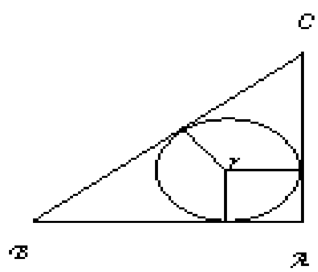


SIMILAR TRIANGLES

Geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art.

1. ABC is a right-angled triangle, right-angled at A. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6cm and 8 cm. Find the radius of the in circle.

(Ans: r=2)

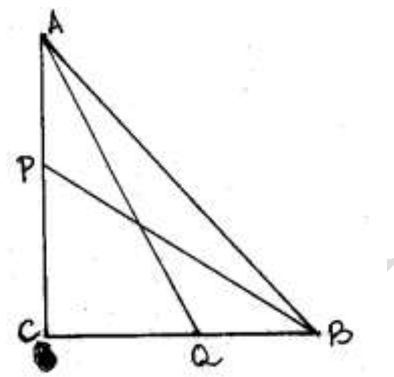


Ans: BC = 10cm
y + z = 8cm
x + z = 6cm
x + y = 10
$\Rightarrow$ x + y + z = 12
z = 12 – 10
z = 2 cm
$\therefore$ radius = 2cm

2. ABC is a triangle. PQ is the line segment intersecting AB in P and AC in Q such that PQ parallel to BC and divides triangle ABC into two parts equal in area. Find BP: AB.

Ans: Refer example problem of text book.

3. In a right triangle ABC, right angled at C, P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio 2: 1.
Prove that $9AQ^2 = 9AC^2 + 4BC^2$
$9BP^2 = 9BC^2 + 4AC^2$
$9(AQ^2 + BP^2) = 13AB^2$



Ans: Since P divides AC in the ratio 2 : 1

$CP = \frac{2}{3}AC \qquad QC = \frac{2}{3}BC$

$AQ^2 = QC^2 + AC^2$

$AQ^2 = \frac{4}{9}BC^2 + AC^2$

$9AQ^2 = 4BC^2 + 9AC^2$ …………………. (1)

Similarly we get $9BP^2 = 9BC^2 + 4AC^2$ ……..…………(2)

Adding (1) and (2) we get $9(AQ^2 + BP^2) = 13AB^2$

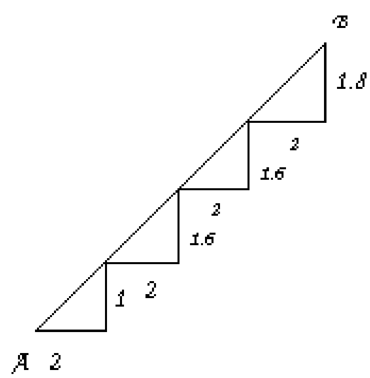
4. P and Q are the mid points on the sides CA and CB respectively of triangle ABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

Self Practice

5. In an equilateral triangle ABC, the side BC is trisected at D.
Prove that $9AD^2 = 7AB^2$

Self Practice

6. There is a staircase as shown in figure connecting points A and B. Measurements of steps are marked in the figure. Find the straight distance between A and B. (Ans:10)



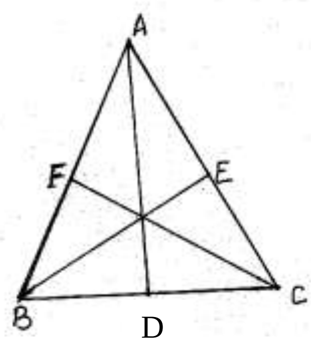
Ans: Apply Pythagoras theorem for each right triangle add to get length of AB.

7. Find the length of the second diagonal of a rhombus, whose side is 5cm and one of the diagonals is 6cm. (Ans: 8cm)

Ans: Length of the other diagonal = 2(BO)
where BO = 4cm
$\therefore$ BD = 8cm.

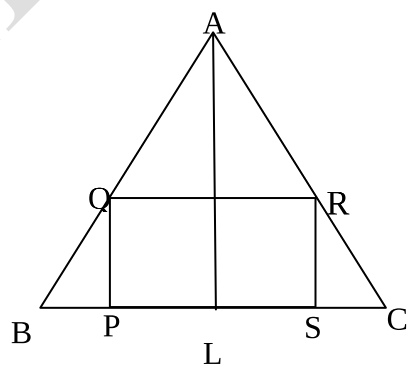
8. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans: To prove $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$
In any triangle sum of squares of any two sides is equal to twice the square of half of third side, together with twice the square of median bisecting it.
If AD is the median



$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$
$2(AB^2 + AC^2) = 4AD^2 + BC^2$
Similarly by taking BE & CF as medians we get
$\Rightarrow 2(AB^2 + BC^2) = 4BE^2 + AC^2$
& $2(AC^2 + BC^2) = 4CF^2 + AB^2$
Adding we get
$\Rightarrow 3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$

9. ABC is an isosceles triangle is which AB=AC=10cm. BC=12. PQRS is a rectangle inside the isosceles triangle. Given PQ=SR= y cm, PS=QR=2x. Prove that $x = 6 - \frac{3y}{4}$.



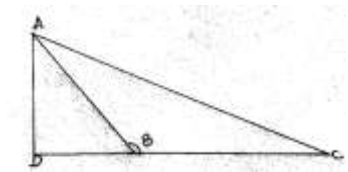
Ans: AL = 8 cm
$\Delta BPQ \sim \Delta BAL$
$\Rightarrow \frac{BQ}{PQ} = \frac{BL}{AL}$
$\Rightarrow \frac{6-x}{y} = \frac{6}{8}$

$\Rightarrow x = 6 - \frac{3y}{4}$.

Hence proved

10. If ABC is an obtuse angled triangle, obtuse angled at B and if AD⊥CB
Prove that AC2 =$AB^2 + BC^2$+2BCxBD

Ans: $AC^2 = AD^2 + CD^2$
$= AD^2 + (BC + BD)^2$
$= AD^2 + BC^2 + 2BC.BD+BD^2$
$= AB^2 + BC^2 + 2BC.BD$

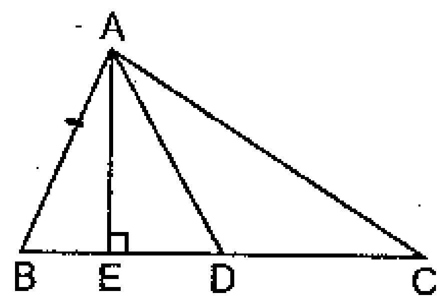


11. If ABC is an acute angled triangle , acute angled at B and AD⊥BC
prove that $AC^2 =AB^2 + BC^2 -2BCxBD$

Ans: Proceed as sum no. 10.

12. Prove that in any triangle the sum of the squares of any two sides is equal to twice the square of half of the third side together with twice the square of the median, which bisects the third side.

Ans: To prove $AB^2 + AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$



Draw AE ⊥ BC
Apply property of Q. No.10 & 11.
In Δ ABD since ∠D > 90^0
$\therefore AB^2 = AD^2 + BD^2 + 2BD \times DE$ ….(1)
Δ ACD since ∠ D < 90^o
$AC^2 = AD^2 + DC^2 - 2DC \times DE$ ….(2)
Adding (1) & (2)
$AB^2 + AC^2 = 2(AD^2 + BD^2)$
$= 2(AD^2 + \left(\frac{1}{2}BC\right)^2)$
Or $AB^2 + AC^2 = 2 (AD^2 + BD^2)$
Hence proved

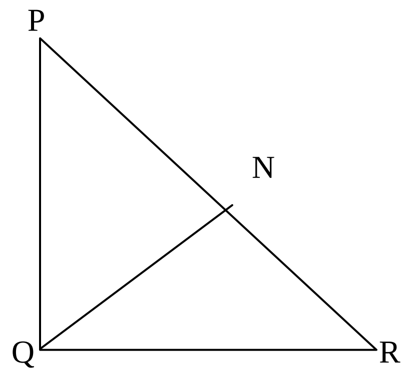
13. If A be the area of a right triangle and b one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4+4A^2}}$.

Ans: Let QR = b

A = Ar(ΔPQR)

$A = \frac{1}{2} \times b \times PQ$

$PQ = \frac{2A}{b}$ ……………………….(1)



Δ PNQ ~ ΔPQR (AA)

$\Rightarrow \frac{PQ}{PR} = \frac{NQ}{QR}$ …………………(2)

From Δ PQR

$PQ^2 + QR^2 = PR^2$

$\frac{4A^2}{b^2} + b^2 = PR^2$

$PR = \sqrt{\frac{4A^2+b^4}{b^2}} = \frac{\sqrt{4A^2+b^4}}{b}$

Equation (2) becomes

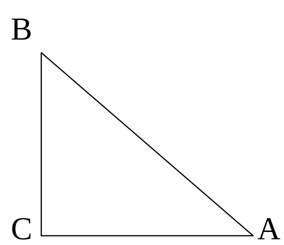
$\frac{2A}{bxPR} = \frac{NQ}{b}$

$NQ = \frac{2A}{PR}$

$NQ = \frac{2Ab}{\sqrt{4A^2+b^4}}$ Ans

14. ABC is a right triangle right-angled at C and AC=$\sqrt{3}$ BC. Prove that ∠ABC=60°.

Ans: $\text{Tan } B = \frac{AC}{BC}$



$\text{Tan } B = \frac{\sqrt{3}BC}{BC}$

$\text{Tan } B = \sqrt{3}$

⇒ Tan B = Tan 60

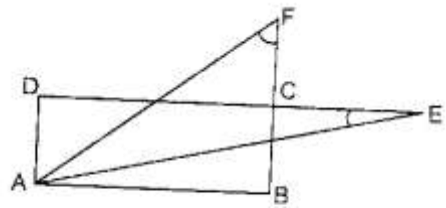
⇒ B = 60°

⇒ ∠ABC = 60°

Hence proved

15. ABCD is a rectangle. Δ ADE and Δ ABF are two triangles such that ∠E=∠F as shown in the figure. Prove that AD x AF=AE x AB.

Ans: Consider Δ ADE and Δ ABF



∠D = ∠B = 90^o

∠E = ∠F (given)

∴ Δ ADE ≅ Δ ABF

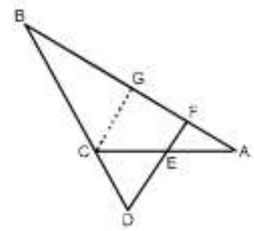
$\frac{AD}{AB} = \frac{AE}{AF}$

⇒ AD x AF = AB x AE

Proved

16. In the given figure, ∠AEF=∠AFE and E is the mid-point of CA. Prove that

$\frac{BD}{CD} = \frac{BF}{CE}$



Ans: Draw CG ∥ DF

In Δ BDF

CG ∥ DF

∴ $\frac{BD}{CD} = \frac{BF}{GF}$ ………….(1) BPT

In ΔAFE

∠AEF=∠AFE

⇒AF=AE

⇒AF=AE=CE…………..(2)

In Δ ACG

E is the mid point of AC

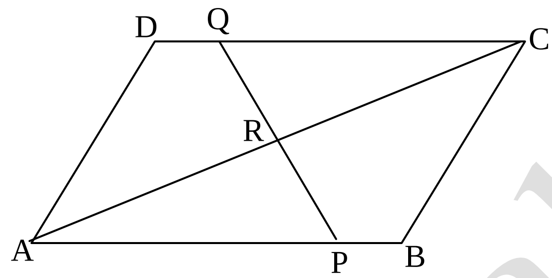
⇒ FG = AF

∴ From (1) & (2)

$\frac{BD}{CD} = \frac{BF}{CE}$

Hence proved

17. ABCD is a parallelogram in the given figure, AB is divided at P and CD and Q so that AP:PB=3:2 and CQ:QD=4:1. If PQ meets AC at R, prove that AR=$\frac{3}{7}$ AC.



Ans: $\Delta APR \sim \Delta CQR$ (AA)

$\Rightarrow \frac{AP}{CQ} = \frac{PR}{QR} = \frac{AR}{CR}$

$\Rightarrow \frac{AP}{CQ} = \frac{AR}{CR}$ & AP=$\frac{3}{5}$AB

$\Rightarrow \frac{3AB}{5CQ} = \frac{AR}{CR}$ & CQ=$\frac{4}{5}$CD = $\frac{4}{5}$ AB

$\Rightarrow \frac{AR}{CR} = \frac{3}{4}$

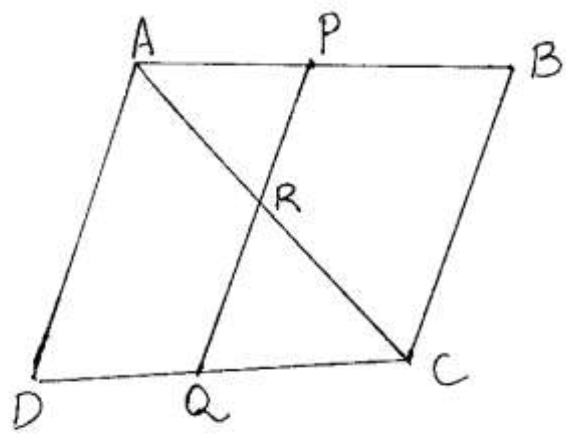
$\Rightarrow \frac{CR}{AR} = \frac{4}{3}$

$\frac{CR + AR}{AR} = \frac{4}{3} + 1$

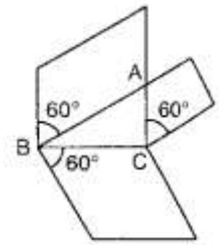
$\Rightarrow \frac{AC}{AR} = \frac{7}{3}$

$\Rightarrow$ AR = $\frac{3}{7}$AC

Hence proved



18. Prove that the area of a rhombus on the hypotenuse of a right-angled triangle, with one of the angles as 60^o, is equal to the sum of the areas of rhombuses with one of their angles as 60^o drawn on the other two sides.



Ans: Hint: Area of Rhombus of side a & one angle of 60^o

$= \frac{\sqrt{3}}{2} \text{ x a x a} = \frac{\sqrt{3}}{2} a^2$

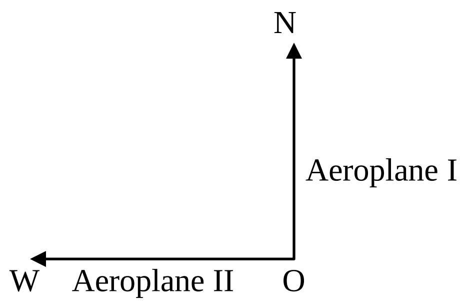
19. An aeroplane leaves an airport and flies due north at a speed of 1000 km/h. At the same time, another plane leaves the same airport and flies due west at a speed of 1200 km/h. How far apart will be the two planes after 1½ hours. (Ans:$300\sqrt{61}$Km)

Ans: ON = 1500km (dist = s x t)
OW = 1800 km

$NW = \sqrt{1500^2 + 1800^2}$

$= \sqrt{5490000}$

$= 300\sqrt{61}$ km



20. ABC is a right-angled isosceles triangle, right-angled at B. AP, the bisector of ∠BAC, intersects BC at P. Prove that $AC^2 = AP^2 + 2(1+\sqrt{2})BP^2$

Ans: $AC = \sqrt{2}$ AB (Since AB = BC)

$\frac{AB}{AC} = \frac{BP}{CP}$ (Bisector Theorem)

$\Rightarrow CP = \sqrt{2}\, BP$

$AC^2 - AP^2 = AC^2 - (AB^2 + BP^2)$

$= AC^2 - AB^2 - BP^2$

$= BC^2 - BP^2$

$= (BP + PC)^2 - BP^2$

$= (BP + \sqrt{2}\, BP)^2 - BP^2$

$= 2BP^2 + 2\sqrt{2}\, BP^2$

$= 2(\sqrt{2} + 1) BP^2 \Rightarrow AC^2 = AP^2 + 2(1+\sqrt{2})BP^2$

Proved