Thales is the first mathematician credited with giving the first known proof "a circle is bisected by its diameter.

One of Thales' most famous pupils was Pythagoras and his group discovered many geometric properties and developed the theory of geometry to a great extent.

At that time Euclid, a teacher of mathematics at Alexandria in Egypt, collected all the known work and arranged it in his famous treatise, called 'Elements'. He divided the 'Elements' into thirteen chapters, each called a book.

Some definitions from book -1 of Elements are:

1. A **point** is that which has no part.
2. A **line** is breathless length.
3. The ends of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The edges of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.

⇒ An axiom or a postulate is a mathematical statement which is assumed to be true without proof. These assumptions are actually obvious universal truths.

⇒ Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.

⇒ Some of the Euclid's axioms are:

(i) Things which are equal to same thing are equal to one another.

(ii) If equals are added to equals, the wholes are equal.

(iii) If equals are subtracted from equals, the remainders are equals.

(iv) Things which coincide with one another are equal to one another.

(v) The whole is greater than the part.

(vi) Things which are double of the same thing are equal to one another.

(vii) Things which are halves of the same thing are equal to one another.

⇒ Euclid's five postulates are:

(i) A straight line may be drawn from any point to any other point.

(ii) A terminated line can be produced indefinitely.

(iii) A circle can be drawn with any centre and any radius.

(iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

⇒ A system of axioms is called consistent, if it is impossible to deduce from these axioms a statement that contradicts any axioms or previously proved statement.

⇒ Euclid's first postulate can also be stated as below: **Given** two distinct points, there is a unique line that passes through them.

⇒ Two distinct lines cannot have more than one point in common.

⇒ **Playfair's Axiom:** For every line *l* and for every point P not lying on *l*, there exists a unique line *m*, passing through P and parallel to *l*. [~ 5th Postulate]

⇒ Two distinct intersecting lines cannot be parallel to the same line.

Sample Question 1 : Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared?

Solution: Let x kg be the weight each of Ram and Ravi. On gaining 2 kg, weight of Ram and Ravi will be $(x + 2)$ each.

According to Euclid's second axiom, when equals are added to equals, the wholes are equal.

So, weight of Ram and Ravi are again equal.

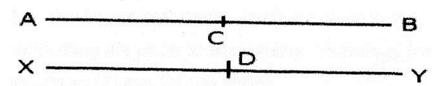
Sample Question 2 : Solve the equation $a - 15 = 25$ and state which axiom do you use here.

Solution: $a - 15 = 25$. On adding 15 to both sides, we have $a - 15 + 15 = 25 + 15 = 40$ (using Euclid's second axiom). Or $a = 40$

Sample Question 3 : if $\angle 1 = \angle 3$, $\angle 2 = \angle 4$ and $\angle 3 = \angle 4$, write the relation between $\angle 1$ and $\angle 2$, using an Euclid's axiom.

Solution: Here, $\angle 3 = \angle 4$ and $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$. Euclid's first axiom says, the things which are equal to equal thing are equal to one another. So, $\angle 1 = \angle 2$

Sample Question 4: In fig. We have: AC = XD, C is the mid-point of AB and D is the mid-point of XY. Using Euclid's axiom, show that AB = XY.



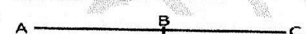
Solution: AB = 2AC (C is the mid-point of AB)

XY = 2XD (D is the mid-point of XY)

Also, AC = XD (Given)

Therefore, AB = XY, because things which are double of the same things are equal to one another.

Sample Question 5: If A, B and C are three points on a line, and B lies between A and C then prove that AB + BC = AC.



Solution: In the figure given above, AC coincides with AB + BC. Also, Euclid's Axiom (4) says that things which coincide with one another are equal to one another. So, it can be deduced that AB + BC = AC

Sample Question 6: Prove that an equilateral triangle can be constructed on any given line segment.

Solution: Draw a line segment AB. Now draw two circles with centre A and B of radius AB. Then draw the line segments AC and BC to form Δ ABC

Now, AB = AC, since they are the radii of the same circle (1)

Similarly, AB = BC (Radii of the same circle) (2)

From these two facts, and Euclid's axiom that things which are equal to the same thing are equal to one another, you can conclude that AB = BC = AC. So, Δ ABC is an equilateral triangle

Sample Question 7: Prove that two distinct lines cannot have more than one point in common.

Solution: Here we are given two lines l and m

If possible let the two lines intersect in two distinct points, say P and Q.

So, you have two lines passing through two distinct points P and Q

But it is the axiom that only one line can pass through two distinct points.

So, our supposition that two lines can pass through two distinct points is wrong.

Hence, two distinct lines cannot have more than one point in common.

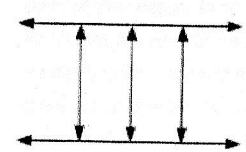
Q. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) Parallel lines (ii) perpendicular lines (iii) line Segment (iv) radius of a circle (v) square

Solution: (i) Parallel Lines

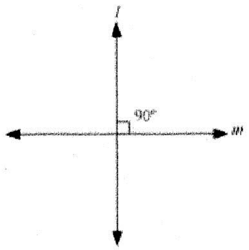
If the perpendicular distance between two lines is always constant, then these are called parallel lines. In other words, the lines which never intersect each other are called parallel lines.

To define parallel lines, we must know about point, lines, and distance between the lines and the point of intersection.



(ii) Perpendicular lines

If two lines intersect each other at 90° , then these are called perpendicular lines. We are required to define line and the angle before defining perpendicular lines.

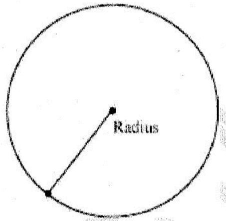


(iii) Line segment

A straight line drawn from any point to any other point is called as line segment. To define a line segment, we must know about point and line segment.

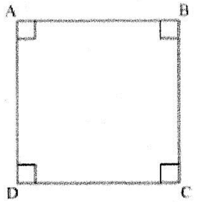
(iv) Radius of a circle

It is the distance between the centers of a circle to any point lying on the circle. To define the radius of a circle, we must know about point and circle.



(v) Square

A square is a quadrilateral having all sides of equal length and all angles of same measure, i.e., 90°. To define square, we must know about quadrilateral, side, and angle.



Q. Consider the two 'postulates' given below:

(i) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(ii) There exists at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent?

Do they follow from Euclid's postulates? Explain.

Solution: (a) The postulate (i) contains undefined term i.e. '*between A and B*. It is unclear where point C lies, does it lie on line segment AB, or above AB or below AB or very far above AB. There are other undefined terms such as point, line etc.

(b) Both the postulates are consistent since they do not conflict each other and refer to two different situations.

(c) These postulates do not follow Euclid's postulates. However they follow his axioms. E.g. Axiom 5.1: *Given two distinct points, there is a unique line that passes through them.*

Q. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}AB$. Explain by drawing the figure.

Solution: It is given that, AC = BC

AC + AC = BC + AC (Equals are added on both sides)(1)

Here, (BC + AC) coincides with AB.

It is known that things which coincide with one another are equal to one another.

∴ BC + AC = AB ... (2)

It is also known that things which are equal to the same thing are equal to one another. Therefore, from equations (1) and (2), we obtain

AC + AC = AB ⇒ 2AC = AB ⇒ $AC = \frac{1}{2}AB$

Q. If point C is a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Solution: Let there be two mid-points, C and D.

C is the mid-point of AB.

AC = CB

AC + AC = BC + AC (Equals are added on both sides) ... (1)

Here, (BC + AC) coincides with AB. It is known that things which coincide with one another are equal to one another.

∴ BC + AC = AB ... (2)

It is also known that things which are equal to the same thing are equal to one another. Therefore, from equations (1) and (2), we obtain

AC + AC = AB

⇒ 2AC = AB ... (3)

Similarly, by taking D as the mid-point of AB, it can be proved that

2AD = AB ... (4)

From equation (3) and (4), we obtain

2AC = 2AD (Things which are equal to the same thing are equal to one another.)

⇒ AC = AD (Things which are double of the same things are equal to one another.)

This is possible only when point C and D are representing a single point.

Hence, our assumption is wrong and there can be only one mid-point of a given line segment.

Q. In the following figure, if AC = BD, then prove that AB = CD.

Solution: From the figure, it can be observed that A = AB + BC and BD = BC + CD

It is given that AC = BD ⇒

AB + BC = BC + CD (1)

According to Euclid's axiom, when equals are subtracted from equals, the remainders are also equ

Subtracting BC from equation (1), we obtain

AB + BC − BC = BC + CD − BC ⇒ AB = CD

Q. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the questio is not about the fifth postulate.)

Answer: **Axiom 5** states that the whole is greater than the part. This axiom is known as a universal tru because it holds true in any field, and not just in the field of mathematics. Let us take two cases – one in the field of mathematics, and one other than that.

Case I : Consider a group of numbers 15, 8, 4, 2, 1 such that 15 = 8 + 4 + 2 + 1 and

15 is greater than any of its part (8, 4, 2, 1)

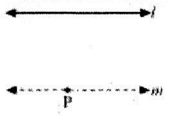
Therefore, it is rightly said that the whole is greater than the part.

Case II

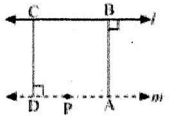
Let us consider the continent Asia. Then, let us consider a country India which belongs to Asia. Indi is a part of Asia and it can also be observed that Asi is greater than India. That is why we can say that th whole is greater than the part. This is true for anything in any part of the world and is thus a universal truth.

Q. How would you rewrite Euclid's fifth postulate so that it would be easier to understand?

Answer: Two lines are said to be parallel if they are equidistant from one other and they do not have an point of intersection. In order to understand it easily let us take any line *l* and a point P not on *l*. Then, b Playfair's axiom (equivalent to the fifth postulate), there is a unique line *m* through P which is parallel to *l*.



The distance of a point from a line is the length of the perpendicular from the point to the line. Let AB be the distance of any point on *m* from *l* and CD be the distance of any point on *l* from *m*. It can be observed that AB = CD. In this way, the distance will be the same for any point on *m* from *l* and any point on *l* from *m*. Therefore, these two lines are everywhere equidistant from one another.

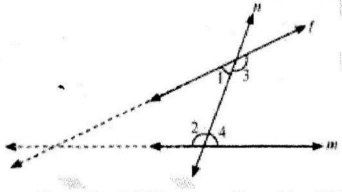


Q. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

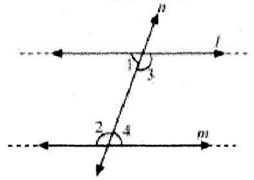
Answer: Yes.

According to Euclid's 5[th] postulate, when *n* line falls on *l* and *m* and if

$\angle 1 + \angle 2 < 180°$, then $\angle 3 + \angle 4 > 180°$, producing line *l* and *m* further will meet in the side of ∠1 and ∠2 which is less than $180°$.



If $\angle 1 + \angle 2 = 180°$, then $\angle 3 + \angle 4 = 180°$



The lines *l* and *m* neither meet at the side of ∠1 and ∠2 nor at the side of ∠3 and ∠4. This means that the lines *l* and *m* will never intersect each other. Therefore, it can be said that the lines are parallel.

Q. There exist pair of straight lines that are everywhere equidistant from one another. Is this statement a direct consequence of Euclid's fifth postulate? Explain.

Solution: Take any line l and a point P not on l. Then, by Playfair's axiom, which is equivalent to the fifth postulate, we know that there is a unique line m through P which is parallel to l.

Now, the distance of a point from a line is the length of the perpendicular from the point to the line. This distance will be the same for any point on m from l and any point on l from m.

So, these two lines are everywhere equidistant from one another.

Q. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Solution. If a straight line l falls on two straight lines m and n such that the sum of interior angles on same side of l is 180°, then by Euclid's 5th postulate, the lines will not meet on this side of l.

MCQ-Practice Paper

1. The three steps from solids to points are :

(A) Solids - surfaces - lines - points

(B) Solids - lines - surfaces - points

(C) Lines - points - surfaces - solids

(D) Lines - surfaces - points - solids

2. The number of dimensions, a solid has :

(A) 1 (B) 2 (C) 3 (D) 0

3. The number of dimensions, a surface has :

(A) 1 (B) 2 (C) 3 (D) 0

4. The number of dimension, a point has :

(A) 0 (B) 1 (C) 2 (D) 3

5. Euclid divided his famous treatise "The Elements" into :

(A) 13 chapters (B) 12 chapters

(C) 11 chapters (D) 9 chapters

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6. The total number of propositions in the Elements are : (A) 465 (B) 460 (C) 13 (D) 55

7. Boundaries of solids are :

(A) surfaces (B) curves (C) lines (D) points

8. Boundaries of surfaces are :

(A) surfaces (B) curves (C) lines (D) points

9. In Indus Valley Civilization (about 300 B.C.), the bricks used for construction work were having dimensions in the ratio

(A) 1 : 3 : 4 (B) 4 : 2 : 1 (C) 4 : 4 : 1 (D) 4 : 3 : 2

10. A pyramid is a solid figure, the base of which is

(A) only a triangle (B) only a square

(C) only a rectangle (D) any polygon

11. The side faces of a pyramid are :

(A) Triangles (B) Squares (C) Polygons (D) Trapeziums

12. It is known that if $x + y = 10$ then $x + y + z = 10 + z$. The Euclid's axiom that illustrates this statement is :

(A) First Axiom (B) Second Axiom

(C) Third Axiom (D) Fourth Axiom

13. In ancient India, the shapes of altars used for house hold rituals were :

(A) Squares and circles (B) Triangles and rectangles

(C) Trapeziums and pyramids (D) Rectangles and squares

14. The number of interwoven isosceles triangles in Sriyantra (in the Atharvaveda) is:

(A) Seven (B) Eight (C) Nine (D) Eleven

15. Greek's emphasised on :

(A) Inductive reasoning (B) Deductive reasoning

(C) Both A and B (D) Practical use of geometry

16. In Ancient India, Altars with combination of shapes like rectangles, triangles and trapeziums were used for :

(A) Public worship (B) Household rituals

(C) Both A and B (D) None of A, B, C

17. Euclid belongs to the country:

(A) Babylonia (B) Egypt (C) Greece (D) India

18. Thales belongs to the country :

(A) Babylonia (B) Egypt (C) Greece (D) Rome

19. Pythagoras was a student of :

(A) Thales (B) Euclid (C) Both A and B (D) Archimedes

20. Which of the following needs a proof ?

(A) Theorem (B) Axiom (C) Definition (D) Postulate

21. Euclid stated that all right angles are equal to each other in the form of

(A) an axiom (B) a definition

(C) a postulate (D) a proof

22. 'Lines are parallel if they do not intersect' is stated in the form of

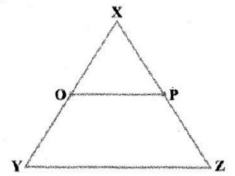
(A) an axiom (B) a definition (C) a postulat

Answer:

1. (A) **2.** (C) **3.** (B) **4.** (A) **5.** (A) **6.** (A) **7.** (A) **8.** (B) **9.** (B) **10.** (D) **11.** (A) **12.** (B) **13.** (A) **14.** (C) **15.** (B) **16.** (A) **17.** (C) **18.** (C) **19.** (A) **20.** (A) **21.** (C) **22.** (B)

More Practice:

Q. if OX = ½ XY, PX = ½ XZ and OX = PX, show that XY = XZ.



Q. we have BX = ½ AB and BY = ½ BC and AB = BC. Show that BX = BY.