MATHEMATICS
Time Allowed: 3hrs
CLASS - IX
Maximum Marks : 80
General Instructions :

1. The question paper comprises of thirty questions divided into four Sections- A, B, C and D.
2. Section A comprises of six questions Q1 to Q6 of one mark each.
3. Section B comprises of six questions Q7 to Q12 of two marks each.
4. Section C comprises of ten questions Q13 to Q22 of three marks each.
5. Section D comprises of eight questions Q23 to Q30 of four marks each.
6. All questions are compulsory.
7. Use of calculators is not permitted.

## SECTION - A

1 Find the product of $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$.
2 Evaluate: $\frac{(2.3)^{3}-0.027}{(2.3)^{2}+0.69+0.09}$
3 Find the distance of the point $(5,-12)$ from the origin.
4 Find the coordinates of that points where the line $3 x+5 y=15$ intersects $x$-axis and $y$-axis.
5 If complement of an angle is equal to $\frac{2}{5}$ times its supplement. Find the angle.
6 A coin was tossed 20 times and outcomes were noted. If the experimental probability of getting heads is $\frac{3}{5}$, then how many times tails came up?
SECTION - B

7 Solve: $5^{x-3} \cdot 3^{2 x-8}=225$.
8 If $a^{2}+\frac{1}{a^{2}}=102$, find the value of $a-\frac{1}{a}$.
9 If the point $\mathrm{A}(3,0)$ and $\mathrm{B}(1,2)$ lie on the graph of the line $p x+q y-9=0$, then find the value of $p^{2}-p q+q^{2}$.

10 Prove that an equilateral triangle can be constructed on any given line segment.

11 Find the value of $x$ :


12 Prove that the angles opposite to the equal sides of an isosceles triangle are equal.

## SECTION - C

13 Evaluate : $\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times\left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div\left(\frac{5}{2}\right)^{-3}\right]$

14 If $a+b+c=0$ then, find the value of $\frac{(b+c)^{2}}{b c}+\frac{(c+a)^{2}}{c a}+\frac{(a+b)^{2}}{a b}$.
15 Plot the points A $(0,4)$, and $B(-3,0)$ on the Cartesian plane. Find the IMAGE of Point A taking $x$ axis as mirror and image of point $B$ taking $y$-axis as mirror. Find the area of the figure formed by joining these points.

16 Draw the graph of the linear equation $3 x+4 y=7$ and $3 x-2 y=1$ and find the point of intersection of the lines representing the equations.

17 In fig., if $\mathrm{AC}=\mathrm{DC}$ and $\mathrm{CB}=\mathrm{CE}$ then show that $\mathrm{AB}=\mathrm{DE}$.


18 In the given figure, PM and QN are the bisectors and $\mathrm{PM} \| \mathrm{QN}$. Prove that $\mathrm{AB} \| \mathrm{CD}$.


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19 BE and CF are two equal altitudes of a triangle ABC . Prove that the triangle ABC is isosceles.
20 Prove that the sum of the lengths of the medians of a triangle is less than the perimeter of the triangle.

21 Construct an isosceles triangle whose base is 7.4 cm and the vertical angles twice each of the base angle.

22 If the mean of the following data is 8.05 , find the value of $k$.

| $\mathrm{x}_{\mathrm{i}}$ | 4 | 6 | 8 | 10 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}_{\mathrm{i}}$ | 4 | $2 k+2$ | 14 | 11 | $k$ |

## SECTION - D

23 Express $\frac{1}{1+\sqrt{2}-\sqrt{3}}$ with rational denominator.
24 The polynomials $x^{3}+2 x^{2}-5 a x-8$ and $x^{3}+a x^{2}-12 x-6$ when divided by $(x-2)$ and $(x-3)$ leave the remainder $\boldsymbol{p}$ and $\boldsymbol{q}$ respectively. If $\boldsymbol{q}-\boldsymbol{p}=\mathbf{1 0}$, find the value of $\boldsymbol{a}$.
25 A guest house has a fixed charge for the first two days and an additional charge for each day thereafter. Mr. Sharma paid ₹ 1600 for a room for eight days. If fixed charges are ₹ $x$ and per day charge be ₹ $y$. Write the linear equation representing the above equation. Draw the graph from linear equation.

If two parallel lines are intersected by a transversal, prove that bisectors of the interior angle on same side of transversal intersect each other at right angles.
27 In given fig, $\triangle \mathrm{ABC}$ is right-angled triangle with $\angle \mathrm{C}=90^{\circ}$, M is the mid-point of hypotenuse AB . C is joined to M and produced to a point D such that $\mathrm{DM}=\mathrm{CM}$. Point D is joined to point B . Show that:
(i) $\triangle \mathrm{AMC} \cong \triangle \mathrm{BMD}$
(ii) $\angle \mathrm{DBC}$ is a right angle.
(iii) $\triangle \mathrm{DBC} \cong \triangle \mathrm{ACB}$
(iv) $\mathrm{CM}=\frac{1}{2} \mathrm{AB}$


28 Construct a $\triangle \mathrm{XYZ}$ in which $\angle \mathrm{Y}=30^{\circ}, \angle \mathrm{Z}=90^{\circ}$ and $\mathrm{XY}+\mathrm{YZ}+\mathrm{ZX}=11 \mathrm{~cm}$.
Draw a histogram to represent the following distribution:

| C.I. | $10-15$ | $15-20$ | $20-30$ | $30-50$ | $50-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 6 | 10 | 10 | 8 | 18 |

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30 A die is rolled 200 times and its outcomes are recorded as below:

| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 25 | 35 | 40 | 28 | 42 | 30 |

Find the probability of getting:
(a) an even prime
(b) a multiple of 3
(c) number greater than 6
(d) an odd number
-o000000-

|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | SECTION - A |  |  |
| 1 | $\begin{aligned} & =\sqrt[12]{2^{4}} \cdot \sqrt[12]{2^{3}} \cdot \sqrt[12]{32^{1}} \\ & =\sqrt[12]{2^{4+3+5}} \\ & =2 \end{aligned}$ |  | 1 |
| 2 | $\begin{aligned} & \frac{(2.3)^{3}-(0.3)^{3}}{(2.3)^{2}+2.3 \times 0.3+(0.3)^{2}} \\ & =\frac{(2.3-0.3)\left[(2.3)^{2}+2.3 \times 0.3+(0.3)^{2}\right]}{(2.3)^{2}+2.3 \times 0.3+(0.3)^{2}} \\ & =2 \end{aligned}$ |  | 1 |
| 3 | $\begin{aligned} & =\sqrt{5^{2}+12^{2}} \\ & =\sqrt{25+144} \\ & =13 \end{aligned}$ |  | 1 |
| 4 | When $\mathrm{x}=0, \quad 5 y=15-0$ $y=\frac{15}{5}$ <br> Coordinates of point are $(0,3)$ <br> When $y=0,3 x=15-0$ $x=\frac{15}{3}$ <br> Coordinates of point are $(5,0)$ |  | 1 |
| 5 | $\begin{aligned} & (90-x)=\frac{2}{5}(180-x) \\ & 450-5 x=360-2 x \\ & 3 x=90 \\ & x=30^{\circ} \end{aligned}$ |  | 1 |
| 6 | $\begin{aligned} \text { No. of times tails came up } & =\left(20-20 \times \frac{3}{5}\right) \\ = & 20-12 \\ = & \end{aligned}$ |  | 1 |
|  |  |  |  |

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|  | SECTION - B |  |
| :---: | :---: | :---: |
| 7 | $5^{x-3} \cdot 3^{2 x-8}=5^{2} \cdot 3^{2}$ <br> Comparing exponents, $\begin{aligned} & \Rightarrow x-3=2 \text { and } 2 x-8=2 \\ & \Rightarrow x=5 \end{aligned}$ | 2 |
| 8 | $\begin{aligned} & \Rightarrow a^{2}+\frac{1}{a^{2}}-2=102-2 \\ & \Rightarrow\left(a-\frac{1}{a}\right)^{2}=100 \\ & \Rightarrow\left(a-\frac{1}{a}\right)=10 \end{aligned}$ | 2 |
| 9 | Since, (3,0) is on graph, $3 p-9=0$ $p=3$ <br> Also, $(1,2)$ is on graph, $3+2 q=9$ $\begin{aligned} & 2 q=6 \\ & q=3 \end{aligned}$ | 2 |
| 10 | A circle is drawn with point A as the centre and AB as the radius. Similarly, another circle with point B as the centre and BA as the radius. The two circles meet a point, say C. Now, the line segments $A C$ and $B C$ are drawn to form $\triangle A B C$. | 2 |
| 11 | Construction: AD is extended such that it meets BC at M . <br> Sol: <br> In $\triangle A B M$, <br> $\angle \mathrm{AMC}=45^{\circ}+35^{\circ}$ (Exterior angle property) <br> $\angle \mathrm{AMC}=80^{\circ}$ <br> In $\triangle \mathrm{DMC}$, <br> $\angle \mathrm{ADC}=80^{\circ}+50^{\circ}$ (Exterior angle property) $x=130^{\circ}$ | 2 |
| 12 | To prove: $\angle B=\angle C$ <br> Construction: Draw bisectors of $\angle \mathrm{A}$ intersecting BC at D. <br> Proof: <br> In $\triangle A B D$ and $\triangle A C D$, <br> $\mathrm{AB}=\mathrm{AC}$ (given) <br> $\angle \mathrm{BAD}=\angle \mathrm{CAD}$ (given) <br> $\mathrm{AD}=\mathrm{AD}$ (common) | 2 |


|  | $\triangle \mathrm{BAD} \cong \triangle \mathrm{CAD}(\mathrm{SAS})$ |  |
| :---: | :---: | :---: |
|  | SECTION - C |  |
| 13 | $\begin{aligned} &\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times\left[\left(\frac{25}{9}\right)^{\frac{-3}{2}} \div\left(\frac{5}{2}\right)^{-3}\right] \\ &=\left(\frac{16}{81}\right)^{\frac{3}{4}} \times\left[\left(\frac{9}{25}\right)^{\frac{3}{2}} \times\left(\frac{5}{2}\right)^{3}\right] \\ &=\left(\frac{2}{3}\right)^{4 \times \frac{3}{4}} \times\left[\left(\frac{3}{5}\right)^{2 \times \frac{3}{2}} \times\left(\frac{5}{2}\right)^{3}\right] \\ &=\left(\frac{2}{3}\right)^{3} \times\left[\left(\frac{3}{5}\right)^{3} \times\left(\frac{5}{2}\right)^{3}\right] \end{aligned}$ $=1$ | 3 |
| 14 | Since, $a+b+c=0, \Rightarrow a+b=-c$ or $b+c=-a$ or $c+a=-b$ $\frac{(b+c)^{2}}{b c}+\frac{(c+a)^{2}}{c a}+\frac{(a+b)^{2}}{a b}$ $=\frac{(-a)^{2}}{b c}+\frac{(-b)^{2}}{c a}+\frac{(-c)^{2}}{a b}$ $\begin{gathered} =\frac{a^{3}+b^{3}+c^{3}}{a b c} \\ =\frac{3 a b c}{a b c} \\ =3 \end{gathered}$ | 3 |
| 15 | Image of $A(0,4)$ when $x$-axis is mirror $=(0,-4)$ Image of $\mathrm{B}(-3,0)$ when y -axis is mirror $=(3,0)$ Figure formed is Quadrilateral. $\begin{aligned} \operatorname{Ar}(\triangle \mathrm{ABD}) & =\frac{1}{2} \times 4 \times 6 \\ & =12 \text { sq. units } \\ \operatorname{Ar}(\triangle \mathrm{CBD}) & =\frac{1}{2} \times 4 \times 6 \\ & =12 \text { sq. units } \\ \operatorname{Ar}(\mathrm{ABCD}) & =24 \text { sq. units } \end{aligned}$  | 3 |


| 16 | Correct graphical representation <br> Point of intersection is $(1,1)$ | 3 |
| :---: | :---: | :---: |
| 17 | Proof: Given, $\begin{aligned} & \mathrm{AC}=\mathrm{DC} \\ & \mathrm{CB}=\mathrm{CE} \end{aligned}$ <br> Adding both equations, <br> $\mathrm{AC}+\mathrm{CB}=\mathrm{DC}+\mathrm{CE}$ (if equals are added to equals, wholes remain equal.) $\mathrm{AB}=\mathrm{CD}$ | 3 |
| 18 | In the given figure, PM and QN are the bisectors and $\mathrm{PM} \\| \mathrm{QN}$. Prove that $\mathrm{AB} \\| \mathrm{CD}$ <br> Proof: <br> $\angle 1=\angle 2$ (given, MP\||NP) <br> $2 . \angle 1=2 . \angle 2$ (If equals are multiplied to equals, wholes are equal) $\angle \mathrm{SPB}=\angle \mathrm{PQD}$ <br> Since, corresponding angles are equal, so lines are parallel. <br> Hence, $\mathrm{AB} \\| \mathrm{CD}$. | 3 |
| 19 | Proof: In $\triangle \mathrm{BCE}$ and $\triangle \mathrm{CBF}$, <br> $\angle \mathrm{BEC}=\angle \mathrm{CFB}\left(\right.$ each $\left.90^{\circ}\right)$ <br> $\mathrm{BE}=\mathrm{CF}$ (given) <br> $\mathrm{BC}=\mathrm{BC}$ (common) <br> $\triangle \mathrm{BCE} \cong \triangle \mathrm{CBF}$ (RHS) $\mathrm{AB}=\mathrm{AC}(\mathrm{CPCT})$ | 3 |
| 20 | To Prove: $A B+B C+C A>A D+B E+C F$ <br> Proof: Since, sum of two sides of a triangle is greater than the third sidt <br> So, $A B+B D>A D$ <br> $A C+C D>A D$ <br> $\mathrm{BC}+\mathrm{CE}>\mathrm{BE}$ <br> $\mathrm{AB}+\mathrm{AE}>\mathrm{BE}$ <br> $\mathrm{CA}+\mathrm{AF}>\mathrm{CF}$ <br> $\mathrm{BC}+\mathrm{BF}>\mathrm{CF}$ <br> Adding all equations, $\begin{aligned} 2(\mathrm{AB}+\mathrm{BC}+\mathrm{CA}) & >2(\mathrm{AD}+\mathrm{BE}+\mathrm{CF}) \\ \mathrm{AB}+\mathrm{BC}+\mathrm{CA} & >\mathrm{AD}+\mathrm{BE}+\mathrm{CF} \end{aligned}$ | 3 |


| 21 | Correct construction |  |  | 3 |
| :---: | :---: | :---: | :---: | :---: |
| 22 | $x_{i}$ | $f_{i}$ | $f_{i} x_{i}$ | 3 |
|  | 4 | 4 | 16 |  |
|  | 6 | $2 k+2$ | $12 k+12$ |  |
|  | 8 | 14 | 112 |  |
|  | 10 | 11 | 110 |  |
|  | 12 | $k$ | $12 k$ |  |
|  | Total: | $\Sigma f_{i}=31+3 k$ | $\Sigma f_{i} x_{i}=250+24 k$ |  |
|  |  | $\begin{gathered} \Rightarrow 8.05( \\ \Rightarrow 249.5 \end{gathered}$ | $\begin{aligned} & =\frac{\Sigma f_{i} x_{i}}{\Sigma f_{i}} \\ & k)=250+24 k \\ & 15 k=50+24 k \\ & =0.15 k \\ & =3 \end{aligned}$ |  |
|  |  |  | ON - D |  |
| 23 | $\begin{aligned} & =\frac{1}{(1+\sqrt{2})-(\sqrt{3})} \times \frac{(1+\sqrt{2})+(\sqrt{3})}{(1+\sqrt{2})+(\sqrt{3})} \\ & =\frac{1}{(1+\sqrt{2})-(\sqrt{3})} \times \frac{(1+\sqrt{2})+(\sqrt{3})}{(1+\sqrt{2})+(\sqrt{3})} \\ & =\frac{(1+\sqrt{2}+\sqrt{3})}{(1+2+2 \sqrt{2})-3} \end{aligned}$ |  | $\begin{aligned} & \frac{+\sqrt{3})}{2} \times \frac{\sqrt{2}}{\sqrt{2}} \\ & \frac{2+\sqrt{6}}{4} \end{aligned}$ | 4 |
| 24 | $\mathrm{P}(2)=\mathrm{p}$ $\begin{aligned} & \mathrm{g}(3)=\mathrm{q} \\ & \Rightarrow 9 a-15=q \end{aligned}$ | $(2)^{3}+$ $\Rightarrow(3)^{3}+$ $\Rightarrow 9 a$ | $\begin{aligned} & 5 a(2)-8=p \\ & 10 a=p \\ & -12(3)-6=q \\ & p=10 \\ & 8+10 a=10 \end{aligned}$ | 4 |

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|  | $\Rightarrow a=\frac{33}{19}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 25 | $\begin{aligned} & 1600=x+y(8-2) \\ & 1600=x+6 y \end{aligned}$ <br> Graph can be drawn for above equation. |  |  | 4 |
| 26 | Proof: $\begin{aligned} & \angle 1+\angle 2+\angle 3+\angle 4=180 \text { (co-interior angles ) } \\ & 2(\angle 2+\angle 4)=180 \\ & (\angle 2+\angle 4)=90 \end{aligned}$ <br> In $\triangle Q M R$, <br> By angle sum property, $\angle \mathrm{QMR}=90^{\circ}$ | A M <br> c | B | 4 |
| 27 | ```Proof: \(\Delta \mathrm{AMC} \cong \triangle \mathrm{BMD}(\mathrm{SAS})\) \(\mathrm{AC}=\mathrm{BD}(\mathrm{cpct})\) \(\angle \mathrm{MAC}=\angle \mathrm{MBD}\) (cpct) \(\Rightarrow \mathrm{BD}\|\mid \mathrm{CA}\) (converse of alternate int angle property) \(\angle \mathrm{DBC}=\mathrm{ACB}=90^{\circ}\) (co-interior angles) \(\Delta \mathrm{DBC} \cong \triangle \mathrm{ACB}(\mathrm{SAS})\) \(\mathrm{CM}=\mathrm{DM}=\frac{1}{2} \mathrm{AB}\) (cpct)``` |  |  | 4 |
| 28 | Correct construction |  |  | 4 |
| 29 | CI Frequency <br> $10-15$ 6 <br> $15-20$ 10 <br> $20-30$ 10 <br> $30-50$ 8 <br> $50-80$ 18 <br> Correct histogram for above data. | Width of class <br> 5 <br> 5 <br> 10 <br> 20 <br> 30 | New Frequency <br> 6 <br> 10 <br> 5 <br> 2 <br> 3 | 4 |
| 30 | (a) $\mathrm{P}($ even prime $)=\frac{35}{200}$ <br> (b) $P($ multiple of 3$)=\frac{70}{200}$ |  |  | 4 |

(c) $P($ number greater than 6$)=0$
(d) $P($ an odd number $)=\frac{107}{200}$
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