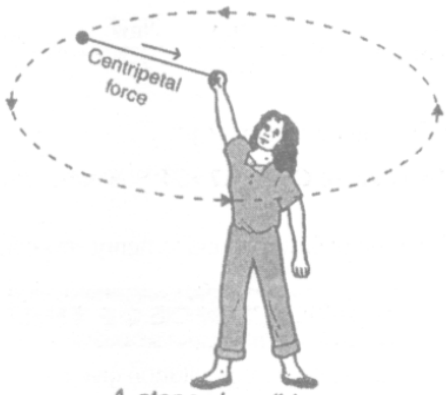
GRAVITATION

INTRODUCTION:

Sir Isaac Newton observed that the earth, basides attracting mall objects on or near its surface, attracts planet and other heavenly bodies far away from. It. He further assumed that the force of attracting exists between a objects big or small, not only on the surface of the Earth, but anywhere and everywhere in the universe irrespective of the distances separating them. He named this force of attracting between any two bodies as **the force of graviton**

GRAVITATION

Example: Consider a girl whirling a stone along a circular path. If the girl releases the stone at some point, the stone flies of along the tangent, at that point on the circular path. Let us discuss this observation carefully.



A stone describing a circular path with a velocity of constant magnitude.

Before the release of thread, the stone was moving with a certain uniform speed and changed its direction at every point. Because of the change in direction, it moved with a variable velocity and has some definite acceleration. The force that causes this acceleration and makes the stone move along the circular path, acts towards the centre, i.e., towards the hand of the girl. **This force is called centripetal force.** When the thread is released, the stone does not experience the centripetal force and files off along a straight line. This straight line is always tangent to the circular path.

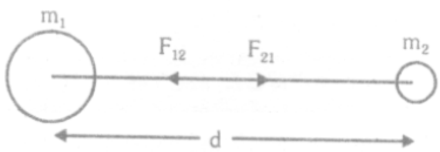
NEWTON'S UNIVERSAL LAW OF GRAVITATION

Every particle in the universe every other particle with of force which is directly proportional to the product of two masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining two masses.

If m_1 and m_2 are the masses of two bodies separated by distance d and F is the force of attraction between them, then

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d_2}$$



$$F \propto \frac{m_1 m_2}{d^2}$$

or $F \propto \frac{G\, m_1 m_2}{d^2}$ Where G is a constant of proportionally and known as the of universal gravitation and is equal to

$$Fd^2 = G\, m_1 m_2$$

$$G = \frac{Fd^2}{m_1 m_2}$$

If $m_1 = m_2 = 1$kg and d = 1m, then

$$G = \frac{F \times 1^2}{1 \times 1} \text{ or } G = F$$

i.e. Universal gravitation constant is the force of attraction (in newton) between two bodies of mass 1 (kg) each lying 1 (m) distance apart.

UNIT OF GRAVITATION CONSTANT

$$G = \frac{Fd^2}{m_1 m_2}$$

In SI unit $G = \frac{Nm^2}{kg \times kg} = \frac{Nm^2}{kg^2} = Nm^2\,kg^{-2}$

In CGS unit of dyn cm^{-2} g^{-2}

The value of G = 6.67 x 10^{-11} N m2 kg^{-2} or 6.67 x 10^{-8} dyn cm^2 g^{-2}

The value of G was found by henry cavendish.

IMPROTANCE OF UNIVERSAL LWO OF GRAVITATION

The Universal low of gravitation given by Newton has explained successfully several phenomena. For example

1. The gravitation force attraction of the Earth is responsible for binding all terrestrial object on the Earth
2. The gravitation force of the Earth is responsible for holding the atmosphere aground the Earth.
3. The gravitation force of the Earth is also responsible for holding the atmosphere around the Earth.
4. The flow of water in rivers is also due to gravitational force the Earth on water.
5. The moon revolves around the Earth on account of gravitational pull of the Earth on the Moon. Even all artificial satellites
6. All the planets revolve around the Sun due to gravitational pull of the sun on the planets . Thus, gravitation force alone is responsible for holding our solar system.
7. The tides formed by the rising and falling of water level in the ocean are due to the gravitational force attraction, which the Sun and the Moon exert on sea water.
8. The predictions about solar and lunar eclipses made on the basis of this law always come out to be true.

GRAVITATIONAL FROCE BETWEEN LIGHT OBJECT AND HEAVY OBJECTS

The formula applied for calculating gravitational force between light objects and heavy object is the. same, i.e.$F = \frac{G\, m_1 m_2}{r^2}$ .Let us take three cases:

1. When two of mass 1 kg each are 1 metre apart .

Sol. i.e. $m_1 = m_1 = 1$ kg, r = 1 m

Taking G = 6.67 x 10^{-11} Nm^2/kg^2, we obtain gravitational force of attraction

$$F = \frac{G\, m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(1)^2}$$

Which is extremely small. Hence we conclude that though every pair of two object exert gravitation pull o each other, yet they cannot move towards eachother because this gravitational pull is to weak.

2. When a body of mass 1 kg is held on the surface of Earth.

Sol. Here,

$m_1 = 1$ kg

m_2 = mass of Earth = 6 x 10^{24} kg

r = distance of body from centre of Earth

= radius of Earth = 6400 km = 6.4×10^3 km = 6.4×10^6 m

Gravitational force of attraction between the body and Earth.

$$F = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8 \text{ N}$$

It means that the Earth exerts a gravitational force of 9.8 N on a body mass one kg. force is much larger compared to the force when both the bodies are lighter. That is why when a body dropped from a height it falls to the Earth.

3. When both the bodie are heavy

Sol. Let us calculate gravitational force of attraction between Earth and the Moon.

Mass of Earth, $m_1 = 6 \times 10^{24}$ kg

Mass of Moon, $m_2 = 7.4 \times 10^{22}$ kg

Distance between Earth and Moon, r = 3.84×10^5 kg = 3.84×10^8

Gravitational constant, G = 6.67 x 10-11Nm^2 /kg^2

The gravitational force between Earth and Moon.

$$F = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times (6 \times 10^{24}) \times (7.4 \times 10^{22})}{(3.84 \times 10^8)^2} = 2.01 x 10^{20} N$$

Which is really. It is this large gravitational force exerted by Earth on Moon, which the moon revolve around the Earth.

Ex. Let us find force of attraction between block lying 1m apart. Let the mass of each block is 40 kg.

Sol. F = ?

m^1 = 40 kg

$m^2 = 40$ kg

d = 1 m

G = 6.67 x 10^{-11} N m^2 kg^{-2}

$$F = \frac{G\, m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8 \text{ N}$$

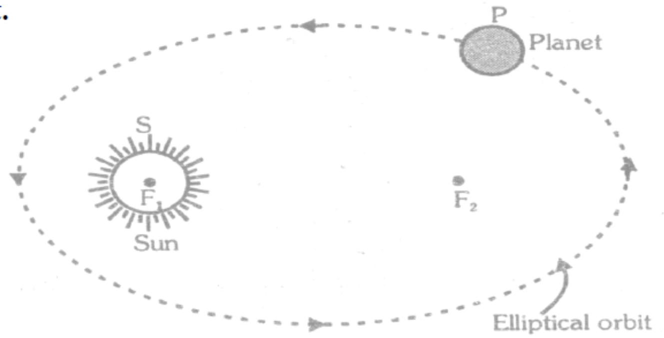
$$F = \frac{G\, m_1 m_2}{d^2} = \frac{6.67 \times 10^{-11} \times 40 \times 40}{1 \times 1}$$

Or F = 1.0672 x 10^{-2} N

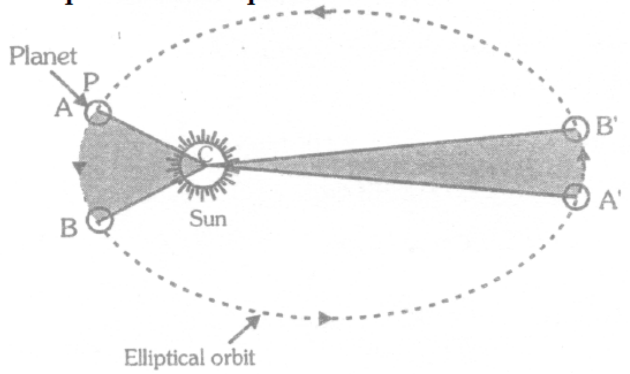
KEPLER'S LEWS OF PLANETRY MOTION

Johannes kepler was a 16th astronomer who established three laws which govern the motion of planets (around the sun). These are kwon as kepler's lows of planetary motion. The same laws also describe the motion of satellites (like the moon) around the planets (like the earth). The Kaplar's laws of planetary motion are give below.

1. **KEPLER'S FIRST LAW : The planets move in elliptical around the sun, with at one of the to foci of the elliptical orbit.**



2. **KEPLER'S SECOND LOW : Each revolves around the sun in such a way that the line joining plant to the sun sweeps over equal areas in equal intervals of time.**



3. **KEPLER'S THIRD LAW : The cube of the mean distance of a plant the sun is directly proportional the square of time it takes to move around the sun.** The law can be expressed as :

$r^3 \propto T^2$

Or $r^3 = \text{constant} \times T^2$

Or $\frac{r^3}{T^2} = \text{constant}$

Where r = Mean distance of planet from the sun

and T = Time period of the planet (around the sun)

Through Kepler gave the laws of planetary motion but he could not give a theory to explain the motion of planets. It was Newton who showed that the cause of the motion of planets is the gravitation force which the sun exerts on them. In fact, Newton used the Kapler's third law of planetary motion of develop the law of universal gravitation

NEWTON'S INVERSE-SQUARE RULE

The force between two bodies is inversely proportional the square of distance between them is called the inverse-square rule.

$$\mathbf{F} \propto \frac{1}{r^2}$$

Consider planet of mass m moving with a velocity (of speed) v around the sun in circular robit of radius r , centripetal force F acts on the orbiting planet (due to the sun) which is given by :

$$F = \frac{mv^2}{r}$$

The mass m of a given planet is constant

$$F \propto \frac{v^2}{r}$$

If the planet takes time T to complete one revolution (of $2\pi r$) around the sun, then its velocity v is given by:

$$v = \frac{2\pi\pi}{T}$$

The factor 2π is a constant

$$v = \frac{r}{T}$$

Now, taking square on both sides

$$v^2 = \frac{r^2}{T^2}$$

If we multiply as well as divide the right side of this relation by r

$$v^3 = \frac{r^3}{T^3} x \frac{1}{r}$$

The factor $\frac{r^3}{T^3}$ is constant by Keplar's third law.

$$v^2 \propto \frac{1}{r}$$

by putting $\frac{1}{r}$ in place of v^2 in relation $F \propto \frac{v^2}{r}$

$$F \propto \frac{1}{r \times r} \text{ or } F \propto \frac{1}{r^2}$$

NEWTON'S THIRD LAWOF MOTION AND GRAVITATION

The Newton third of motion also good for the force of gravitation. This means that when earth exerts a force of attraction on an object, then the object also exerts an equal force on the earth, in the opposite direction.

According to Newton's second law,

orce = Mass x Acceleration

F = me

$$\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}$$

Or $$a = \frac{F}{m}$$

The mass of earth is very large and acceleration produced in the earth very very small and cannot detected with even the most accurate instrument available to us.

FREE FALL

The falling of a body (or object) from a height towards the earth under the gravitational force of earth (with on other force acting on it) is called free fall

ACCELERATION DUE TO GRAVITY

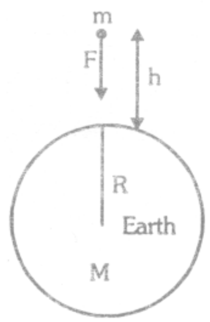
When a body dropped from a certain height . it falls with a constant acceleration. This uniform acceleration produced in a freely falling body due to the gravitational pull of the earth's is known as acceleration due to gravity and it is denoted by g. "the acceleration of a body due to attraction of earth its centre is calle acceleration due to gravity

The value of $g = 9.8 \text{ m/s}^2$

DETERMINATION OF VALUE OF g

When a body of mass m is dropped from a certain distance R from centre of earth mass M, then the exerted by the earth on the body is

$$F = \frac{GMm}{R^2} \quad \ldots\ldots \text{(i)}$$



Let this force produced an acceleration a in mass in

$\therefore$ F = ma

From (i) and (ii),

$$Ma = \frac{GMm}{R}$$

Or $$a = \frac{GM}{R^2}$$

For bodies falling near the surface of earth, this acceleration is called acceleration due to gravity and is represented by g.

$$\therefore \quad g = \frac{GM}{R^2}$$

Where M is the mass of the earth i.e. 6×10^{24} kg and R, radius the earth i.e. 6.4×10^{6} m

$$\therefore \quad g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} (6 \times 10^{24})}{(6.4 \times 10^{24})}$$

Or $g = 9.8 \text{ ms}^{-2}$ or nearly 10 ms^{-2}

VALUE OF g ON MOON

Mass of moon = 7.4 x 10^{22} kg and its radius = = 1,740 km

Or R = 1,740,000 m = 1,74 x 10^6 m

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11}(7.4 \times 10^{22})}{(1.74 \times 10^6)^2} = 1.63 \text{ ms}^{-2}$$

We have already seen that acceleration due to gravity does not depend upon mass of falling body. Mass of the earth

We can determine mass of the earth from equation (i)

$$g = \frac{GM}{R^2} \quad \text{or} \quad M = \frac{gR^2}{G}$$

$$\therefore \quad \text{Mass of the earth } M = \frac{9.8(6.4 \times 10^6)}{6.66 \times 10^{-11}}$$

Or M = 5.99 x 10^{24} kg

AVERAGE DENSITY OF THE EARTH

It can also be determined from equation (i) above

$$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)}{R^2} = G\,\frac{4}{3}\pi Rd$$

$$\text{or} \quad d = \frac{3g}{G4\pi R}$$

Taking the earth to be a sphere of radius R

$$\therefore \quad d = \frac{3 \times 9.8}{6.66 \times 10^{-11} \times 4 \times \pi(6.4 \times 10^6)^2}$$

Or d = 5.5 x 10^3 kg m^{-3}

Calculation of acceleration due to gravity on the moon and to prove that it is $1/6^{th}$ of the acceleration due to gravity on the earth.

Mass of the moon (M) = 7-4 x 10^{22} kg

Radius of the moon (R) = 1.74 x 10^6 m

Gravitational constant (G) = 6.7 X 10^{-11} Nm^2/kg^2

$\therefore$ Acceleration due to gravity on the moon, g = $\frac{GM}{R^2}$

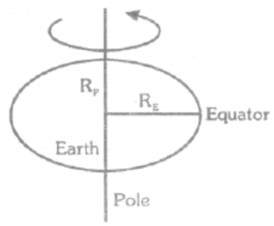
$$g_{Moon} = \frac{6.7 \times 10^{-11} Nm^2kg^{-2} \times 7.4 \times 10^{22}}{(1.7 \times 10^6 m)^2}$$

$$g_{Moon} = \frac{6.7 \times 7.4}{1.74 \times 1.74} \times \frac{10^{-11+22}}{10^{12}} N/kg$$

$$g_{Moon} = 1.63 \text{ ms}^{-2}$$

VERIATION OF ACCELERATION DUE TO GRAVITY ON THE SURFACE OF THE EARTH

The acceleration due to gravity of the earth at the poles is 9.83 m/s^2 and at the equator is 9.78 m/s^2. The generally accepted value of the acceleration due tc gravity, that is 9.81 m/s^2 is the average acceleration. It is not the same at all places of the Earth.



Furthermore, the average acceleration due to gravity is, with respect to the sea level.

Its value can further vary with the height or depth. In general, the acceleration due to gravity at sea level maximum at the geographic poles and minimum at equator. The value of acceleration due to gravity decrease when one moves :

(i) from the poles towards equator,

(ii) away from the earth as on hills, balloons, spaceships, etc.,

(iii) into the earth say in deep mines.

Ex. Mass of 1kg falling towards it with acceleration of 9.8 ms^{-2}. The force acting on it will be

Sol. F = ma = 1 x 9.8 = 9.8 N

Let A be the acceleration with which earth rises towards the object, then

F = MA

Where M = mass of the earth

$M = 6 \times 10^{24}$ kg

$$A = \frac{F}{M}$$

Or $$A = \frac{9.8}{6 \times 10^{24}} = 1.63 \times 10^{-24} \ ms^{-2}$$

Ex. Calculate the force of gravitation due child of mass 25 kg on his fat mother mass 75 kg if the istance between their centres is 1m from each other. Given G = (20/3) x 10^{-11} Nm^2 kg^{-2}

Sol. Here $m_1 = 25$ kg ; m_2 d = 1 m ; $G = \frac{20}{3} \times 10^{-11}$ Nm^2 Kg^{-2}

Using $$F = \frac{Gm_1m_2}{d^2}$$

Or $$F = \frac{20 \times 10^{-11} \times 75 \times 75}{3 \times (1)^2}$$

Or $F = 12{,}500 \times 10^{-11}$

Or $F = 1.25 \times 10^{-7}$ N

$$\therefore \frac{g_{Moon}}{g_{Earth}} = \frac{1.63 ms^{-2}}{9.81 ms^{-2}} = \frac{1}{6} \text{approx}$$

$$\therefore g_{Moon} = \frac{1}{6} g_{Eath}$$

GRVITATION AND GRAVITY

Gravitational is the force of attraction between any two bodies whereas gravity is the force of attraction between two bodies when one of the two bodies earth. Hence gravity is special of gravitation.

EQUATION OF MOATION FREELY FALLING BODIES

When the bodies are falling under influence of gravity, they experience acceleration g.i.e. 9.8 ms^{-2}. However when these are going up against gravity, they move with retardation of 9.8 m^{-2} All the equation of motion already read by us are valid for freely falling body with the difference that a replaced by g. For motion vertically upwards (a) is replased by (-g).

The equation of motion

$v = u + at$

Replace $a = g$

$v = u + gt$ When body falls in downward

$v = u - gt$ When body through upward

$s = ut + \frac{1}{2}at^2$

Relace $a = g$ & $s = h$

$H = ut + \frac{1}{2}gt^2$

$v^2 - u^2 = 2as$

Relace $s = h$

$v^2 - u^2 = 2gh$

Ex.1 A body drops a stone from the edge of the root. If passes a window 2m high in 0.1s. How far is the root above the top of the window ?

Sol. Let a stone be dropped from the edge of the roof A. Let it passes over B with a velocity say u. Consider motion BC.

u = ? ; a = 9.8 ms^{-2} ; s = h = 2m ; t = 0.1s

Using s = ut + $\frac{1}{2}gt^2$, we have

$$2 = u\,(0.1) + \frac{1}{2}x9.8(0.1)^2$$

$$2 = 0.1u + 0.049$$

$$0.1u = u - 2 - 0.049$$

or $u = \frac{1.915}{0.1} = 1951 ms^{-1}$

Root is 19.4 m above the window.

Ex2. A ball thrown up is caught by the ghrower after 4s. whit what velocity was it thrown up ? How high did go ? Where it after 3 s? (g = 9.8 s^{-2})

Sol. Since the time of going up is the same that of commg down, therefore time of going = 4/2 = 2s. starts upward with velocity u.

Hereu = ? ; a = 9.8 m s^{-2} ; t = 2s; v = 0 (at the top); s = h

Using $v = u + at$

Or $u = u - 9.8 \times 2$

Or $u = 19.6$ m s^{-1}

Again $v^2 - u^2 = 2as$

$0(19.6)^2 = 2\,(-9.8)\,h$

$h = 19.6$ m

After 2s. it starts coming downwards (starting with u = 0) Considering motion.

u = 0 ; a = 9.8 m s^{-2} ; t = 3 – 2 = 1s ; s = ?

$$s = ut + \frac{1}{2}at^2$$

or $s = 0 + \frac{1}{2} \times 9.8\,(1)^2$ 4.9 m from top.

Ex. Coconut is hanging on a tree at a height of 15 m from the ground A body launches a projectile ertically upwards with a velocity of 20 m s^{-1}. After what time the projecting pass by coconut ? Explain the two answers in this problem.

Sol. Here u = 20 m s^{-1}; a = - 10 m ; s = 15 m ; t = ?

Using s = ut + $\frac{1}{2}at^2$, we have

$$15 = 20t + \frac{1}{2}(-10)\,t^2$$

Dividing throughout by 5, we have

$3 = 4t - t^2$

or $t^2 = 4t + 3 = 0$

or $(t - 1)(t - 3) = 0$

$t - 1 = 0$ or $t = 1s$

or $t - 3 = 0$ or $t = 3s$

After 1s, it will cross coconut while going up and after 3 s while coming down.

MASS

The amount of matter contained in a body is called its mass

Or

The measure of the quantity of matter in a body is called its mass.

The mass of a body is a scalar quantity. It is independent of surrounding and the position of the body. It is a **constant quality for a given body.**

Mass is measured in kilograms (kg) in SI system.

WEIGHT

Everybody on the surface of earth is attracted towards the centre of earth. The force of attraction depends upon the mass of the body and acceleration due to gravity. The weight of the body is the force with which it is attracted towards the centre of the earth. We know

$F = ma$

The acceleration produced by the froe of attraction of the earth is known as acceleration due to gravity .e.g.

$\therefore \quad F = ma = mg$

But by definition this force is equal to the weight of the body i.e. F = W.

$\therefore \quad W = mg$

SI unit weight is Newton (N) and CGS, it is measured in dyne (dyn).

PRACTICAL UNITS

In SI, the weight is also measured in kg f of kg wt.

Therefore, kilogram froe or kilogram weight is force with which a mass of 1 kg is attracted by centre of earth

1 kg f = 1 kg wt = 9.8 N

In CGS, the practical unit of weight is grams force or a wt org g f or 1 g wt is force with which a mass 1 g of attracted b the centre of the earth.

1 g f = 1 g wt = 980 dyn

WEIGHT OF A BODY ON THE SURFACE OF EARTH AT DIFFERENT PLACE

Since the weight of the body depends upon mass and acceleration due to gravity, g. The value of g change from place to place and h ; pce the weight of the body is different at different place.

EFFECT OF SHAPE OF THE EARTH ON WEIGHT

The earth is not perfectly spherical. Its radius at poles is less thi GM