



## NCERT Class 9 Solved Questions for Chapter: Circle

### EXERCISE 10.1

**Q.1.** Fill in the blanks :

- (i) The centre of a circle lies in \_\_\_\_\_ of the circle. (exterior/interior)
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in \_\_\_\_\_ of the circle. (exterior/interior)
- (iii) The longest chord of a circle is a \_\_\_\_\_ of the circle.
- (iv) An arc is a \_\_\_\_\_ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and \_\_\_\_\_ of the circle.
- (vi) A circle divides the plane, on which it lies in \_\_\_\_\_ parts.

**Sol.** (i) interior (ii) exterior (iii) diameter (iv) semicircle (v) the chord (vi) three

**Q.2.** Write True or False: Give reasons for your answers.

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is a major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
- (v) Sector is the region between the chord and its corresponding arc.
- (vi) A circle is a plane figure.

**Sol.** (i) True (ii) False (iii) False (iv) True (v) False (vi) True

### EXERCISE 10.2

**Q.1.** Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

**Sol. Given :** Two congruent circles with centres O and O'. AB and CD are equal chords of the circles with centres O and O' respectively.

**To Prove :**  $\angle AOB = \angle CO'D$

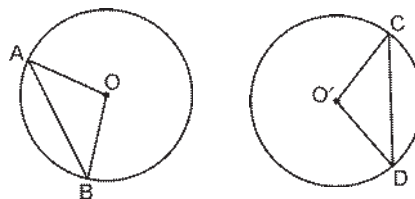
**Proof :** In triangles AOB and CO'D,

$$AB = CD \quad [\text{Given}]$$

$$\left. \begin{array}{l} AO = CO' \\ BO = DO' \end{array} \right\} [\text{Radii of congruent circle}]$$

$$\Rightarrow \triangle AOB \cong \triangle CO'D \quad [\text{SSS axiom}]$$

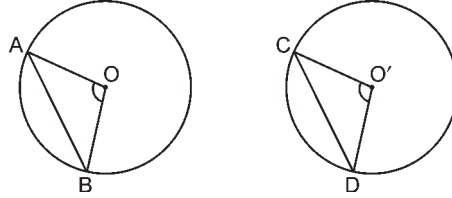
$$\Rightarrow \angle AOB \cong \angle CO'D \quad \text{Proved.} \quad [\text{CPCT}]$$





**Q.2.** Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

**Ans. Given :** Two congruent circles with centres O and O'. AB and CD are chords of circles with centre O and O' respectively such that  $\angle AOB = \angle CO'D$



**To Prove :**  $AB = CD$

**Proof :** In triangles AOB and CO'D,

$$\left. \begin{array}{l} AO = CO' \\ BO = DO' \end{array} \right\} \text{ [Radii of congruent circle]}$$

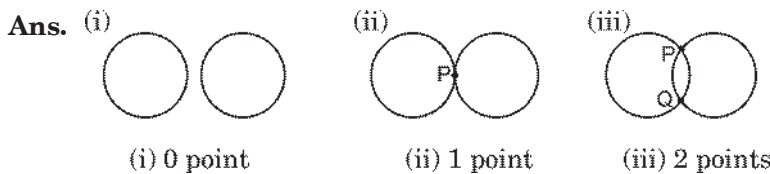
$$\angle AOB = \angle CO'D \quad \text{[Given]}$$

$$\Rightarrow \triangle AOB \cong \triangle CO'D \quad \text{[SAS axiom]}$$

$$\Rightarrow AB = CD \quad \text{Proved. [CPCT]}$$

### EXERCISE 10.3

**Q.1.** Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

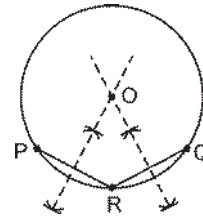


Maximum number of common points = 2 **Ans.**

**Q.2.** Suppose you are given a circle. Give a construction to find its centre.

**Ans. Steps of Construction :**

1. Take arc PQ of the given circle.
  2. Take a point R on the arc PQ and draw chords PR and RQ.
  3. Draw perpendicular bisectors of PR and RQ. These perpendicular bisectors intersect at point O.
- Hence, point O is the centre of the given circle.



**Q.3.** If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

**Ans. Given :** AB is the common chord of two intersecting circles (O, r) and (O', r').

**To Prove :** Centres of both circles lie on the perpendicular bisector of chord AB, i.e., AB is bisected at right angle by OO'.

**Construction :** Join AO, BO, AO' and BO'.

**Proof :** In  $\triangle AOO'$  and  $\triangle BOO'$

$$AO = BO \quad \text{(Radii of the circle (O, r))}$$

$$AO' = BO' \quad \text{(Radii of the circle (O', r'))}$$

$$OO' = OO' \quad \text{(Common)}$$

$$\therefore \triangle AOO' \cong \triangle BOO' \quad \text{(SSS congruency)}$$

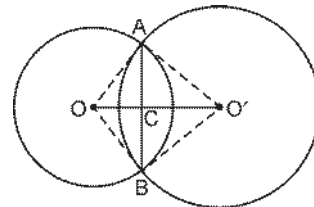
$$\Rightarrow \angle AOO' = \angle BOO' \quad \text{(CPCT)}$$

Now in  $\triangle AOC$  and  $\triangle BOC$

$$\angle AOC = \angle BOC \quad (\angle AOO' = \angle BOO')$$

$$AO = BO \quad \text{(Radii of the circle (O, r))}$$

$$OC = OC \quad \text{(Common)}$$





$\therefore \triangle AOC \cong \triangle BOC$  (SAS congruency)  
 $\Rightarrow AC = BC$  and  $\angle ACO = \angle BCO$  ... (i) (CPCT)  
 $\Rightarrow \angle ACO + \angle BCO = 180^\circ$  .. (ii) (Linear pair)  
 $\Rightarrow \angle ACO = \angle BCO = 90^\circ$  (From (i) and (ii))  
 Hence,  $OO'$  lie on the perpendicular bisector of  $AB$ . **Proved.**

### EXERCISE 10.4

**Q.1.** Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

**Sol.** In  $\triangle AOO'$ ,

$$AO^2 = 5^2 = 25$$

$$AO'^2 = 3^2 = 9$$

$$OO'^2 = 4^2 = 16$$

$$AO'^2 + OO'^2 = 9 + 16 = 25 = AO^2$$

$$\Rightarrow \angle AO'O = 90^\circ$$

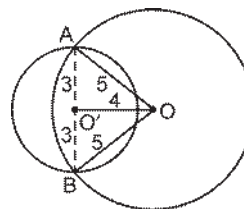
[By converse of pythagoras theorem]

Similarly,  $\angle BO'O = 90^\circ$ .

$$\Rightarrow \angle AO'B = 90^\circ + 90^\circ = 180^\circ$$

$\Rightarrow AO'B$  is a straight line. whose mid-point is  $O$ .

$$\Rightarrow AB = (3 + 3) \text{ cm} = 6 \text{ cm Ans.}$$



**Q.2.** If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

**Sol. Given :**  $AB$  and  $CD$  are two equal chords of a circle which meet at  $E$ .

**To prove :**  $AE = CE$  and  $BE = DE$

**Construction :** Draw  $OM \perp AB$  and  $ON \perp CD$  and join  $OE$ .

**Proof :** In  $\triangle OME$  and  $\triangle ONE$

$$OM = ON \text{ [Equal chords are equidistant]}$$

$$OE = OE \text{ [Common]}$$

$$\angle OME = \angle ONE \text{ [Each equal to } 90^\circ]$$

$$\therefore \triangle OME \cong \triangle ONE \text{ [RHS axiom]}$$

$$\Rightarrow EM = EN \text{ ... (i) [CPCT]}$$

$$\text{Now } AB = CD \text{ [Given]}$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} CD$$

$$\Rightarrow AM = CN \text{ .. (ii) [Perpendicular from centre bisects the chord]}$$

Adding (i) and (ii), we get

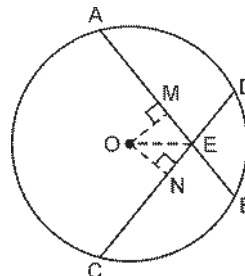
$$EM + AM = EN + CN$$

$$\Rightarrow AE = CE \text{ .. (iii)}$$

$$\text{Now, } AB = CD \text{ .. (iv)}$$

$$\Rightarrow AB - AE = CD - CE \text{ [From (iii)]}$$

$$\Rightarrow BE = CD - CE \text{ **Proved.**}$$



**Q.3.** If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

**Sol. Given :**  $AB$  and  $CD$  are two equal chords of a circle which meet at  $E$  within the circle and a line  $PQ$  joining the point of intersection to the centre.

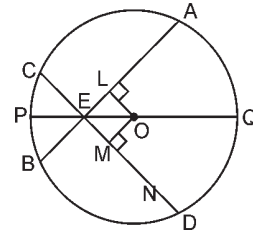
**To Prove :**  $\angle AEQ = \angle DEQ$



**Construction :** Draw  $OL \perp AB$  and  $OM \perp CD$ .

**Proof :** In  $\triangle OLE$  and  $\triangle OME$ , we have

$$\begin{aligned} OL &= OM \text{ [Equal chords are equidistant]} \\ OE &= OE \text{ [Common]} \\ \angle OLE &= \angle OME \text{ [Each = } 90^\circ\text{]} \\ \therefore \triangle OLE &\cong \triangle OME \text{ [RHS congruence]} \\ \Rightarrow \angle LEO &= \angle MEO \text{ [CPCT]} \end{aligned}$$



**Q.4.** If a line intersects two concentric circles (circles with the same centre) with centre  $O$  at  $A, B, C$  and  $D$ , prove that  $AB = CD$  (see Fig.)

**Sol. Given :** A line  $AD$  intersects two concentric circles at  $A, B, C$  and  $D$ , where  $O$  is the centre of these circles.

**To prove :**  $AB = CD$

**Construction :** Draw  $OM \perp AD$ .

**Proof :**  $AD$  is the chord of larger circle.

$\therefore AM = DM$  ..(i) [ $OM$  bisects the chord]

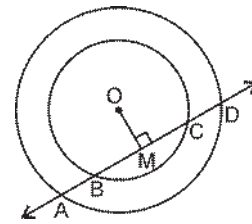
$BC$  is the chord of smaller circle

$\therefore BM = CM$  ..(ii) [ $OM$  bisects the chord]

Subtracting (ii) from (i), we get

$$AM - BM = DM - CM$$

$$\Rightarrow AB = CD \text{ Proved.}$$



**Q.5.** Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

**Sol.** Let Reshma, Salma and Mandip be represented by  $R, S$  and  $M$  respectively.

Draw  $OL \perp RS$ ,

$$OL^2 = OR^2 - RL^2$$

$$OL^2 = 5^2 - 3^2 \text{ [RL = 3 m, because } OL \perp RS\text{]} \\ = 25 - 9 = 16$$

$$OL = \sqrt{16} = 4$$

$$\text{Now, area of triangle } ORS = \frac{1}{2} \times KR \times OS$$

$$= \frac{1}{2} \times KR \times OS$$

$$\text{Also, area of } \triangle ORS = \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4 = 12 \text{ m}^2$$

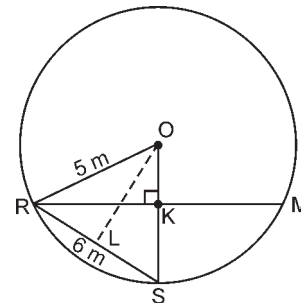
$$\Rightarrow \frac{1}{2} \times KR \times 5 = 12$$

$$\Rightarrow KR = \frac{12 \times 2}{5} = \frac{24}{5} = 4.8 \text{ m}$$

$$\Rightarrow RM = 2KR$$

$$\Rightarrow RM = 2 \times 4.8 = 9.6 \text{ m}$$

Hence, distance between Reshma and Mandip is 9.6 m **Ans.**





**Q.6.** A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

**Sol.** Let Ankur, Syed and David be represented by A, S and D respectively.

Let  $PD = SP = SQ = QA = AR = RD = x$

In  $\triangle OPD$ ,

$$OP^2 = 400 - x^2$$

$$\Rightarrow OP = \sqrt{400 - x^2}$$

$$\Rightarrow AP = 2\sqrt{400 - x^2} + \sqrt{400 - x^2}$$

[ $\because$  centroid divides the median in the ratio 2 : 1]

$$= 3\sqrt{400 - x^2}$$

Now, in  $\triangle APD$ ,

$$PD^2 = AD^2 - AP^2$$

$$\Rightarrow x^2 = (2x)^2 - (3\sqrt{400 - x^2})^2$$

$$\Rightarrow x^2 = 4x^2 - 9(400 - x^2)$$

$$\Rightarrow x^2 = 4x^2 - 3600 + 9x^2$$

$$\Rightarrow 12x^2 = 3600$$

$$\Rightarrow x^2 = \frac{3600}{12} = 300$$

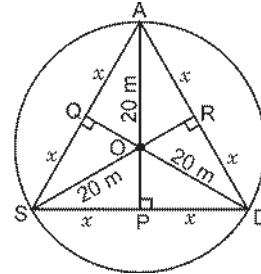
$$\Rightarrow x = 10\sqrt{3}$$

$$\text{Now, } SD = 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$$

$\therefore$  ASD is an equilateral triangle.

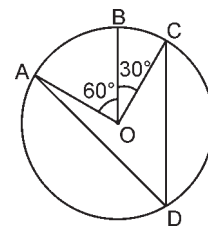
$$\Rightarrow SD = AS = AD = 20\sqrt{3}$$

Hence, length of the string of each phone is  $20\sqrt{3}$  m **Ans.**



### EXERCISE 10.5

**Q.1.** In the figure, A, B and C are three points on a circle with centre O such that  $\angle BOC = 30^\circ$  and  $\angle AOB = 60^\circ$ . If D is a point on the circle other than the arc ABC, find  $\angle ADC$ .



**Sol.** We have,  $\angle BOC = 30^\circ$  and  $\angle AOB = 60^\circ$

$$\angle AOC = \angle AOB + \angle BOC = 60^\circ + 30^\circ = 90^\circ$$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$$\therefore 2\angle ADC = \angle AOC$$

$$\Rightarrow \angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 90^\circ \Rightarrow \angle ADC = 45^\circ \text{ **Ans.**}$$

**Q.2.** A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



**Sol.** We have,  $OA = OB = AB$

Therefore,  $\triangle OAB$  is an equilateral triangle.

$$\Rightarrow \angle AOB = 60^\circ$$

We know that angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc on the remaining part of the circle.

$$\therefore \angle AOB = 2\angle ACB$$

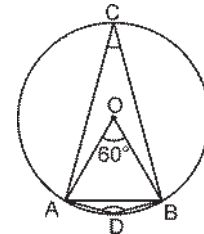
$$\Rightarrow \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 60^\circ$$

$$\Rightarrow \angle ACB = 30^\circ$$

$$\text{Also, } \angle ADB = \frac{1}{2} \text{ reflex } \angle AOB$$

$$= \frac{1}{2} (360^\circ - 60^\circ) = \frac{1}{2} \times 300^\circ = 150^\circ$$

Hence, angle subtended by the chord at a point on the minor arc is  $150^\circ$  and at a point on the major arc is  $30^\circ$  **Ans.**



**Q.3.** In the figure,  $\angle PQR = 100^\circ$ , where  $P, Q$  and  $R$  are points on a circle with centre  $O$ . Find  $\angle OPR$ .

**Sol.** Reflex angle  $POR = 2\angle PQR$

$$= 2 \times 100^\circ = 200^\circ$$

$$\text{Now, angle } POR = 360^\circ - 200^\circ = 160^\circ$$

Also,  $PO = OR$  [Radii of a circle]

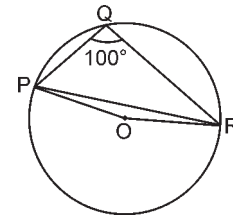
$$\therefore \angle OPR = \angle ORP$$

[Opposite angles of isosceles triangle]

$$\text{In } \triangle OPR, \angle POR = 160^\circ$$

$$\therefore \angle OPR = \angle ORP = 10^\circ$$

[Angle sum property of a triangle]. **Ans.**



**Q.4.** In the figure,  $\angle ABC = 69^\circ$ ,  $\angle ACB = 31^\circ$ , find  $\angle BDC$ .

**Sol.** In  $\triangle ABC$ , we have

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

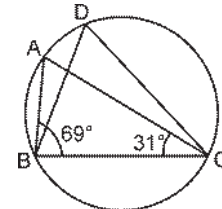
[Angle sum property of a triangle]

$$\Rightarrow 69^\circ + 31^\circ + \angle BAC = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 100^\circ = 80^\circ$$

Also,  $\angle BAC = \angle BDC$  [Angles in the same segment]

$$\Rightarrow \angle BDC = 80^\circ \text{ **Ans.**}$$



**Q.5.** In the figure,  $A, B, C$  and  $D$  are four points on a circle.  $AC$  and  $BD$  intersect at a point  $E$  such that  $\angle BEC = 130^\circ$  and  $\angle ECD = 20^\circ$ . Find  $\angle BAC$ .

**Sol.**  $\angle BEC + \angle DEC = 180^\circ$  [Linear pair]

$$\Rightarrow 130^\circ + \angle DEC = 180^\circ$$

$$\Rightarrow \angle DEC = 180^\circ - 130^\circ = 50^\circ$$

Now, in  $\triangle DEC$ ,

$$\Rightarrow \angle DEC + \angle DCE + \angle CDE = 180^\circ$$

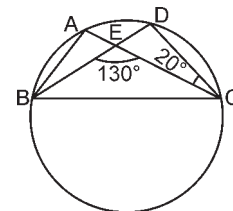
[Angle sum property of a triangle]

$$\Rightarrow 50^\circ + 20^\circ + \angle CDE = 180^\circ$$

$$\Rightarrow \angle CDE = 180^\circ - 70^\circ = 110^\circ$$

Also,  $\angle CDE = \angle BAC$  [Angles in same segment]

$$\Rightarrow \angle BAC = 110^\circ \text{ **Ans.**}$$





**Q.6.** ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If  $\angle DBC = 70^\circ$ ,  $\angle BAC = 30^\circ$ , find  $\angle BCD$ . Further, if  $AB = BC$ , find  $\angle ECD$ .

**Sol.**  $\angle CAD = \angle DBC = 70^\circ$  [Angles in the same segment]

$$\begin{aligned} \text{Therefore, } \angle DAB &= \angle CAD + \angle BAC \\ &= 70^\circ + 30^\circ = 100^\circ \end{aligned}$$

$$\text{But, } \angle DAB + \angle BCD = 180^\circ$$

[Opposite angles of a cyclic quadrilateral]

$$\text{So, } \angle BCD = 180^\circ - 100^\circ = 80^\circ$$

Now, we have  $AB = BC$

Therefore,  $\angle BCA = 30^\circ$  [Opposite angles of an isosceles triangle]

$$\text{Again, } \angle DAB + \angle BCD = 180^\circ$$

[Opposite angles of a cyclic quadrilateral]

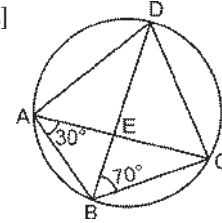
$$\Rightarrow 100^\circ + \angle BCA + \angle ECD = 180^\circ \quad [\because \angle BCD = \angle BCA + \angle ECD]$$

$$\Rightarrow 100^\circ + 30^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow 130^\circ + \angle ECD = 180^\circ$$

$$\Rightarrow \angle ECD = 180^\circ - 130^\circ = 50^\circ$$

Hence,  $\angle BCD = 80^\circ$  and  $\angle ECD = 50^\circ$  **Ans.**



**Q.7.** If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

**Sol. Given :** ABCD is a cyclic quadrilateral, whose diagonals AC and BD are diameter of the circle passing through A, B, C and D.

**To Prove :** ABCD is a rectangle.

**Proof :** In  $\triangle AOD$  and  $\triangle COB$

$$AO = CO \quad [\text{Radii of a circle}]$$

$$OD = OB \quad [\text{Radii of a circle}]$$

$$\angle AOD = \angle COB \quad [\text{Vertically opposite angles}]$$

$$\therefore \triangle AOD \cong \triangle COB \quad [\text{SAS axiom}]$$

$$\therefore \angle OAD = \angle OCB \quad [\text{CPCT}]$$

But these are alternate interior angles made by the transversal AC, intersecting AD and BC.

$$\therefore AD \parallel BC$$

Similarly,  $AB \parallel CD$ .

Hence, quadrilateral ABCD is a parallelogram.

$$\text{Also, } \angle ABC = \angle ADC \quad \dots(i) \quad [\text{Opposite angles of a } \parallel\text{gm are equal}]$$

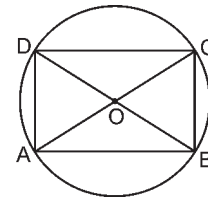
$$\text{And, } \angle ABC + \angle ADC = 180^\circ \quad \dots(ii)$$

[Sum of opposite angles of a cyclic quadrilateral is  $180^\circ$ ]

$$\Rightarrow \angle ABC = \angle ADC = 90^\circ \quad [\text{From (i) and (ii)}]$$

$\therefore$  ABCD is a rectangle. [A  $\parallel\text{gm}$  one of whose angles is

$90^\circ$  is a rectangle] **Proved.**



**Q.8.** If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

**Sol. Given :** A trapezium ABCD in which  $AB \parallel CD$  and  $AD = BC$ .

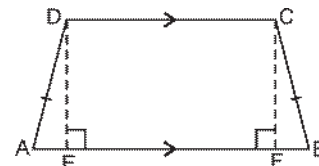
**To Prove :** ABCD is a cyclic trapezium.

**Construction :** Draw  $DE \perp AB$  and  $CF \perp AB$ .

**Proof :** In  $\triangle DEA$  and  $\triangle CFB$ , we have

$$AD = BC \quad [\text{Given}]$$

$$\angle DEA = \angle CFB = 90^\circ \quad [DE \perp AB \text{ and } CF \perp AB]$$





$DE = CF$   
[Distance between parallel lines remains constant]

$\therefore \triangle DEA \cong \triangle CFB$  [RHS axiom]  
 $\Rightarrow \angle A = \angle B$  ... (i) [CPCT]  
 and,  $\angle ADE = \angle BCF$  .. (ii) [CPCT]  
 Since,  $\angle ADE = \angle BCF$  [From (ii)]  
 $\Rightarrow \angle ADE + 90^\circ = \angle BCF + 90^\circ$   
 $\Rightarrow \angle ADE + \angle CDE = \angle BCF + \angle DCF$   
 $\Rightarrow \angle D = \angle C$  .. (iii)  
 [  $\angle ADE + \angle CDE = \angle D$ ,  $\angle BCF + \angle DCF = \angle C$  ]  
 $\therefore \angle A = \angle B$  and  $\angle C = \angle D$  [From (i) and (iii)] (iv)  
 $\angle A + \angle B + \angle C + \angle D = 360^\circ$  [Sum of the angles of a quadrilateral is  $360^\circ$ ]

$\Rightarrow 2(\angle B + \angle D) = 360^\circ$  [Using (iv)]  
 $\Rightarrow \angle B + \angle D = 180^\circ$   
 $\Rightarrow$  Sum of a pair of opposite angles of quadrilateral ABCD is  $180^\circ$ .  
 $\Rightarrow$  ABCD is a cyclic trapezium **Proved.**

**Q.9.** Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that  $\angle ACP = \angle QCD$ .

**Sol. Given :** Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively.

**To Prove :**  $\angle ACP = \angle QCD$ .

**Proof :**  $\angle ACP = \angle ABP$  ... (i)

[Angles in the same segment]

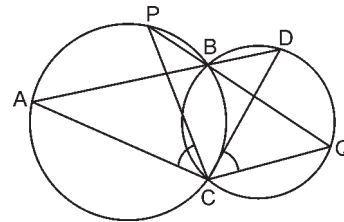
$\angle QCD = \angle QBD$  .. (ii)

[Angles in the same segment]

But,  $\angle ABP = \angle QBD$  .. (iii) [Vertically opposite angles]

By (i), (ii) and (iii) we get

$\angle ACP = \angle QCD$  **Proved.**



**Q.10.** If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

**Sol. Given :** Sides AB and AC of a triangle ABC are diameters of two circles which intersect at D.

**To Prove :** D lies on BC.

**Proof :** Join AD

$\angle ADB = 90^\circ$  ... (i) [Angle in a semicircle]

Also,  $\angle ADC = 90^\circ$  .. (ii)

Adding (i) and (ii), we get

$\angle ADB + \angle ADC = 90^\circ + 90^\circ$

$\Rightarrow \angle ADB + \angle ADC = 180^\circ$

$\Rightarrow$  BDC is a straight line.

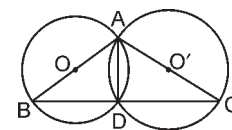
$\therefore$  D lies on BC

Hence, point of intersection of circles lie on the third side BC. **Proved.**

**Q.11.** ABC and ADC are two right triangles with common hypotenuse AC. Prove that  $\angle CAD = \angle CBD$ .

**Sol. Given :** ABC and ADC are two right triangles with common hypotenuse AC.

**To Prove :**  $\angle CAD = \angle CBD$







**Proof :** Let O be the mid-point of AC.

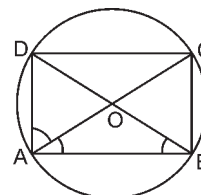
Then  $OA = OB = OC = OD$

Mid point of the hypotenuse of a right triangle is equidistant from its vertices with O as centre and radius equal to OA, draw a circle to pass through A, B, C and D.

We know that angles in the same segment of a circle are equal.

Since,  $\angle CAD$  and  $\angle CBD$  are angles of the same segment.

Therefore,  $\angle CAD = \angle CBD$ . **Proved.**



**Q.12.** Prove that a cyclic parallelogram is a rectangle.

**Sol. Given :** ABCD is a cyclic parallelogram.

**To prove :** ABCD is a rectangle.

**Proof :**  $\angle ABC = \angle ADC$  ... (i)

[Opposite angles of a ||gm are equal]

But,  $\angle ABC + \angle ADC = 180^\circ$  ... (ii)

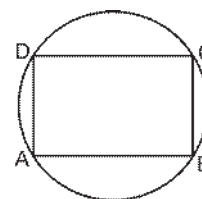
[Sum of opposite angles of a cyclic quadrilateral is  $180^\circ$ ]

$\Rightarrow \angle ABC = \angle ADC = 90^\circ$  [From (i) and (ii)]

$\therefore$  ABCD is a rectangle

[A ||gm one of whose angles is  $90^\circ$  is a rectangle]

Hence, a cyclic parallelogram is a rectangle. **Proved.**



### EXERCISE 10.6 (Optional)

**Q.1.** Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

**Sol. Given :** Two intersecting circles, in which  $OO'$  is the line of centres and A and B are two points of intersection.

**To prove :**  $\angle OAO' = \angle OBO'$

**Construction :** Join AO, BO,  $AO'$  and  $BO'$ .

**Proof :** In  $\triangle AOO'$  and  $\triangle BOO'$ , we have

$$AO = BO \quad [\text{Radii of the same circle}]$$

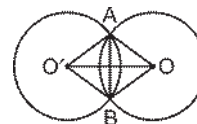
$$AO' = BO' \quad [\text{Radii of the same circle}]$$

$$OO' = OO' \quad [\text{Common}]$$

$$\therefore \triangle AOO' \cong \triangle BOO' \quad [\text{SSS axiom}]$$

$$\Rightarrow \angle OAO' = \angle OBO' \quad [\text{CPCT}]$$

Hence, the line of centres of two intersecting circles subtends equal angles at the two points of intersection. **Proved.**



**Q.2.** Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

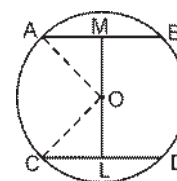
**Sol.** Let O be the centre of the circle and let its radius be r cm.

Draw  $OM \perp AB$  and  $OL \perp CD$ .

$$\text{Then, } AM = \frac{1}{2} AB = \frac{5}{2} \text{ cm}$$

$$\text{and, } CL = \frac{1}{2} CD = \frac{11}{2} \text{ cm}$$

Since,  $AB \parallel CD$ , it follows that the points O, L, M are





collinear and therefore,  $LM = 6$  cm.

Let  $OL = x$  cm. Then  $OM = (6 - x)$  cm

Join  $OA$  and  $OC$ . Then  $OA = OC = r$  cm.

Now, from right-angled  $\triangle OMA$  and  $\triangle OLC$ , we have

$OA^2 = OM^2 + AM^2$  and  $OC^2 = OL^2 + CL^2$  [By Pythagoras Theorem]

$$\Rightarrow r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2 \quad \dots(i) \text{ and } r^2 = x^2 + \left(\frac{11}{2}\right)^2 \quad \dots (ii)$$

$$\Rightarrow (6 - x)^2 + \left(\frac{5}{2}\right)^2 = x^2 + \left(\frac{11}{2}\right)^2 \quad [\text{From (i) and (ii)}]$$

$$\Rightarrow 36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$\Rightarrow -12x = \frac{121}{4} - \frac{25}{4} - 36$$

$$\Rightarrow -12x = \frac{96}{4} - 36$$

$$\Rightarrow -12x = 24 - 36$$

$$\Rightarrow -12x = -12$$

$$\Rightarrow x = 1$$

Substituting  $x = 1$  in (i), we get

$$r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (6 - 1)^2 + \left(\frac{5}{2}\right)^2$$

$$\Rightarrow r^2 = (5)^2 + \left(\frac{5}{2}\right)^2 = 25 + \frac{25}{4}$$

$$\Rightarrow r^2 = \frac{125}{4}$$

$$\Rightarrow r = \frac{5\sqrt{5}}{2}$$

Hence, radius  $r = \frac{5\sqrt{5}}{2}$  cm. **Ans.**

**Q.3.** The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

**Sol.** Let  $PQ$  and  $RS$  be two parallel chords of a circle with centre  $O$ .

We have,  $PQ = 8$  cm and  $RS = 6$  cm.

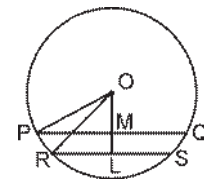
Draw perpendicular bisector  $OL$  of  $RS$  which meets  $PQ$  in  $M$ . Since,  $PQ \parallel RS$ , therefore,  $OM$  is also perpendicular bisector of  $PQ$ .

Also,  $OL = 4$  cm and  $RL = \frac{1}{2}RS \Rightarrow RL = 3$  cm

and  $PM = \frac{1}{2}PQ \Rightarrow PM = 4$  cm

In  $\triangle ORL$ , we have

$$OR^2 = RL^2 + OL^2 \quad [\text{Pythagoras theorem}]$$





$$\Rightarrow OR^2 = 3^2 + 4^2 = 9 + 16$$

$$\Rightarrow OR^2 = 25 \Rightarrow OR = \sqrt{25}$$

$$\Rightarrow OR = 5 \text{ cm}$$

$$\therefore OR = OP \quad [\text{Radii of the circle}]$$

$$\Rightarrow OP = 5 \text{ cm}$$

Now, in  $\triangle OPM$

$$OM^2 = OP^2 - PM^2 \quad [\text{Pythagoras theorem}]$$

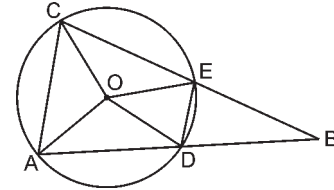
$$\Rightarrow OM^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$OM = \sqrt{9} = 3 \text{ cm}$$

Hence, the distance of the other chord from the centre is 3 cm. **Ans.**

**Q.4.** Let the vertex of an angle  $ABC$  be located outside a circle and let the sides of the angle intersect equal chords  $AD$  and  $CE$  with the circle. Prove that  $\angle ABC$  is equal to half the difference of the angles subtended by the chords  $AC$  and  $DE$  at the centre.

**Sol. Given :** Two equal chords  $AD$  and  $CE$  of a circle with centre  $O$ . When meet at  $B$  when produced.



$$\text{To Prove : } \angle ABC = \frac{1}{2} (\angle AOC - \angle DOE)$$

**Proof :** Let  $\angle AOC = x$ ,  $\angle DOE = y$ ,  $\angle AOD = z$

$$\angle EOC = z \quad [\text{Equal chords subtends equal angles at the centre}]$$

$$\therefore x + y + 2z = 360^\circ \quad [\text{Angle at a point}] \quad \dots (i)$$

$$OA = OD \Rightarrow \angle OAD = \angle ODA$$

$\therefore$  In  $\triangle OAD$ , we have

$$\angle OAD + \angle ODA + z = 180^\circ$$

$$\Rightarrow 2\angle OAD = 180^\circ - z \quad [ \because \angle OAD = \angle ODA ]$$

$$\Rightarrow \angle OAD = 90^\circ - \frac{z}{2} \quad \dots (ii)$$

$$\text{Similarly } \angle OCE = 90^\circ - \frac{z}{2} \quad \dots (iii)$$

$$\Rightarrow \angle ODB = \angle OAD + \angle ODA \quad [\text{Exterior angle property}]$$

$$\Rightarrow \angle OEB = 90^\circ - \frac{z}{2} + z \quad [\text{From (ii)}]$$

$$\Rightarrow \angle ODB = 90^\circ + \frac{z}{2} \quad \dots (iv)$$

$$\text{Also, } \angle OEB = \angle OCE + \angle COE \quad [\text{Exterior angle property}]$$

$$\Rightarrow \angle OEB = 90^\circ - \frac{z}{2} + z \quad [\text{From (iii)}]$$

$$\Rightarrow \angle OEB = 90^\circ + \frac{z}{2} \quad \dots (v)$$

Also,  $\angle OED = \angle ODE = 90^\circ - \frac{y}{2}$  ... (vi)

O from (iv), (v) and (vi), we have

$$\angle BDE = \angle BED = 90^\circ + \frac{z}{2} - \left(90^\circ - \frac{y}{2}\right)$$

$$\Rightarrow \angle BDE = \angle BED = \frac{y+z}{2}$$

$$\Rightarrow \angle BDE = \angle BED = y + z \quad \dots \text{(vii)}$$

$$\therefore \angle BDE = 180^\circ - (y + z)$$

$$\Rightarrow \angle ABC = 180^\circ - (y + z) \quad \dots \text{(viii)}$$

$$\text{Now, } \frac{y-z}{2} = \frac{360^\circ - y - 2z - y}{2} = 180^\circ - (y + z) \quad \dots \text{(ix)}$$

From (viii) and (ix), we have

$$\angle ABC = \frac{x-y}{2} \quad \text{Proved.}$$

**Q.5.** Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

**Sol. Given :** A rhombus ABCD whose diagonals intersect each other at O.

**To prove :** A circle with AB as diameter passes through O.

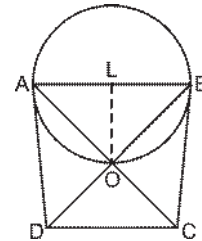
**Proof :**  $\angle AOB = 90^\circ$

[Diagonals of a rhombus bisect each other at  $90^\circ$ ]

$\Rightarrow \triangle AOB$  is a right triangle right angled at O.

$\Rightarrow AB$  is the hypotenuse of  $\triangle AOB$ .

$\Rightarrow$  If we draw a circle with AB as diameter, then it will pass through O. because angle in a semicircle is  $90^\circ$  and  $\angle AOB = 90^\circ$  **Proved.**



**Q.6.** ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that  $AE = AD$ .

**Sol. Given :** ABCD is a parallelogram.

**To Prove :**  $AE = AD$ .

**Construction :** Draw a circle which passes through ABC and intersect CD (or CD produced) at E.

**Proof :** For fig (i)

$$\angle AED + \angle ABC = 180^\circ$$

[Linear pair] ... (ii)

But  $\angle ACD = \angle ADC = \angle ABC + \angle ADE$

$$\Rightarrow \angle ABC + \angle ADE = 180^\circ \quad \text{[From (ii)]} \quad \dots \text{(iii)}$$

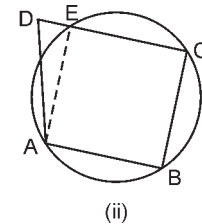
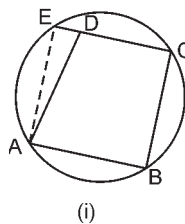
From (i) and (iii)

$$\angle AED + \angle ABC = \angle ABC + \angle ADE$$

$$\Rightarrow \angle AED = \angle ADE$$

$$\Rightarrow \angle AD = \angle AE \quad \text{[Sides opposite to equal angles are equal]}$$

Similarly we can prove for Fig (ii) **Proved.**





**Q.7.** *AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is rectangle.*

**Sol. Given :** A circle with chords AB and CD which bisect each other at O.

**To Prove :** (i) AC and BD are diameters  
(ii) ABCD is a rectangle.

**Proof :** In  $\triangle OAB$  and  $\triangle OCD$ , we have

$OA = OC$	[Given]
$OB = OD$	[Given]
$\angle AOB = \angle COD$	[Vertically opposite angles]
$\Rightarrow \triangle AOB \cong \triangle COD$	[SAS congruence]
$\Rightarrow \angle ABO = \angle CDO$ and $\angle BAO = \angle BCO$	[CPCT]
$\Rightarrow AB \parallel DC$	... (i)

Similarly, we can prove  $BC \parallel AD$  ... (ii)

Hence, ABCD is a parallelogram.

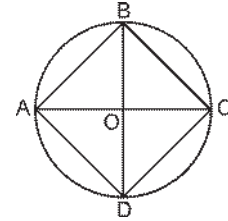
But ABCD is a cyclic parallelogram.

$\therefore$  ABCD is a rectangle. [Proved in Q. 12 of Ex. 10.5]

$\Rightarrow \angle ABC = 90^\circ$  and  $\angle BCD = 90^\circ$

$\Rightarrow AC$  is a diameter and  $BD$  is a diameter

[Angle in a semicircle is  $90^\circ$ ] **Proved.**



**Q.8.** *Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are*

$90^\circ - \frac{1}{2}A$ ,  $90^\circ - \frac{1}{2}B$  and  $90^\circ - \frac{1}{2}C$ .

**Sol. Given :**  $\triangle ABC$  and its circumcircle. AD, BE, CF are bisectors of  $\angle A$ ,  $\angle B$ ,  $\angle C$  respectively.

**Construction :** Join DE, EF and FD.

**Proof :** We know that angles in the same segment are equal.

$$\therefore \angle 5 = \frac{\angle C}{2} \text{ and } \angle 6 = \frac{\angle B}{2} \dots(i)$$

$$\angle 1 = \frac{\angle A}{2} \text{ and } \angle 2 = \frac{\angle C}{2} \dots(ii)$$

$$\angle 4 = \frac{\angle A}{2} \text{ and } \angle 3 = \frac{\angle B}{2} \dots(iii)$$

From (i), we have

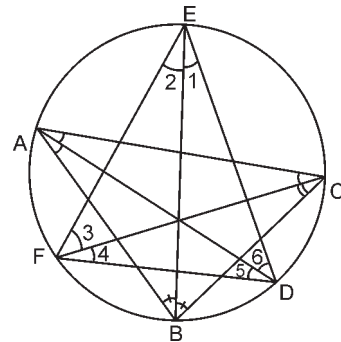
$$\angle 5 + \angle 6 = \frac{\angle C}{2} + \frac{\angle B}{2}$$

$$\Rightarrow \angle D = \frac{\angle C}{2} + \frac{\angle B}{2} \dots(iv)$$

$$[\because \angle 5 + \angle 6 = \angle D]$$

But  $\angle A + \angle B + \angle C = 180^\circ$

$$\Rightarrow \angle B + \angle C = 180^\circ - \angle A$$



$$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

$\therefore$  (iv) becomes,

$$\angle D = 90^\circ - \frac{\angle A}{2}$$

Similarly, from (ii) and (iii), we can prove that

$$\angle E = 90^\circ - \frac{\angle B}{2} \text{ and } \angle F = 90^\circ - \frac{\angle C}{2} \quad \textbf{Proved.}$$

**Q.9.** Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

**Sol.** **Given :** Two congruent circles which intersect at A and B. PAB is a line through A.

**To Prove :** BP = BQ.

**Construction :** Join AB.

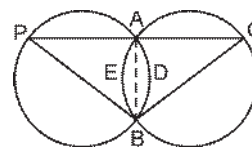
**Proof :** AB is a common chord of both the circles.

But the circles are congruent —

$$\Rightarrow \text{arc ADB} = \text{arc AEB}$$

$$\Rightarrow \angle APB = \angle AQB \quad \text{Angles subtended}$$

$$\Rightarrow BP = BQ \quad [\text{Sides opposite to equal angles are equal}] \quad \textbf{Proved.}$$



**Q.10.** In any triangle ABC, if the angle bisector of  $\angle A$  and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

**Sol.** Let angle bisector of  $\angle A$  intersect circumcircle of  $\Delta ABC$  at D.

Join DC and DB.

$$\angle BCD = \angle BAD$$

[Angles in the same segment]

$$\Rightarrow \angle BCD = \angle BAD = \frac{1}{2} \angle A$$

[AD is bisector of  $\angle A$ ] ... (i)

$$\text{Similarly } \angle DBC = \angle DAC = \frac{1}{2} \angle A \quad \dots \text{ (ii)}$$

From (i) and (ii)  $\angle DBC = \angle BCD$

$$\Rightarrow BD = DC \quad [\text{sides opposite to equal angles are equal}]$$

$\Rightarrow$  D lies on the perpendicular bisector of BC.

Hence, angle bisector of  $\angle A$  and perpendicular bisector of BC intersect on the circumcircle of  $\Delta ABC$  **Proved.**

