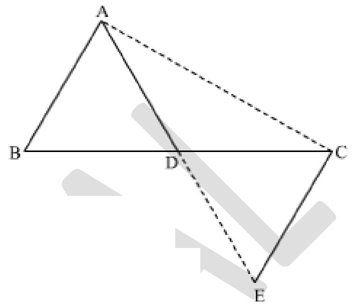
CBSE IX Congruence of Triangle Solved Questions

Q. Prove that Sum of Two Sides of a triangle is greater than twice the length of median drawn to third side.

Given: Δ ABC in which AD is a median.

To prove: AB + AC > 2AD.

Construction: Produce AD to E, such that AD = DE. Join EC.



Proof: In ΔADB and ΔEDC,

AD = DE (Construction)

BD = BD (D is the mid point of BC)

∠ADB = ∠EDC (Vertically opposite angles)

∴ ΔADB ≅ ΔEDC (SAS congruence criterion)

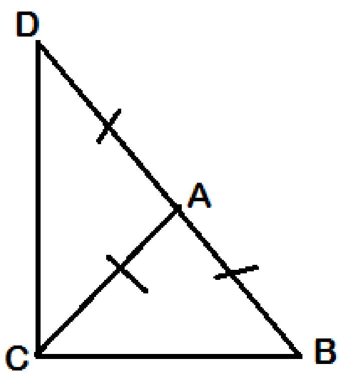
⇒ AB = ED (CPCT)

In ΔAEC,

AC + ED > AE (Sum of any two sides of a triangles is greater than the third side)

∴ AC + AB > 2AD (AE = AD + DE = AD + AD = 2AD & ED = AB)

Q. ABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see the given figure). Show that ∠BCD is a right angle.



In ΔABC,

AB = AC (Given)

⇒ ∠ACB = ∠ABC (Angles opposite to equal sides of a triangle are also equal)

In ΔACD, AC = AD

⇒ ∠ADC = ∠ACD (Angles opposite to equal sides of a triangle are also equal)

In ΔBCD,

∠ABC + ∠BCD + ∠ADC = 180º (Angle sum property of a triangle)

⇒ ∠ACB + ∠ACB +∠ACD + ∠ACD = 180º

⇒ 2(∠ACB + ∠ACD) = 180º

⇒ 2(∠BCD) = 180º

⇒ ∠BCD = 90º

Q. given: two triangles ABC and PQR in which AB=PQ, BC=QR , median AM =median PN prove that triangle ABC is congruent to triangle PQR.

In Δ ABM and Δ PQN

AB = PQ (Given)

AM = PN (Given)

And BM = QN (As M and N are the midpoint of sides BC and QR respectively and given BC= QR)

Δ ABM $\cong$ Δ PQN (By SSS rule)

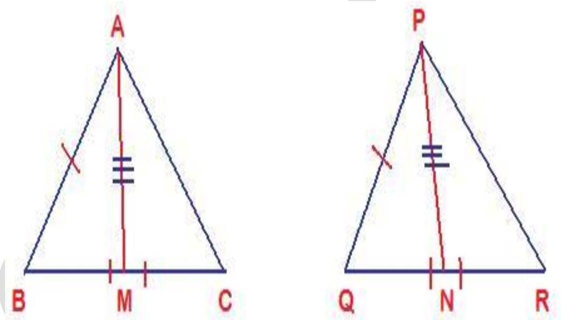
SO, $\angle$ ABM = $\angle$ PQN (by CPCT)

Now In Δ ABC and Δ PQR

AB = PQ (Given)

BC = QR (Given)

And $\angle$ ABC = $\angle$ PQR (As we proved)



Δ ABC $\cong$ Δ PQR (By SAS rule)

(Hence proved)

Q. The vertex angle of an isosceles triangle is twice the sum of its base angles. Find the measure of all the angles.

Let ABC be an isosceles Δ.Let the measure of each of the base angles = x

Let $\angle B = \angle C = x$

Now, vertex angle = $\angle A = 2x$

Now,$\angle A + \angle B + \angle C = 180°$ [angle sum property]

$\Rightarrow 2x + x + x = 180° \Rightarrow 4x = 180 \Rightarrow x = 180/4 = 45^0$

So, measure of each of the base angles = 45°

Now, measure of the vertex angle = 90°

Q. Prove that the triangle formed by joining the midpoints of the sides of an equilateral triangle is also equilateral.

Let DEF be the midpoints of sides of a triangle ABC(with D on BC, E on AB and F on AC).

Now, considering triangles AEF and ABC, angles

EAF = BAC and AE / AB = 1/2 and AF/AC = 1/2.

Hence, both triangles are similar by the SAS (Side - Angle - Side) criterion and correspondingly as AE/AB=AF/AC=EF/BC (similar triangle properties), EF =BC/2.

The cases DF=AC/2 and DE=AB/2 can be proved in the same way.

So, AB=BC=AC (from the given data)

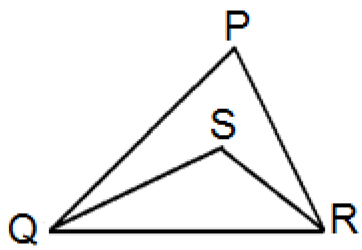
2DF=2EF=2DE

DE=EF=DF

So triangle DEF is also Equilateral Triangle

The triangle formed by joining the mid-points of the equilateral triangle is also an equilateral triangle

Q. In triangle PQR, PQ> PR. QS and RS are the bisectors of angle Q and angle R. Prove that SQ> SR



In ∆PQR, we have,

PQ > PR [given]

⇒ ∠PRQ > ∠PQR [angle opposite to longer side of a ∆ is greater]

⇒12∠PRQ > 12∠PQR ....(1)

Since, SR bisects ∠R, then∠SRQ = 1/2∠PRQ ....(2)

Since SQ bisects ∠P, then∠SQR = 1/2∠PQR ....(3)

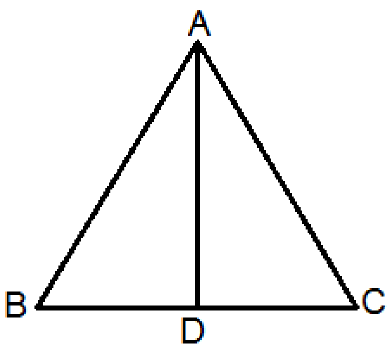
Now, from (1), we have 1/2∠PRQ > 1/2∠PQR

⇒∠SRQ > ∠SQR [using (2) and (3)]

Now, in ∆SQR, we have ∠SRQ > ∠SQR [proved above]

⇒ SQ > SR [side opposite to greater angle of a ∆ is longer]

Q. In triangle ABC (A at the top) , D is any point on the side BC. Prove that AB+BC+CA 2AD



In triangle ABD,

AB+BD >AD (Sum of two sides of a triangle is greater than the third side) ... (1)

In triangle ACD,

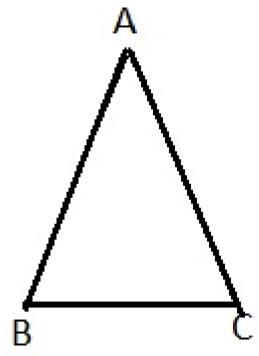
AC+CD>AD (Sum of two sides of a triangle is greater than the third side) ...(2)

Adding eq. (1) and (2)

AB+(BD+CD)+AC> AD+AD

AB+BC+AC> 2AD

Q. In triangle ABC, if AB is the greatest side, then prove that angle c is greater than 60 degrees



It is given that, AB is the longest side of the ΔABC.

$AB > BC$ and $AB > AC$.Now, $AB > BC \Rightarrow \angle C > \angle A$ (angle opposite to longer side is greater) ....(1)

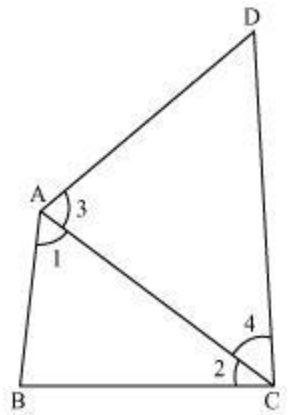
Also,$AB > AC \Rightarrow \angle C > \angle B$ (angle opposite to longer side is greater) ....(2)

adding (1) and (2) ,

we get$\angle C + \angle C > \angle A + \angle B$

$\Rightarrow 2\angle C > \angle A + \angle B \Rightarrow 2\angle C + \angle C > \angle A + \angle B + \angle C \Rightarrow 3\angle C > 180° \Rightarrow \angle C > 60°$

Q. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see the given figure). Show that ∠A > ∠C and ∠B > ∠D.



Let us join AC.

In ΔABC,

AB < BC (AB is the smallest side of quadrilateral ABCD)

∴ ∠2 < ∠1 (Angle opposite to the smaller side is smaller) ... (1)

In ΔADC,

AD < CD (CD is the largest side of quadrilateral ABCD)

∴ ∠4 < ∠3 (Angle opposite to the smaller side is smaller) ... (2)

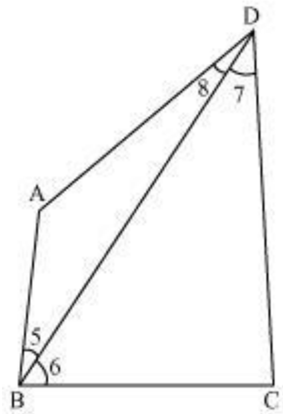
On adding equations (1) and (2), we obtain

$\angle 2 + \angle 4 < \angle 1 + \angle 3$

$\Rightarrow \angle C < \angle A$

$\Rightarrow \angle A > \angle C$

Let us join BD.



In ΔABD,

AB < AD (AB is the smallest side of quadrilateral ABCD)

∴ ∠8 < ∠5 (Angle opposite to the smaller side is smaller) ... (3)

In ΔBDC,

BC < CD (CD is the largest side of quadrilateral ABCD)

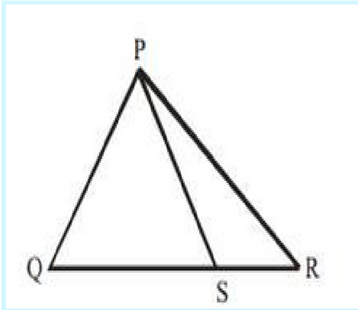
∴ ∠7 < ∠6 (Angle opposite to the smaller side is smaller) ... (4)

On adding equations (3) and (4), we obtain

∠8 + ∠7 < ∠5 + ∠6

⇒ ∠D < ∠B ⇒ ∠B > ∠D

Q. If S. is any point on the side QR of triangle PQR, prove that PQ+QR+RP> 2PS



In ΔPQS,

PQ + QS > PS (i) ……………..(Sum of two sides of a triangle is greater than the third side)

In ΔPSR,

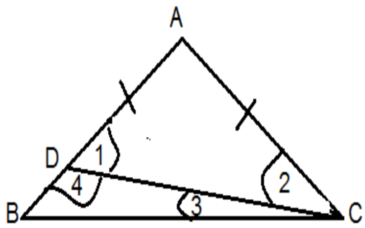
PR + SR > PS ……(ii)… Sum of two sides of a triangle is greater than the third side)

Adding (i) and (ii), we get

PQ + QS + PR + SR > 2PS

PQ + QR + PR > 2PS (QS + SR = QR) Hence proved.

Q. Prove that the difference of any two sides of a triangle is less than the third side.



Construction: Take a Point D on AB such that AD = AC and join CD

Prove that : AB – AC < BC , AB – BC < AC and BC-AC <AB

Proof: In Δ ACD, Ext <4 > <2

but , AD = AC ⇒ <1 = <2

So , < 4 > < 1 ----------------(i)

Now , In ∆ BCD, ext <1 > <3 -------------(ii)

Then from (i) and (ii)

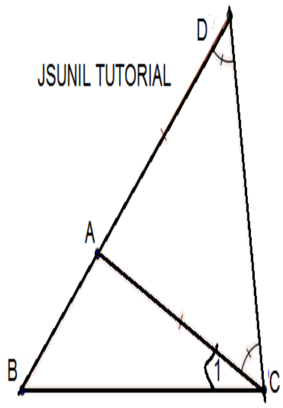
< 4 > <3 ⇒ BC > BD

But, BD = AB – AD and AD = AC ⇒ BD = AB – AC

So, BC > AB – AC

Q.Prove that Sum of any two sides of triangle is greater than third side .

Solution:.



Construction: Extend BA to D Such that AD = AC

Proof : In ∆ACD, *DA=CA*.

Therefore, *∠ADC=∠ACD* [isosceles triangle have two equal angles]

∠ADC + *<1* > *∠ACD*

Thus, *∠BCD* >*∠BDC* [by Euclid's fifth common notion.]

In ∆*DCB*

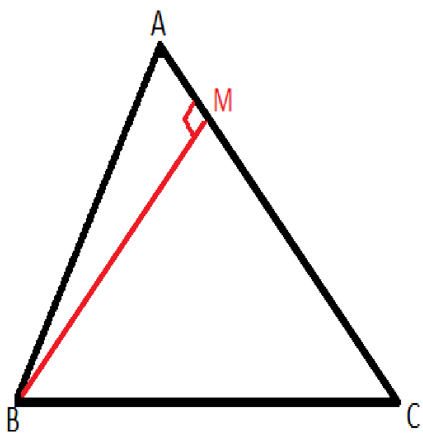
∠BCD > *∠BDC*, So, *BD>BC*.

But *BD=BA+AD*, and *AD=AC*.

Thus, *BA+AC>BC*.

A similar argument shows that *AC+BC>BA* and *BA+BC>AC*.

OR, Another way to prove



Draw a triangle, ∆ ABC and line perpendicular to AC passing through vertex B.

Prove that BA + BC > AC

From the diagram, AM is the shortest distance from vertex A to BM. and CM is the shortest distance from vertex C to BM.

i.e. AM < BA and CM < BC

By adding these inequalities, we have

AM + CM < BA + BC

=> AC < BA + BC (∵ AM + CM = AC)

BA + BC > AC (Hence Proved)

Q. if one acute angle in a right angled triangle is double the other then prove that the hypotenuse is double the shortest side

Given: In ∆ ABC , <B = 90^0 and <ACB = 2 <CAB

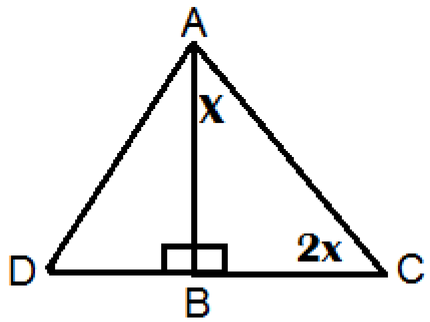
Prove that AC = 2BC

Construction: Produce CB to D such that BC = BD Join to AD

Proof : in triangle ABD, and ABC

BD = BC ; AB = AB and <B = <B = 90^0

By SAS congruency, ∆ ABD ≅ ∆ABC



By CPCT, AD = AC

<DAB = <BAC = X^0

So, < DAC = $2X^0$ ⇒ <ACB = <ACD

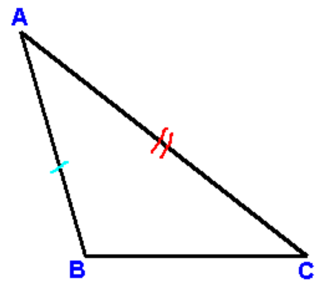
Now in ∆ ADC, <DAC = <ACD= $2X^0$

So, AD = DC

⇒ AC = DC = 2BC Proved

Q. Prove that in a triangle the side opposite to the largest angle is the longest.

Solution:



Given , in ∆ ABC, <ABC < <ACB

There is a triangle ABC, with angle ABC > ACB.

Assume line AB = AC

Then angle ABC = ACB, This is a contradiction

Assume line AB > AC

Then angle ABC < ACB, This also contradiction our hypothesis

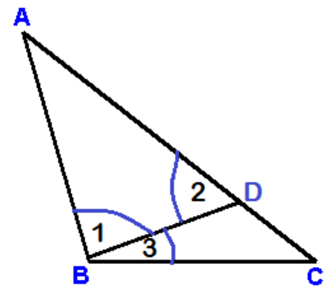
So we are left with only one possibility ,AC> AB, which must be true

Hence proved: AB < AC

Q. Prove that in a triangle the angle opposite to the longer side is the longest.

Solution:

Given, in Δ ABC, AC > AB.



Construction: Take a point D on AC such that AB = AD

Proof: Angle ADB > DCB

< ADB = <ABD

So < ABD > <DCB (or ACB)

< ABC > <ABD, so < ABC > <ACB

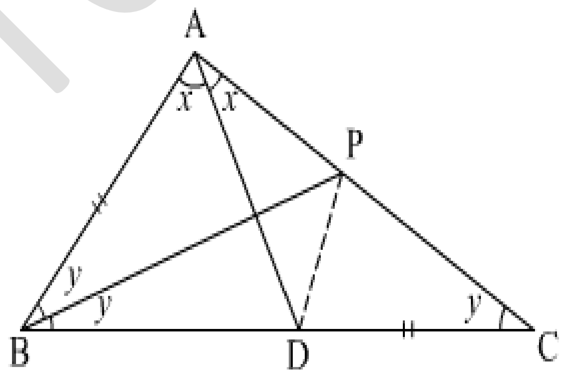
Q. In a Δ ABC ,<B = 2<C. D is a point on BXC such that AD bisect < BAC and AB = CD. Prove that < BAC = 72 degree

In ΔABC, we have

∠B = 2∠C or, ∠B = 2y, where ∠C = y

AD is the bisector of ∠BAC. So, let ∠BAD = ∠CAD = x

Let BP be the bisector of ∠ABC. Join PD.



In ΔBPC, we have

∠CBP = ∠BCP = y ⇒ BP = PC ... (1)

Now, in ΔABP and ΔDCP, we have

∠ABP = ∠DCP = y

AB = DC [Given]

and, BP = PC [Using (1)]

So, by SAS congruence criterion, we have

Δ ABP ≅ Δ DCP

<BAP = < CPD and AP = DP

<CDP = 2x then <ADP = < DAP = x   [<A = 2x]

In ΔABD, we have

∠ADC = ∠ABD + BAD ⇒ $x + 2x = 2y + x \Rightarrow x = y$

In ΔABC, we have

$\angle A + \angle B + \angle C = 180°$

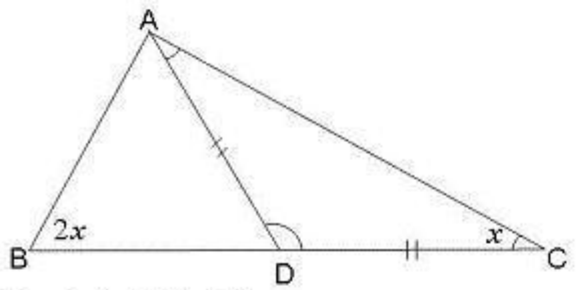
$\Rightarrow 2x + 2y + y = 180°$

$\Rightarrow 5x = 180°$

$\Rightarrow x = 36°$

Hence, $\angle BAC = 2x = 72°$

<u>You may also use this way:</u>



Let in $\triangle ABC, \angle C = x$

$\therefore \quad \angle B = 2x$

$\therefore \quad \angle BAC = 180 - 3x \quad \ldots(i)$

$\therefore \frac{1}{2}\angle BAC = \frac{180 - 3x}{2}$

$\therefore \quad \angle CAD = \frac{180 - 3x}{2} \quad [\because \text{AD bisects } \angle BAC]$

In $\triangle ADC$,

$AD = DC$

$\Rightarrow \angle ACD = \angle CAD$

$\therefore \quad x = \frac{180 - 3x}{2}$

$\therefore 2x = 180 - 3x$

$\therefore 5x = 180$

$\Rightarrow x = 36$

Substituting the value of x in (i), we get

$\angle BAC = 180 - 3x$

$= 180 - 3 \times 36$

$= 180 - 108$

$\Rightarrow \angle BAC = 72°$

Hence proved