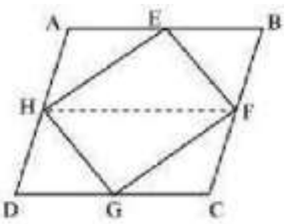
Class-IX Math SOLVED CBSE TEST PAPER - 02

Chapter: Area of Parallelogram and Triangles

1. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD
show that ar (EFGH) = ½ ar (ABCD)



In parallelogram ABCD,

AD || BC

=> AH || BF

and AD = BC

=> ½ AD = ½ BC

=> AH = BF

Therefore, ABFH is a parallelogram.

=> AB II HF

Since ΔHEF and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

∴ ar (ΔHEF) = ½ ar (ABFH) -------------(i)

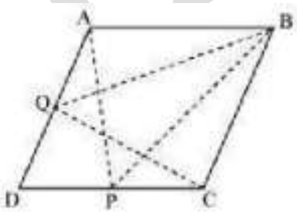
Similarly, it can be proved that

ar (ΔHGF) = ½ ar (HDCF) ----------- (ii)

On adding equations (1) and (2), we obtain

ar (EFGH) = ½ ar (ABCD)

2. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).



ΔBQC and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

∴ar (ΔBQC) = ½ ar (ABCD) ... (1)

Similarly, ΔAPB and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

∴ ar (ΔAPB) = ½ ar (ABCD) ... (2)

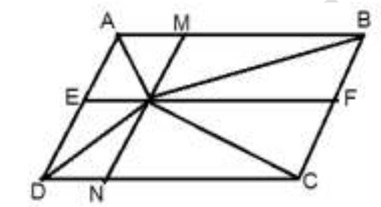
From equation (1) and (2), we obtain

ar (ΔBQC) = ar (ΔAPB)

3. In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) ar (APB) + ar (PCD) = ½ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)



Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

AB || EF (By construction) ---------------(1)

ABCD is a parallelogram.

∴ AD || BC (Opposite sides of a parallelogram)

⇒ AE || BF --------------------------------- (2)

From equations (1) and (2),

AB || EF and AE || BF

Therefore, quadrilateral ABFE is a parallelogram.

Now, ΔAPB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

∴ Area (ΔAPB) = ½ Area (ABFE) --------------- (3)

Similarly, Area (ΔPCD) = ½ Area (EFCD) -----------(4)

Adding equations (3) and (4),

ar (APB) + ar (PCD) = ½ ar (ABCD) -----------(5)

Similarly by drawing MN, passing through point P and parallel to line segment AD. we can prove that

ar (APD) + ar (PBC) = ½ ar (ABCD) -----------(6)

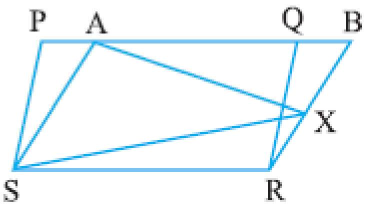
From (5) and (6)

ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

4. In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that
(i) ar (PQRS) = ar (ABRS) (ii) ar (ΔPXS) = ar (PQRS)

Answer: Parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

=> ar(PQRS) = 1/2 ar (ABRS) ... (1)



(ii) ΔAXS and parallelogram ABRS.lie on the same base AS and are between the same parallel lines AS and BR,

∴ Area (ΔAXS) = ½ Area (ABRS) ... (2)

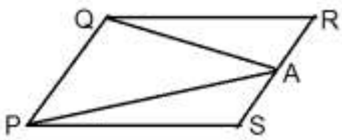
From equations (1) and (2), we obtain

Area (ΔAXS) = Area (PQRS)

5. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer:

From the figure, it can be observed that point A divides the field into three parts. These parts are triangular in shape − ΔPSA, ΔPAQ, and ΔQRA



Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = Area of PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

∴ Area (ΔPAQ) = ½ Area (PQRS) ... (2)

From equations (1) and (2), we obtain

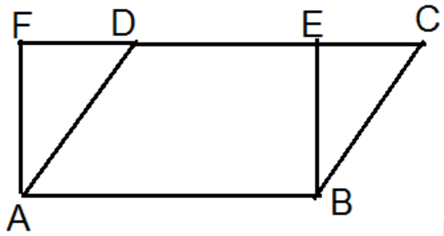
Area (ΔPSA) + Area (ΔQRA) = ½ Area (PQRS) ... (3)

Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

6. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Solution: Given that IIgm ABCD and rectangle ABEF are on the same base AB and have equal areas.

To Prove: The perimeter of the parallelogram ABCD is greater than that of rectangle ABEF.



Proof: In Δ ADF, ∠ AFD = 90°

∠ ADF is an acute angle. (< 90°)

∴ ∠ AFD > ∠ ADF

⇒ AD > AF (Side opposite to greater angle of a triangle is longer)

Adding side AB on both side

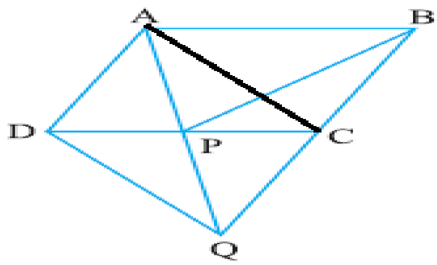
⇒ AB + AD > AB + AF ⇒ 2(AB + AD) > 2(AB + AF)

7. In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, Show that ar (BPC) = ar(DPQ).

Given: ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. AQ intersects DC at P.

To Prove: ar(Δ BPC) = ar(Δ DPQ).

Construction: Join AC.



Proof: Δ QAC and Δ QDC are on the same base QC and between the same parallels AD and QC.

∴ ar(Δ QAC) = ar(Δ QDC) ...(1)

=> ar(Δ QAC) - ar(Δ QPC) = ar(Δ QDC) - ar(Δ QPC)

⇒ ar(Δ PAC) = ar(Δ QDP) ...(2)

∴ Δ PAC and Δ PBC are on the same base PC and between the same parallels AB and DC.

∴ ar(Δ PAC) = ar(Δ PBC) ...(3)

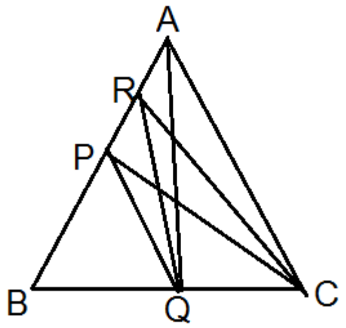
From (2) and (3), ar(Δ PBC) = ar(Δ QDP)

⇒ ar(Δ BPC) = ar(Δ DPQ).

8. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that (i) ar(PRQ) = ar(ARC) (ii) ar(RQC) = ar(ABC) (iii) ar(PBQ) = ar(ARC)

Solution:

Part- 1



: (i) ar(PRQ) = ½ ar(APQ) (RQ is median of Δ APQ)

=> ar(PRQ) = ½ x ½ ar(ABQ) (QP is median of Δ ABQ)

=> ar(PRQ) = ½ x ½ x ½ ar(ABC) (AQ is median of Δ ABC)

=> ar(PRQ) = 1/8 ar(ABC) (AQ is median of Δ ABC)

Similarly

ar(ARC) = 1/8 ar(ABC)

Thus, ar(PRQ) = ar(ARC)

Part- 2

ar(RQC) = ar (RBQ) = ar(PRQ) + ar(BPQ)

=> ar(RQC) = 1/8 ar(ABC) + ¼ ar(ABC)

=> ar(RQC) = 3/8 ar(ABC)

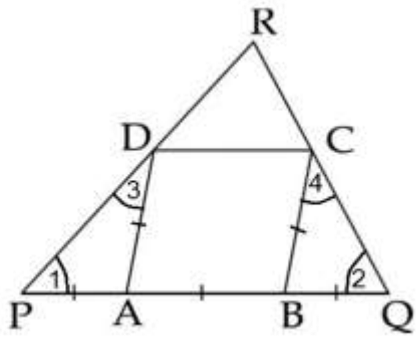
Part- 03

ar(PBQ) = ¼ ar(ABC)

ar(ARC) = ¼ ar(ABC)

=> ar(PBQ) = ar(ARC)

9. In the figure, ABCD is a rhombus whose side AB is produced to points P and Q such that AP = AB = BQ. PD and QC are produced to meet at a point R. Show that <PRQ = 90^0.



Solution: AP = AB

<1 = < 3

CQ = BQ

< 2 = < 4

Now, ext < BAD = <1 +< 3 = 2 < 1

and ext < ABC = <2 +< 4 = 2 < 2

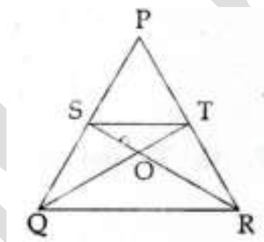
adding them , < BAD + < ABC = 180^0

2 (< 1 + < 2) = 180^0

< 1 + < 2 = 180^0 /2 = 90^0

= > <PRQ = 180^0 - (< 1 + < 2) = 180^0 - 90^0 = 90^0

10. In triangle PQR, S and T are points on PQ and PR respectively. If ar (QSR) = ar(QTR). Show that <OST = <ORQ



Solution:

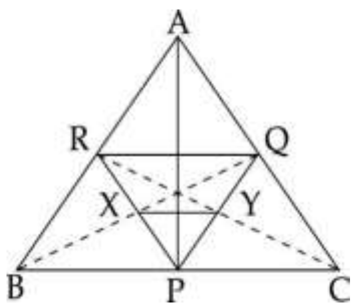
ar (QSR) = ar (QTR

These are on same base QR and between QR and ST

=> ST II QR

So, <OST = <ORQ alternate angles

11. P, Q and R are the mid points of sides BC, AC and AB of Δ ABC. If BQ and PR intersect at X and CR and PQ intersect at Y, then show that XY = ¼ BC



R and Q are midpoint of AB and AC

=> RQ = ½ BC and RQ II BC

=> RQ = BP and RQ = BP

So, BPQR is iigm and BQ and RP bisect at X

Similarly PCQR is IIgm and PQ and RC bisect at y

Now, x and y are midpoint of RP and PQ

½ RQ = xy => XY = ¼ BC [RQ = ½ BC]