CBSE Area of triangles by Heron's formula Solved

OABC is a rhombus whose three vertices A,B,C lies on circle with centre O. Radius of a circle is 10cm. Find the area of rhombus .

OABC is a rhombus and O is the centre of the circle,

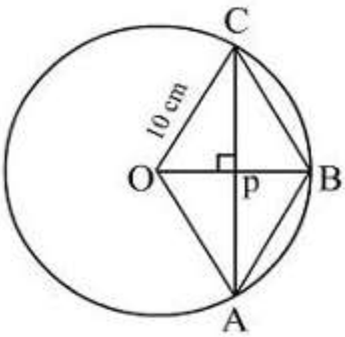
OB and AC are the diagonals of the rhombus.

Radius of the circle = 10 cm

$\therefore OA = OC = OB = 10$ cm

In a rhombus diagonals bisect at 90°

$\therefore OP = \frac{1}{2}OB = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$



In ΔOPC,

$OC^2 = OP^2 + CP^2$

$\therefore (10)^2 = 5^2 + CP^2$

$\therefore 100 = 25 + CP^2$

$\therefore CP^2 = 100 - 25 = 75$

$\therefore CP = \sqrt{75} = \sqrt{25 \times 3} \text{ cm} = 5\sqrt{3} \text{ cm}$

$\Rightarrow AC = 2 \times 5\sqrt{3} \text{ cm} = 10\sqrt{3} \text{ cm}$

Area of the rhombus $= \frac{1}{2} \times OB \times AC$

$= \frac{1}{2} \times 10 \times 10\sqrt{3} \text{ cm}^2$

$= 50\sqrt{3} \text{ cm}^2$

A school started a campaign against the habit of taking junk food. Ten students were asked to prepare banners in triangular shape .The sides of the banner are 112, 78 AND 50 cm.if cost of banners is Rs 3 per 100 cm2, find the total cost of banners. What value is being promoted in this question?

The sides of each banner are 112 cm, 78 cm and 50 cm

$\therefore s = \frac{112 + 78 + 50}{2} = \frac{240}{2} = 120 \text{ cm}$

$\therefore$ Area of 1 banner $= \sqrt{120(120-112)(120-78)(120-50)} \text{ cm}^2$

$= \sqrt{120 \times 8 \times 42 \times 70} \text{ cm}^2$

$= \sqrt{2 \times 6 \times 10 \times 4 \times 2 \times 7 \times 6 \times 7 \times 10} \text{ cm}^2$

$= 2 \times 6 \times 10 \times 2 \times 7 \text{ cm}^2 \quad = 1680 \text{ cm}^2$

$\therefore$ Total area of 10 banners $= 10 \times 1680 \text{ cm}^2$

$= 16800 \text{ cm}^2$

Cost of banner = Rs 3 per 100 cm^2

$\therefore$ Cost of 10 banners $= \text{Rs } \frac{3}{100} \times 16800$

$= \text{Rs } 504$

$\Rightarrow$ Total cost of banners = Rs 504

The value which is promoted in the question is that we should not eat junk food, we should eat healthy food.

The area of an equilateral triangle is 16√3 m sq. Find its perimeter.

Area of an equilateral triangle $= 16\sqrt{3} \text{ m}^2$

Let the side of the equilateral triangle be x m.

$\therefore$ Area of equilateral triangle $= \frac{\sqrt{3}}{4} x^2$

$\therefore \frac{\sqrt{3}}{4} x^2 = 16\sqrt{3}$

$\therefore x^2 = \frac{16\sqrt{3} \times 4}{\sqrt{3}}$

$\therefore x^2 = 16 \times 4$

$\Rightarrow x = \sqrt{16 \times 4} = 4 \times 2 = 8 \text{ m}$

$\therefore$ Perimeter of the equilateral triangle $= 3 \times x$ m

$= 3 \times 8 = 24$ m

The base of a triangular field is three times its altitude. If the cost of sowing the field at Rs 58 per hectare is Rs 783, find its height and base.

Cost of sowing field = Rate of sowing $\times$ Area of triangular field

$\therefore$ Area of triangular field $= \frac{\text{Cost of sowing field}}{\text{Rate of sowing}}$

$\therefore$ Area of triangular field $= \frac{783}{58} = 13.5$ hectare

1 hectare $= 10000 \text{ m}^2$

$\therefore$ 13.5 hectare $= 13.5 \times 10000$

$= 135000 \text{ m}^2$

$\therefore$ Area of triangular field $= 135000 \text{ m}^2$

Let altitude of the field be x m

$\therefore$ Base of the field = $3x$ m

Area of the field = $\frac{1}{2} \times x \times 3x$ m^2

$\therefore 135000 = \frac{1}{2} \times 3x^2$

$\therefore x^2 = \frac{\overset{45000}{\cancel{135000}} \times 2}{\cancel{3}}$

$\therefore x = \sqrt{90000} = 300$

$\therefore$ Altitude of the field = 300 m

Base of the field = 3×300 m = 900 m

Perimeter of a triangle is 450m and its sides are in the ratio 13:12:5, Find the area of the triangle and altitude of the smallest side.

The sides of a triangle are in the ratio 13:12:5.

$\therefore$ The sides of the triangle are $13x$ cm, $12x$ cm and $5x$ cm.

Perimeter of the triangle = 450 cm

$\therefore 13x + 12x + 5x = 450$

$\therefore 30x = 450$

$\therefore x = \frac{450}{30} = 15$

$\Rightarrow$ The sides of the triangle are $13 \times 15 = 195$ cm, $12 \times 15 = 180$ cm and $5 \times 15 = 75$ cm

Semi perimeter $s = \frac{450}{2} = 225$ cm

$$\begin{aligned}\therefore \text{Area of the triangle} &= \sqrt{225(225-195)(225-180)(225-75)} \\ &= \sqrt{225 \times 30 \times 45 \times 150} \text{ cm}^2 \\ &= \sqrt{15 \times 15 \times 3 \times 10 \times 15 \times 3 \times 15 \times 10} \text{ cm}^2 \\ &= 15 \times 3 \times 10 \times 15 \\ &= 6750 \text{ cm}^2 \quad \ldots(1)\end{aligned}$$

Let the altitude to the smallest side i.e., 75 cm be x cm.

$\therefore$ Area of triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$$6750 = \frac{1}{2} \times 75 \times x \quad \ldots[\text{From}(1)]$$

$\therefore$ Altitude to the smallest side = 180 cm

The area of 4 walls of a room is 120m^2. The length is twice its breadth. If the height of the room is 4 m . Find the area of floor.

Area of four walls = $120\ m^2$

Let the breadth of the room be x m

$\therefore$ Length of the room = $2x$ m

Height of the room = 4 m

Area of four walls of a room = $2(l+b)\times h$

$\therefore 120 = 2(2x+x)\times 4$

$\therefore 120 = 8\times 3x$

$\therefore 24x = 120$

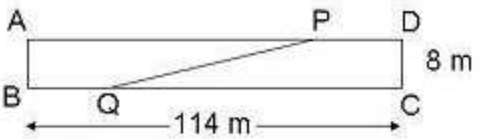
or $x = \frac{120}{24} = 5$ m

$\therefore$ Breadth of the room = 5 m

Length of the room = $5\times 2 = 10$ m

$\Rightarrow$ Area of the floor = $10\times 5\ m^2 = 50\ m^2$

A trapezium with its parallel sides in the ratio 15:4 is cut off from a rectangle whose sides measure 114 m and 8 m respectively. The area of the trapezium is 5/12 of the area of the rectangle. Find the length of the parallel sides of trapezium.



Let ABCD be the rectangle whose sides are 114 m and 8 m.

$\therefore$ Area of the rectangle = $(114\times 8)\ m^2$

Let trapezium PQCD is cut from the rectangle.

$\therefore$ Height of the trapezium = 8 m

Ratio of the parallel sides of trapezium = 15:4

$\therefore$ The lengths of the parallel sides are $15x$ m and $4x$ m respectively.

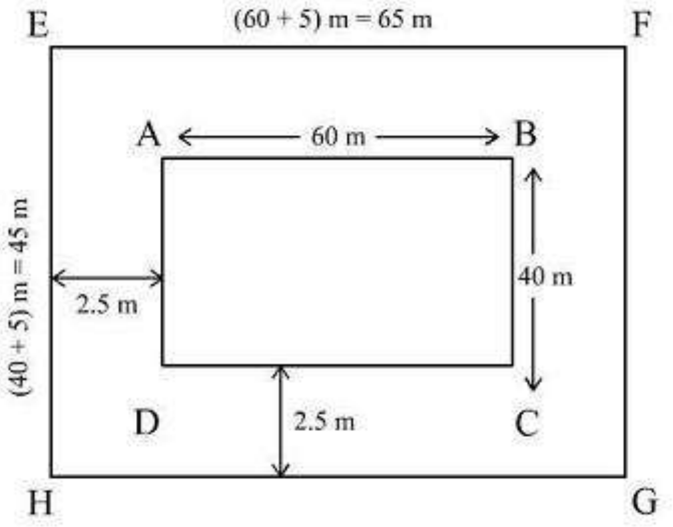
Area of trapezium = $\frac{1}{2}(15x+4x)\times 8\ m^2 \quad = \frac{1}{2}\times 19x\times 8\ m^2$

Area of trapezium = $\frac{5}{12}$ (Area of rectangle)

$\therefore \frac{1}{2}\times 19x\times 8 = \frac{5}{12}\times 114\times 8 \qquad \therefore x = \frac{5\times 38\times 2\times 2}{4\times 8\times 19} = 5$

$\therefore$ Lengths of the parallel sides of the trapezium are $15\times 5 = 75$ m and $4\times 5 = 20$ m

A lawn is in the shape of the rectangle of length 60m and width 40m.Outside the lawn there is a footpath of uniform width 2.5m,bordering the lawn.Find the area of the path.Please explain with a supporting figure.



Let ABCD be the lawn and EFGH be the lawn including 2.5 m path.

$\therefore AB = 60\ m$

$BC = 40\ m$

$EF = (60 + 2.5 + 2.5)\ m$

$= 65\ m$

$FG = (40 + 2.5 + 2.5)\ m$

$= 45\ m$

Area of the foot path = Area of EFGH − Area of ABCD

$= (65 \times 45 - 60 \times 40)\ m^2$

$= 2925 - 2400\ m^2$

$= 525^2$

The semi-perimeter of a triangle is 132 cm. The product of the difference of semi-perimeter and its respective sides is 13200 cm^3. Find the area of the triangle.

Let the sides of the triangle be a cm, b cm and c cm.

Semiperimeter $(s) = 132$ cm ...(1)

$(s-a)(s-b)(s-c) = 13200\ cm^3$ ...(2)

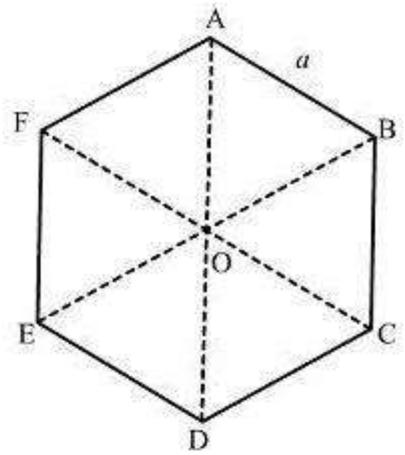
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132 \times 13200}\ cm^2$ [From (1) and (2)]

$= \sqrt{132 \times 132 \times 100}\ cm^2 = 132 \times 10\ cm^2 = 1320\ cm^2$

$\therefore$ Area of the triangle $= 1320\ cm^2$

Find area of a regular hexagon?



A regular hexagon contains 6 equilateral triangles as shown in the figure.

Let each side of the equilateral triangle be a.

$\therefore s = \frac{a+a+a}{2} = \frac{3a}{2}$

Area of equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)}$$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$

$$= \frac{a}{2} \times \frac{a}{2}\sqrt{3}$$

$$= \frac{\sqrt{3}}{4}a^2$$

$$\therefore \text{Area of ABCDEF} = \cancel{6}^{3} \times \frac{\sqrt{3}}{\cancel{4}_{2}}a^2$$

$$= \frac{3\sqrt{3}}{2}a^2$$

From a point in the interior of an equilateral triangle, perpendiculars are drawn on three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Let each side of the equilateral $\triangle ABC$ be a cm.

O be the point in the interior of $\triangle ABC$.

$OQ \perp BC$, $OR \perp AC$ and $OP \perp AB$

$OQ = 14$ cm, $OR = 10$ cm, $OP = 6$ cm

Join OA, OB and OC.

Area of $\Delta ABC = ar(BOC) + ar(COA) + ar(AOB)$

$\therefore \frac{\sqrt{3}}{4}a^2 = \frac{1}{2} \times BC \times OQ + \frac{1}{2} AC \times OR + \frac{1}{2} \times AB \times OP$

$\therefore \frac{\sqrt{3}}{4}a^2 = \frac{1}{\cancel{2}} \times a \times \cancel{14}^{7} + \frac{1}{\cancel{2}} \times a \times \cancel{10}^{5} + \frac{1}{\cancel{2}} \times a \times \cancel{6}^{3}$

$\therefore \frac{\sqrt{3}}{4}a^2 = 7a + 5a + 3a$

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 15a$

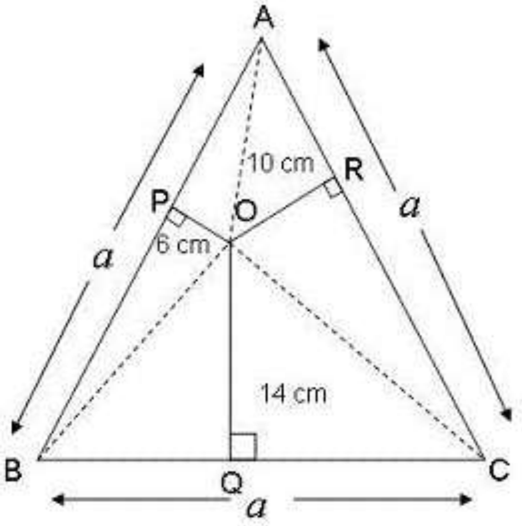
$\Rightarrow \frac{a^2}{a} = 15 \times \frac{4}{\sqrt{3}}$

$\therefore a = \frac{15 \times 4}{\sqrt{3}} = \frac{60}{\sqrt{3}}$ cm

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times \left(\frac{60}{\sqrt{3}}\right)^2$ cm^2

$= \frac{\sqrt{3} \times 60 \times \cancel{60}^{205}}{\cancel{4}_1 \times \cancel{3}}$ cm^2

$= 300\sqrt{3}$ cm^2



Find the area of a parallelogram whose adjacent sides are of length 17 cm and 12 cm and the diagonal is of length 25 cm. Also find the length of perpendicular drawn from D on side AB.

Let diagonal DB divide parallelogram into two congruent triangles.

$\therefore ar(ABCD) = 2 \times ar(DBC)$

Area of ΔDBC:

$a = 12$ cm, $b = 17$ cm, $c = 25$ cm

$s = \frac{12 + 17 + 25}{2} = \frac{54}{2} = 27$ cm

$\therefore ar(DBC) = \sqrt{27(27-12)(27-17)(27-25)}$ cm^2

$= \sqrt{27 \times 15 \times 10 \times 2}$ cm^2

$= \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 2 \times 2}$ cm^2

$= 3 \times 3 \times 5 \times 2$ cm^2

$= 90$ cm^2

$\therefore ar(ABCD) = 2 \times 90$ cm$^2 = 180$ cm^2

$ar(ABCD) = DE \times AB = DE \times 17$

$\therefore DE \times 17 = 180$

$\therefore DE = \frac{180}{17}$ cm $= 10.59$ cm