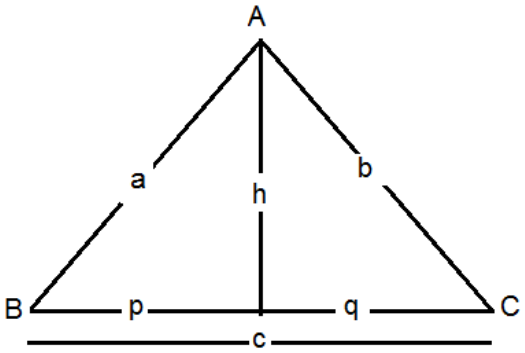
IX Proof of Heron's formula

Let a, b, c are length of the sides and h is height to side of length c of Δ ABC.



We have $S = (a + b + c)/2$

So, $2s = a + b + c$

$\Rightarrow 2(s - a) = - a + b + c$
$\Rightarrow 2(s - b) = a - b + c$
$\Rightarrow 2(s - c) = a + b - c$

Let $p + q = c$ as indicated.

Then, $h^2 = a^2 - p^2$ ----------(1)

Also, $h^2 = b^2 - q^2$ ---------- (ii)

From (i) and (ii)

$\Rightarrow a^2 - p^2 = b^2 - q^2$

$\Rightarrow q^2 = - a^2 + p^2 + b^2$

Since, $q = c - p \Rightarrow q^2 = (c-p)^2 \Rightarrow q^2 = c^2 + p^2 - 2pc$

Then, $c^2 + p^2 - 2pc = - a^2 + p^2 + b^2$

$\Rightarrow - 2pc = - a^2 + b^2 - c^2 = - (a^2 - b^2 + c^2)$

$\Rightarrow p = (a^2 - b^2 + c^2)/2c$

Now, Put this value of p in equation (i)

$h^2 = a^2 - p^2$

$h^2 = (a - p) (a + p)$

$h^2 = \{a - (a^2 - b^2 + c^2)/2c \} \{a + (a^2 - b^2 + c^2)/2c \}$

$h^2 = \{(2ac - a^2 + b^2 - c^2)/2c\}x\{(2ac + a^2 - b^2 + c^2)/2c\}$

$h^2 = \{(b^2 - (a - c)^2 \}\{(a + c)^2 - b^2\}/4c^2$

$h^2 = \{(b - a + c) (b + a - c)\}\{(a + c + b)(a + c - b)$

$h^2 = \{ 2(s - a) \times 2(s - c) \times 2s \times 2(s - b)\}/4c^2$

$h^2 = \{ 4 s (s - a) \times (s - c) \times (s - b)\}/c^2$

By: http://jsuniltutorial.weebly.com/ix-area-of-triangles-by-herons-formula.html

IX Proof of Heron's formula

$h = 2/c \sqrt{s\,(s - a)\,x\,(s - b)\,x(s - c)}$

$\frac{1}{2}\,h\,c = \sqrt{s\,(s - a)\,x\,(s - b)\,x(s - c)}$

Area of triangle $= \sqrt{s\,(s - a)\,x\,(s - b)\,x(s - c)}$

CBSE Test paper-1

1. Two sides of a triangle are 8cm and 11cm and its perimeter is 32cm.The third side is :

(a) 4cm (b) 13cm (c) 14cm (d) 16cm

2. The base of a triangle is 12cm and height is 8cm .Its area is:

(a) $24cm^2$ (b) $96cm^2$ (c) $48cm^2$ (d) none

3. The sides of a triangular plot are in the ratio 3:5:7 and its perimeter is 300m . The sides of a triangle are.

(a) 60m,100m,40m (b) 50m,80m,60m (c) 45m,75m,95m (d) none

4. What will be the area of quadrilateral ABCD if AB =3cm, BC=4cm, CD=4cm, DA=5cm and AC=5cm.

(a) 12.5cm (b) 15.2cm (c) 18.2cm (d)19.2cm

5. An isosceles triangle has perimeter 30cm and each of equal side is 12cm .Area of triangle is:

(a) $8\sqrt{15}cm^2$ (b) $7\sqrt{12}cm^2$ (c) $9\sqrt{15}cm^2$ (d)none

Complete the following sentences

6. Area of an equilateral triangle with side 'a' is _____.

7. If a, b, and c are the three sides of a triangle then by Hero's formula area is_____.

8. In Heron's formula semi perimeter is equal to _____.

9. Area of a right angled triangle is _____.

10. The area of a parallelogram is $392m^2$.If its altitude is twice the corresponding base, determine the base and height.

11. The adjacent sides of a parallelogram are 36cm and 27cm in length .If the distance between the shorter sides is 12cm, find the distance between the longer sides.

12. A rectangular lawn, 75m by 60m, has two roads , each 4m wide, running through the middle of the lawn, one parallel to length and other parallel to breadth. Find the cost of gravelling the roads at Rs 5.50 per m^2

IX Proof of Heron's formula

13. Using Heron's formula, find the area of an equilateral triangle if its side is 'a 'units.

14. Find the percentage increase in the area of a triangle if its each side is doubled.

15. Find the area of quadrilateral ABCD whose sides in meters are 9, 40, 28 and 15 respectively and the angle between first two sides is a right angle.

16. The difference between the sides containing a right angle in a right angled triangle is 14cm. The area of a triangle is $120cm^2$.Calculate the perimeter of a triangle.

17. A field is in the shape of a trapezium whose parallel sides are 35 m and 15 m. The non-parallel sides are 16 m and 18 m. Find the area of the field.

19. The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 900 m. Find its area.

20. An isosceles triangle has perimeter 44 cm and each of the equal sides is 14cm. Find the area of the triangle.

21. The perimeter of a rhombus is 240cm and one of its diagonals is 80cm. Find its area using Heron.s formula. Section C 3 marks each

22. Find area of equilateral triangle of side 4a using Heron.s formula. Using this formula find area of an equilateral triangle whose perimeter is 540cm.

23. The sides of a quadrilateral are 5cm, 12cm, 15cm and 20cm. The angle between first two sides is 90˚ Find the area of quadrilateral.

24. One side of a right triangle is 8 cm and the difference between other two sides is 4 cm. Find its area by Heron.s formula.

25. Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius?