In Δ AEB and AED

DE = BE ; AE = AE and AB = AD

By SSS , AEB ≅ AED

By CPCT, <AEB = < AED

but, <AEB + < AED = 180^0

=> <AEB = < AED = 90^0

Thus, AC ⊥ BD

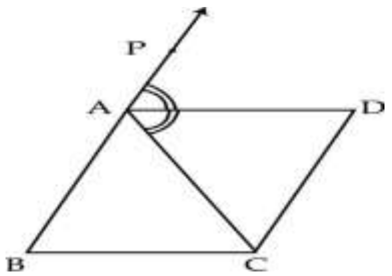
9. ABC is an isosceles triangle in which AB = AC. AD bisects exterior <PAC and CD II AB. Show that (i) <DAC = < BCA (ii) ABCD is a parallelogram

Solution:

ABC is an isosceles triangle in which AB = AC

=> <ACB = < ABC ----------------(i)

But, CD II AB => <ACB = < DAC --- (ii) [proved part 1st.]



Now, AD bisects exterior <PAC

=> < DAC = <DAP = ½ <PAC--- (iii)

From (i) and (ii) and (iii)

< DAP = <ABC

These are the pairs of corresponding angles

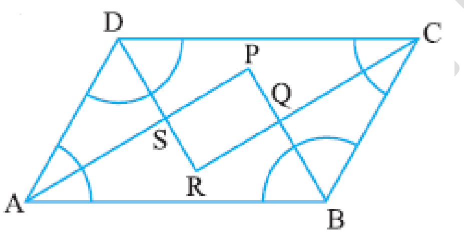
AD II BC

In qud. ABCD , AD II BC and CD II AB

Thus, ABCD is parallelogram

10. Show that the bisectors of angles of a parallelogram form a rectangle.

Solution:



Let P, Q, R and S be the points of intersection of the bisectors of ∠A and ∠B, ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A respectively of parallelogram ABCD

In ΔASD,

Since DS bisects ∠D and AS bisects ∠A, therefore,

∠DAS + ∠ADS = ½∠A + ½∠D = ½(∠A + ∠D)

= ½ × 180° = 90°

Also, ∠DAS + ∠ADS + ∠DSA = 180° (Angle sum property of a triangle)

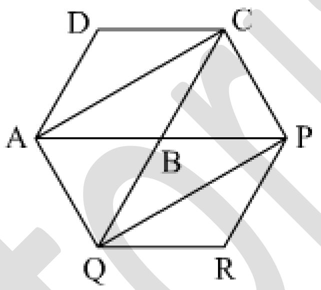
or, 90° + ∠ DSA = 180°

or, ∠DSA = 90°

So, ∠PSR = 90° (Being vertically opposite to ∠DSA)

11. The side AB of parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar (ABCD) = ar (PBQR)

Solution:



ΔAQC and ΔAQP are on the same base AQ and between the same parallel AQ and CP.

=> ar (ΔACQ) = ar (ΔAQP)

(Triangles on the same base and between the same parallels have equal area)

⇒ ar (ΔACQ) – (ΔABQ) = ar (ΔAQP) – ar (ΔABQ)

⇒ ar(ΔABC) = ar (ΔPBQ) -----------(i)

We know that, the diagonal of a parallelogram divides it into two triangle having equal areas

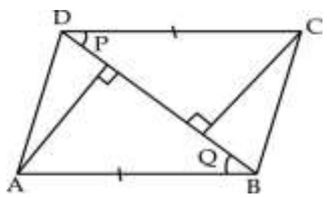
=> ½ ar(ABCD = ar(ΔABC)

and ½ (PBQR) = ar (ΔPBQ) ------------------ (ii)

From (i) and (ii) , ar (ABCD) = ar (PBQR)

12. In the figure ABCD is a parallelogram and AP, CQ are perpendiculars drawn from vertices A and C on diagonal BD.

Show that (i) ΔAPB ≅ ΔCQD (ii) AP = CQ



Solution: In ΔAPB and ΔCQD

< P = < Q ;

AB = CD

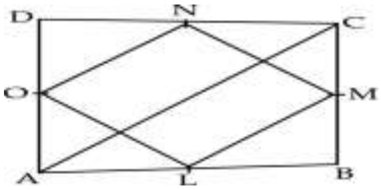
and < < ABP = < CDQ

By AAS ; ΔAPB ≅ ΔCQD

By CPCT, AP = CQ

13. ABCD is a quadrilateral in which L, M, N and O are the mid points of the sides AB, BC, CD and DA respectively as in the figure below. Show that

(i) ON||AC and ON = 1/2 AC (ii) ON = LM (iii) LMNO is a ||gm.



Solution:

In Δ ACD, N and O are the mid points of the sides CD and DA respectively

=> ON||AC and ON = 1/2 AC ----------------(i)

Similarly, In Δ ABC, L and M are the mid points of the sides AB and BC respectively

=> LM||AC and LM = 1/2 AC ------------------(ii)

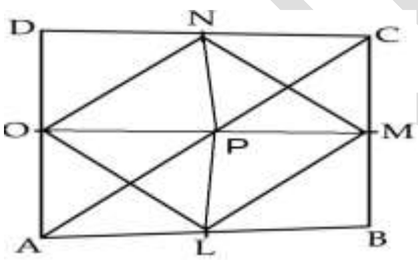
From (i) and (ii) ON = LM and ON II LM

Thus, LMNO is a IIgm.

14. Show that the line segments joining the mid points of the opposite sides of a quadrilateral bisect each other.

Solution: ABCD is a quadrilateral in which L, M, N and O are the mid points of the sides AB, BC, CD and DA respectively.

Prove that OP = PM and PL = PN



In Δ ACD, N and O are the mid points of the sides CD and DA respectively

=> ON||AC and ON = 1/2 AC ----------------(i)

Similarly, In Δ ABC, L and M are the mid points of the sides AB and BC respectively

=> LM||AC and LM = 1/2 AC ------------------(ii)

From (i) and (ii) ON = LM and ON II LM

Thus, LMNO is a IIgm.

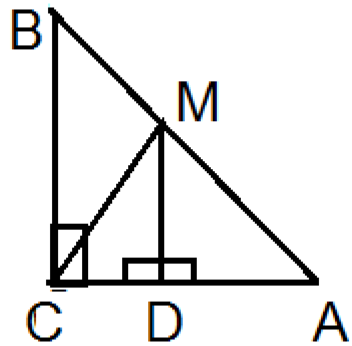
As we know that diagonal of IIgm bisect each other

=> OP = PM and PL = PN

15. ABC is a triangle right angled at C. A line through the midpoint M of hypotenuse AB and parallel to BC intersects AC at D. Show that:

(i) D is mid point of AC (ii) MD ⊥AC

(iii) CM = MA = 1/2 AB



In ΔABC, M is the mid-point of AB and MD II BC.

=> CD = AD (Converse of mid-point theorem)

D is the mid-point of AC [proved part 1st]

(ii) DM II BC and AC is a transversal line for them,

=> ∠MDC + ∠DCB = 180^o (Co-interior angles)

∠MDC + 90^o = 180^o => ∠MDC = 90^o

=> MD ⊥ AC [proved part 2nd]

Now, Join MC.

In ΔAMD and ΔCMD,

AD = CD (D is the mid-point of side AC)

∠ADM = ∠CDM (Each 90^o)

DM = DM (Common)

∴ΔAMD ≅ ΔCMD (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

However, AM = AB (M is the mid-point of AB)

Therefore, it can be said that CM = AM = AB