Exercise 13.1

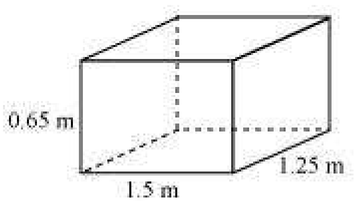
Question 1:

A plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine:

(i) The area of the sheet required for making the box.

(ii) The cost of sheet for it, if a sheet measuring 1 m^2 costs Rs 20.

Answer:



It is given that, length (l) of box = 1.5 m

Breadth (b) of box = 1.25 m

Depth (h) of box = 0.65 m

(i) Box is to be open at top.

Area of sheet required

$= 2lh + 2bh + lb$

$= [2 \times 1.5 \times 0.65 + 2 \times 1.25 \times 0.65 + 1.5 \times 1.25]\ m^2$

$= (1.95 + 1.625 + 1.875)\ m^2 = 5.45\ m^2$

(ii) Cost of sheet per m^2 area = Rs 20

Cost of sheet of 5.45 m^2 area = Rs (5.45×20)

= Rs 109

Question 2:

The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs 7.50 per m^2.

Answer:

It is given that

Length (l) of room = 5 m

Breadth (b) of room = 4 m

Height (h) of room = 3 m

It can be observed that four walls and the ceiling of the room are to be white-washed. The floor of the room is not to be white-washed.

Area to be white-washed = Area of walls + Area of ceiling of room

$= 2lh + 2bh + lb$

$= [2 \times 5 \times 3 + 2 \times 4 \times 3 + 5 \times 4]\ m^2$

$= (30 + 24 + 20)\ m^2$

$= 74\ m^2$

Cost of white-washing per m^2 area = Rs 7.50

Cost of white-washing 74 m^2 area = Rs (74 × 7.50)

= Rs 555

Question 3:

The floor of a rectangular hall has a perimeter 250 m. If the cost of panting the four walls at the rate of Rs.10 per m^2 is Rs.15000, find the height of the hall.

[**Hint:** Area of the four walls = Lateral surface area.]

Answer:

Let length, breadth, and height of the rectangular hall be l m, b m, and h m respectively.

Area of four walls = $2lh + 2bh$

$= 2(l + b)\ h$

Perimeter of the floor of hall = $2(l + b)$

= 250 m

$\therefore$ Area of four walls = $2(l + b)\ h = 250h\ m^2$

Cost of painting per m^2 area = Rs 10

Cost of painting $250h\ m^2$ area = Rs ($250h \times 10$) = Rs $2500h$

However, it is given that the cost of paining the walls is Rs 15000.

$\therefore 15000 = 2500h$

$h = 6$

Therefore, the height of the hall is 6 m.

Question 4:

The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Answer:

Total surface area of one brick = $2(lb + bh + lh)$

= $[2(22.5 \times 10 + 10 \times 7.5 + 22.5 \times 7.5)]$ cm^2

= $2(225 + 75 + 168.75)$ cm^2

= (2×468.75) cm^2

= 937.5 cm^2

Let n bricks can be painted out by the paint of the container.

Area of n bricks = $(n \times 937.5)$ cm^2 = $937.5n$ cm^2

Area that can be painted by the paint of the container = 9.375 m^2 = 93750 cm^2

$\therefore$ 93750 = $937.5n$

$n = 100$

Therefore, 100 bricks can be painted out by the paint of the container.

Question 5:

A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

(i) Which box has the greater lateral surface area and by how much?

(ii) Which box has the smaller total surface area and by how much?

Answer:

(i) Edge of cube = 10 cm

Length (l) of box = 12.5 cm

Breadth (b) of box = 10 cm

Height (h) of box = 8 cm

Lateral surface area of cubical box = $4(\text{edge})^2$

= $4(10 \text{ cm})^2$

= 400 cm^2

Lateral surface area of cuboidal box = $2[lh + bh]$

= [2(12.5 × 8 + 10 × 8)] cm^2

= (2 × 180) cm^2

= 360 cm^2

Clearly, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box.

Lateral surface area of cubical box − Lateral surface area of cuboidal box = 400 cm^2 − 360 cm^2 = 40 cm^2

Therefore, the lateral surface area of the cubical box is greater than the lateral surface area of the cuboidal box by 40 cm^2.

(ii) Total surface area of cubical box = 6$(\text{edge})^2$ = 6$(10 \text{ cm})^2$ = 600 cm^2

Total surface area of cuboidal box

= 2[*lh* + *bh* + *lb*]

= [2(12.5 × 8 + 10 × 8 + 12.5 × 100] cm^2

= 610 cm^2

Clearly, the total surface area of the cubical box is smaller than that of the cuboidal box.

Total surface area of cuboidal box − Total surface area of cubical box = 610 cm^2 − 600 cm^2 = 10 cm^2

Therefore, the total surface area of the cubical box is smaller than that of the cuboidal box by 10 cm^2.

Question 6:

A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

(i) What is the area of the glass?

(ii) How much of tape is needed for all the 12 edges?

Answer:

(i) Length (*l*) of green house = 30 cm

Breadth (*b*) of green house = 25 cm

Height (*h*) of green house = 25 cm

Total surface area of green house

$= 2[lb + lh + bh]$

$= [2(30 \times 25 + 30 \times 25 + 25 \times 25)]\ cm^2$

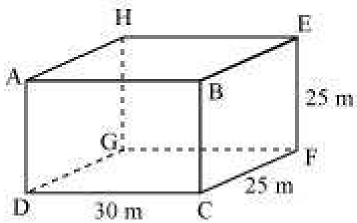
$= [2(750 + 750 + 625)]\ cm^2$

$= (2 \times 2125)\ cm^2$

$= 4250\ cm^2$

Therefore, the area of glass is $4250\ cm^2$.

(ii)



It can be observed that tape is required along side AB, BC, CD, DA, EF, FG, GH, HE, AH, BE, DG, and CF.

Total length of tape = $4(l + b + h)$

$= [4(30 + 25 + 25)]$ cm

= 320 cm

Therefore, 320 cm tape is required for all the 12 edges.

Question 7:

Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for $1000\ cm^2$, find the cost of cardboard required for supplying 250 boxes of each kind.

Answer:

Length (l_1) of bigger box = 25 cm

Breadth (b_1) of bigger box = 20 cm

Height (h_1) of bigger box = 5 cm

Total surface area of bigger box = $2(lb + lh + bh)$

$= [2(25 \times 20 + 25 \times 5 + 20 \times 5)]\ cm^2$

$= [2(500 + 125 + 100)]\ cm^2$

$= 1450\ cm^2$

Extra area required for overlapping $= \left(\frac{1450 \times 5}{100}\right) cm^2$

$= 72.5\ cm^2$

While considering all overlaps, total surface area of 1 bigger box

$= (1450 + 72.5)\ cm^2 = 1522.5\ cm^2$

Area of cardboard sheet required for 250 such bigger boxes

$= (1522.5 \times 250)\ cm^2 = 380625\ cm^2$

Similarly, total surface area of smaller box $= [2(15 \times 12 + 15 \times 5 + 12 \times 5]\ cm^2$

$= [2(180 + 75 + 60)]\ cm^2$

$= (2 \times 315)\ cm^2$

$= 630\ cm^2$

Therefore, extra area required for overlapping $= \left(\frac{630 \times 5}{100}\right) cm^2 = 31.5\ cm^2$

Total surface area of 1 smaller box while considering all overlaps

$= (630 + 31.5)\ cm^2 = 661.5\ cm^2$

Area of cardboard sheet required for 250 smaller boxes $= (250 \times 661.5)\ cm^2$

$= 165375\ cm^2$

Total cardboard sheet required $= (380625 + 165375)\ cm^2$

$= 546000\ cm^2$

Cost of $1000\ cm^2$ cardboard sheet = Rs 4

Cost of $546000\ cm^2$ cardboard sheet $= \text{Rs}\left(\frac{546000 \times 4}{1000}\right) = \text{Rs } 2184$

Therefore, the cost of cardboard sheet required for 250 such boxes of each kind will be Rs 2184.

Question 8:

Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Answer:

Length (l) of shelter = 4 m

Breadth (b) of shelter = 3 m

Height (h) of shelter = 2.5 m

Tarpaulin will be required for the top and four wall sides of the shelter.

Area of Tarpaulin required = $2(lh + bh) + l\,b$

= $[2(4 \times 2.5 + 3 \times 2.5) + 4 \times 3]$ m^2

= $[2(10 + 7.5) + 12]$ m^2

= 47 m^2

Therefore, 47 m^2 tarpaulin will be required.

Exercise 13.2

Question 1:

The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder. Assume π = $\frac{22}{7}$

Answer:

Height (*h*) of cylinder = 14 cm

Let the diameter of the cylinder be *d*.

Curved surface area of cylinder = 88 cm^2

⇒ $2\pi rh = 88\ cm^2$ (*r* is the radius of the base of the cylinder)

⇒ $\pi dh = 88\ cm^2$ ($d = 2r$)

⇒ $\frac{22}{7} \times d \times 14\ \text{cm} = 88\ \text{cm}^2$

⇒ $d = 2$ cm

Therefore, the diameter of the base of the cylinder is 2 cm.

Question 2:

It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square meters of the sheet are required for the same? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (*h*) of cylindrical tank = 1 m

Base radius (*r*) of cylindrical tank $= \left(\frac{140}{2}\right)\ \text{cm} = 70\ \text{cm} = 0.7\ \text{m}$

Area of sheet required = Total surface area of tank $= 2\pi r(r+h)$

$$= \left[2 \times \frac{22}{7} \times 0.7(0.7+1)\right] \text{ m}^2$$

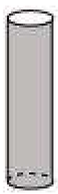
$$= (4.4 \times 1.7) \text{ m}^2$$

$$= 7.48 \text{ m}^2$$

Therefore, it will require 7.48 m^2 area of sheet.

Question 3:

A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm.



(i) Inner curved surface area,

(ii) Outer curved surface area,

(iii) Total surface area. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of cylindrical pipe $= \left(\frac{4}{2}\right) \text{cm} = 2 \text{ cm}$

Outer radius (r_2) of cylindrical pipe $= \left(\frac{4.4}{2}\right) \text{cm} = 2.2 \text{ cm}$

Height (*h*) of cylindrical pipe = Length of cylindrical pipe = 77 cm

(i) CSA of inner surface of pipe $= 2\pi r_1 h$

$$= \left(2 \times \frac{22}{7} \times 2 \times 77\right) \text{ cm}^2$$

$$= 968 \text{ cm}^2$$

(ii) CSA of outer surface of pipe $= 2\pi r_2 h$

$$= \left(2 \times \frac{22}{7} \times 2.2 \times 77\right) \text{ cm}^2$$

$$= (22 \times 22 \times 2.2) \text{ cm}^2$$

$$= 1064.8 \text{ cm}^2$$

(iii) Total surface area of pipe = CSA of inner surface + CSA of outer surface + Area of both circular ends of pipe

$$= 2\pi r_1 h + 2\pi r_2 h + 2\pi\left(r_2^2 - r_1^2\right)$$

$$= \left[968 + 1064.8 + 2\pi\left\{(2.2)^2 - (2)^2\right\}\right] \text{ cm}^2$$

$$= \left(2032.8 + 2 \times \frac{22}{7} \times 0.84\right) \text{ cm}^2$$

$$= (2032.8 + 5.28) \text{ cm}^2$$

$$= 2038.08 \text{ cm}^2$$

Therefore, the total surface area of the cylindrical pipe is 2038.08 cm^2.

Question 4:

The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

It can be observed that a roller is cylindrical.

Height (h) of cylindrical roller = Length of roller = 120 cm

Radius (r) of the circular end of roller = $\left(\frac{84}{2}\right) \text{ cm} = 42 \text{ cm}$

CSA of roller = $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 42 \times 120\right) \text{ cm}^2$

$= 31680 \text{ cm}^2$

Area of field = 500 × CSA of roller

= (500 × 31680) cm^2

= 15840000 cm^2

= 1584 m^2

Question 5:

A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs.12.50 per m^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (*h*) cylindrical pillar = 3.5 m

Radius (*r*) of the circular end of pillar = $\frac{50}{2} = 25 \text{ cm}$

= 0.25 m

CSA of pillar = 2π*rh*

$= \left(2 \times \frac{22}{7} \times 0.25 \times 3.5\right) \text{m}^2$

$= (44 \times 0.125) \text{ m}^2$

$= 5.5 \text{ m}^2$

Cost of painting 1 m^2 area = Rs 12.50

Cost of painting 5.5 m^2 area = Rs (5.5 × 12.50)

= Rs 68.75

Therefore, the cost of painting the CSA of the pillar is Rs 68.75.

Question 6:

Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the height of the circular cylinder be *h*.

Radius (*r*) of the base of cylinder = 0.7 m

CSA of cylinder = 4.4 m^2

$2\pi rh$ = 4.4 m^2

$$\left(2\times\frac{22}{7}\times 0.7\times h\right)\text{ m} = 4.4\text{ m}^2$$

h = 1 m

Therefore, the height of the cylinder is 1 m.

Question 7:

The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) Its inner curved surface area,

(ii) The cost of plastering this curved surface at the rate of Rs 40 per m^2.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

Inner radius (*r*) of circular well $= \left(\frac{3.5}{2}\right)\text{ m} = 1.75\text{ m}$

Depth (*h*) of circular well = 10 m

Inner curved surface area = $2\pi rh$

$$= \left(2\times\frac{22}{7}\times 1.75\times 10\right)\text{ m}^2$$

= (44 × 0.25 × 10) m^2

= 110 m^2

Therefore, the inner curved surface area of the circular well is 110 m^2.

Cost of plastering 1 m^2 area = Rs 40

Cost of plastering 100 m^2 area = Rs (110 × 40)

= Rs 4400

Therefore, the cost of plastering the CSA of this well is Rs 4400.

Question 8:

In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (h) of cylindrical pipe = Length of cylindrical pipe = 28 m

Radius (r) of circular end of pipe = $\frac{5}{2}$ = 2.5 cm = 0.025 m

CSA of cylindrical pipe = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 0.025 \times 28\right) \text{ m}^2$$

= 4.4 m^2

The area of the radiating surface of the system is 4.4 m^2.

Question 9:

Find

(i) The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

(ii) How much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Height (h) of cylindrical tank = 4.5 m

Radius (r) of the circular end of cylindrical tank = $\left(\frac{4.2}{2}\right) \text{ m} = 2.1 \text{ m}$

(i) Lateral or curved surface area of tank = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 2.1 \times 4.5\right) \text{ m}^2$$

$= (44 \times 0.3 \times 4.5)\ m^2$

$= 59.4\ m^2$

Therefore, CSA of tank is $59.4\ m^2$.

(ii) Total surface area of tank = $2\pi r\,(r + h)$

$$= \left[2\times\frac{22}{7}\times 2.1\times(2.1+4.5)\right]\ m^2$$

$= (44 \times 0.3 \times 6.6)\ m^2$

$= 87.12\ m^2$

Let A m^2 steel sheet be actually used in making the tank.

$$\therefore A\left(1-\frac{1}{12}\right)=87.12\ m^2$$

$$\Rightarrow A=\left(\frac{12}{11}\times 87.12\right)\ m^2$$

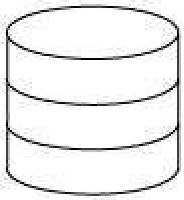
$$\Rightarrow A = 95.04\ m^2$$

Therefore, $95.04\ m^2$ steel was used in actual while making such a tank.

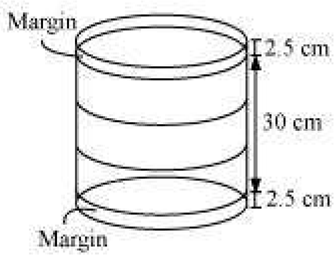
Question 10:

In the given figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame.

Find how much cloth is required for covering the lampshade. $\left[\text{Assume } \pi = \frac{22}{7}\right]$



Answer:



Height (h) of the frame of lampshade = (2.5 + 30 + 2.5) cm = 35 cm

Radius (r) of the circular end of the frame of lampshade = $\left(\frac{20}{2}\right)$ cm $= 10$ cm

Cloth required for covering the lampshade **=** $2\pi rh$

$= \left(2 \times \frac{22}{7} \times 10 \times 35\right)$ cm^2

= 2200 cm^2

Hence, for covering the lampshade, 2200 cm^2 cloth will be required.

Question 11:

The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the circular end of cylindrical penholder = 3 cm

Height (h) of penholder = 10.5 cm

Surface area of 1 penholder = CSA of penholder + Area of base of penholder

$= 2\pi rh + \pi r^2$

$= \left[2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times (3)^2\right] \text{ cm}^2$

$= \left(132 \times 1.5 + \frac{198}{7}\right) \text{ cm}^2$

$= \left(198 + \frac{198}{7}\right) \text{ cm}^2$

$= \frac{1584}{7} \text{ cm}^2$

Area of cardboard sheet used by 1 competitor $= \frac{1584}{7} \text{cm}^2$

Area of cardboard sheet used by 35 competitors

$= \left(\frac{1584}{7} \times 35\right) \text{ cm}^2$ = 7920 cm^2

Therefore, 7920 cm^2 cardboard sheet will be bought.

Exercise 13.3

Question 1:

Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the base of cone = $\left(\frac{10.5}{2}\right)$ cm = 5.25 cm

Slant height (l) of cone = 10 cm

CSA of cone = πrl

$$= \left(\frac{22}{7} \times 5.25 \times 10\right) \text{ cm}^2 = (22 \times 0.75 \times 10) \text{ cm}^2 = 165 \text{ cm}^2$$

Therefore, the curved surface area of the cone is 165 cm^2.

Question 2:

Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of the base of cone = $\left(\frac{24}{2}\right)$ m = 12 m

Slant height (l) of cone = 21 m

Total surface area of cone = $\pi r(r + l)$

$$= \left[\frac{22}{7} \times 12 \times (12 + 21)\right] \text{ m}^2$$

$$= \left(\frac{22}{7} \times 12 \times 33\right) \text{ m}^2$$

$$= 1244.57 \text{ m}^2$$

Question 3:

Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Slant height (*l*) of cone = 14 cm

Let the radius of the circular end of the cone be *r*.

We know, CSA of cone = π*rl*

$$(308)\ \text{cm}^2 = \left(\frac{22}{7} \times r \times 14\right)\ \text{cm}$$

$$\Rightarrow r = \left(\frac{308}{44}\right)\ \text{cm} = 7\ \text{cm}$$

Therefore, the radius of the circular end of the cone is 7 cm.

(ii) Total surface area of cone = CSA of cone + Area of base

= π*rl* + πr^2

$$= \left[308 + \frac{22}{7} \times (7)^2\right]\ \text{cm}^2$$

$$= (308 + 154)\ \text{cm}^2$$

$$= 462\ \text{cm}^2$$

Therefore, the total surface area of the cone is 462 cm^2.

Question 4:

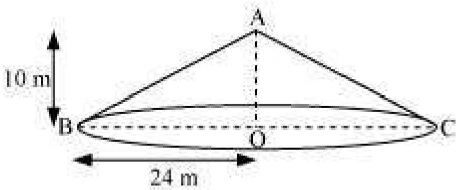
A conical tent is 10 m high and the radius of its base is 24 m. Find

(i) slant height of the tent

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:



(i) Let ABC be a conical tent.

Height (h) of conical tent = 10 m

Radius (r) of conical tent = 24 m

Let the slant height of the tent be l.

In ΔABO,

$AB^2 = AO^2 + BO^2$

$l^2 = h^2 + r^2$

$= (10 \text{ m})^2 + (24 \text{ m})^2$

$= 676 \text{ m}^2$

$\therefore l = 26$ m

Therefore, the slant height of the tent is 26 m.

(ii) CSA of tent = $\pi r l$

$= \left(\frac{22}{7}\times 24\times 26\right) \text{ m}^2$

$= \frac{13728}{7} \text{ m}^2$

Cost of 1 m^2 canvas = Rs 70

Cost of $\frac{13728}{7} \text{ m}^2$ canvas = Rs $\left(\frac{13728}{7}\times 70\right)$

= Rs 137280

Therefore, the cost of the canvas required to make such a tent is Rs 137280.

Question 5:

What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π = 3.14]

Answer:

Height (h) of conical tent = 8 m

Radius (r) of base of tent = 6 m

Slant height (l) of tent = $\sqrt{r^2 + h^2}$

$= \left(\sqrt{6^2 + 8^2}\right) \text{ m} = \left(\sqrt{100}\right) \text{ m} = 10 \text{ m}$

CSA of conical tent = $\pi r l$

= $(3.14 \times 6 \times 10)$ m^2

= 188.4 m^2

Let the length of tarpaulin sheet required be l.

As 20 cm will be wasted, therefore, the effective length will be (l − 0.2 m).

Breadth of tarpaulin = 3 m

Area of sheet = CSA of tent

$[(l - 0.2 \text{ m}) \times 3] \text{ m} = 188.4 \text{ m}^2$

$l - 0.2 \text{ m} = 62.8 \text{ m}$

$l = 63 \text{ m}$

Therefore, the length of the required tarpaulin sheet will be 63 m.

Question 6:

The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs 210 per 100 m^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Slant height (l) of conical tomb = 25 m

Base radius (r) of tomb = $\frac{14}{2} =$ 7 m

CSA of conical tomb = πrl

$= \left(\frac{22}{7} \times 7 \times 25\right)$ m^2

= 550 m^2

Cost of white-washing 100 m^2 area = Rs 210

Cost of white-washing 550 m^2 area = Rs $\left(\frac{210 \times 550}{100}\right)$

= Rs 1155

Therefore, it will cost Rs 1155 while white-washing such a conical tomb.

Question 7:

A joker's cap is in the form of right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of conical cap = 7 cm

Height (h) of conical cap = 24 cm

Slant height (l) of conical cap = $\sqrt{r^2 + h^2}$

$= \left[\sqrt{(7)^2 + (24)^2}\right] \text{ cm} = \left(\sqrt{625}\right) \text{ cm} = 25 \text{ cm}$

CSA of 1 conical cap = πrl

$= \left(\frac{22}{7} \times 7 \times 25\right) \text{ cm}^2 = 550 \text{ cm}^2$

CSA of 10 such conical caps = (10 × 550) cm^2 = 5500 cm^2

Therefore, 5500 cm^2 sheet will be required.

Question 8:

A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{1.04}$ = 1.02).

Answer:

Radius (r) of cone = $\frac{40}{2} = 20 \text{ cm}$ = 0.2 m

Height (h) of cone = 1 m

Slant height (l) of cone = $\sqrt{h^2 + r^2}$

$$= \left[\sqrt{(1)^2 + (0.2)^2}\right] \text{ m} = \left(\sqrt{1.04}\right) \text{ m} = 1.02 \text{ m}$$

CSA of each cone = πrl

= (3.14 × 0.2 × 1.02) m^2 = 0.64056 m^2

CSA of 50 such cones = (50 × 0.64056) m^2

= 32.028 m^2

Cost of painting 1 m^2 area = Rs 12

Cost of painting 32.028 m^2 area = Rs (32.028 × 12)

= Rs 384.336

= Rs 384.34 (approximately)

Therefore, it will cost Rs 384.34 in painting 50 such hollow cones.

Exercise 13.4

Question 1:

Find the surface area of a sphere of radius:

(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius (r) of sphere = 10.5 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2$$

$$= \left(4 \times \frac{22}{7} \times 10.5 \times 10.5\right) \text{cm}^2$$

$$= (88 \times 1.5 \times 10.5) \text{ cm}^2$$

$$= 1386 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 10.5cm is 1386 cm^2.

(ii) Radius(r) of sphere = 5.6 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (5.6)^2\right] \text{cm}^2$$

$$= (88 \times 0.8 \times 5.6) \text{ cm}^2$$

$$= 394.24 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 5.6 cm is 394.24 cm^2.

(iii) Radius (r) of sphere = 14 cm

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (14)^2\right] \text{cm}^2$$

$$= (4 \times 44 \times 14) \text{ cm}^2$$

$$= 2464 \text{ cm}^2$$

Therefore, the surface area of a sphere having radius 14 cm is 2464 cm^2.

Question 2:

Find the surface area of a sphere of diameter:

(i) 14 cm (ii) 21 cm (iii) 3.5 m

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius (r) of sphere = $\frac{\text{Diameter}}{2} = \left(\frac{14}{2}\right) \text{ cm} = 7 \text{ cm}$

Surface area of sphere = $4\pi r^2$

$$= \left(4 \times \frac{22}{7} \times (7)^2\right) \text{cm}^2$$

$$= (88 \times 7) \text{cm}^2$$

$$= 616 \text{ cm}^2$$

Therefore, the surface area of a sphere having diameter 14 cm is 616 cm^2.

(ii) Radius (r) of sphere = $\frac{21}{2} = 10.5 \text{ cm}$

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2$$

$$= 1386 \text{ cm}^2$$

Therefore, the surface area of a sphere having diameter 21 cm is 1386 cm^2.

(iii) Radius (r) of sphere = $\frac{3.5}{2} = 1.75$ m

Surface area of sphere = $4\pi r^2$

$$= \left[4 \times \frac{22}{7} \times (1.75)^2\right] \text{m}^2$$

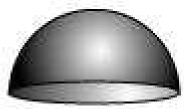
$$= 38.5 \text{ m}^2$$

Therefore, the surface area of the sphere having diameter 3.5 m is 38.5 m^2.

Question 3:

Find the total surface area of a hemisphere of radius 10 cm. [Use π = 3.14]

Answer:



Radius (r) of hemisphere = 10 cm

Total surface area of hemisphere = CSA of hemisphere + Area of circular end of hemisphere

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

$= \left[3 \times 3.14 \times (10)^2\right] \text{ cm}^2$

$= 942 \text{ cm}^2$

Therefore, the total surface area of such a hemisphere is 942 cm^2.

Question 4:

The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Answer:

Radius (r_1) of spherical balloon = 7 cm

Radius (r_2) of spherical balloon, when air is pumped into it = 14 cm

$$\text{Required ratio} = \frac{\text{Initial surface area}}{\text{Surface area after pumping air into balloon}}$$

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{7}{14}\right)^2 = \frac{1}{4}$$

Therefore, the ratio between the surface areas in these two cases is 1:4.

Question 5:

A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r) of hemispherical bowl $= \left(\frac{10.5}{2}\right)$ cm $= 5.25$ cm

Surface area of hemispherical bowl = $2\pi r^2$

$$= \left[2 \times \frac{22}{7} \times (5.25)^2\right] \text{ cm}^2$$

$$= 173.25 \text{ cm}^2$$

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 173.25 cm^2 area $= \text{Rs} \left(\frac{16 \times 173.25}{100}\right)$ = Rs 27.72

Therefore, the cost of tin-plating the inner side of the hemispherical bowl is Rs 27.72.

Question 6:

Find the radius of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the sphere be r.

Surface area of sphere = 154

$\therefore 4\pi r^2 = 154 \text{ cm}^2$

$$r^2 = \left(\frac{154 \times 7}{4 \times 22}\right) \text{ cm}^2 = \left(\frac{7 \times 7}{2 \times 2}\right) \text{ cm}^2$$

$$r = \left(\frac{7}{2}\right) \text{ cm} = 3.5 \text{ cm}$$

Therefore, the radius of the sphere whose surface area is 154 cm^2 is 3.5 cm.

Question 7:

The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface area.

Answer:

Let the diameter of earth be *d*. Therefore, the diameter of moon will be $\frac{d}{4}$.

Radius of earth = $\frac{d}{2}$

Radius of moon = $\frac{1}{2}\times\frac{d}{4}=\frac{d}{8}$

Surface area of moon = $4\pi\left(\frac{d}{8}\right)^2$

Surface area of earth = $4\pi\left(\frac{d}{2}\right)^2$

Required ratio $=\frac{4\pi\left(\frac{d}{8}\right)^2}{4\pi\left(\frac{d}{2}\right)^2}$

$=\frac{4}{64}=\frac{1}{16}$

Therefore, the ratio between their surface areas will be 1:16.

Question 8:

A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl. $\left[\text{Assume } \pi=\frac{22}{7}\right]$

Answer:

Inner radius of hemispherical bowl = 5 cm

Thickness of the bowl = 0.25 cm

∴ Outer radius (r) of hemispherical bowl = (5 + 0.25) cm

= 5.25 cm

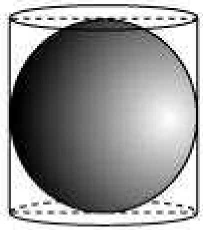
Outer CSA of hemispherical bowl = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times (5.25 \text{ cm})^2 = 173.25 \text{ cm}^2$$

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

Question 9:

A right circular cylinder just encloses a sphere of radius r (see figure). Find

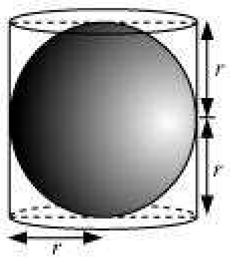


(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in (i) and (ii).

Answer:



(i) Surface area of sphere = $4\pi r^2$

(ii) Height of cylinder = $r + r = 2r$

Radius of cylinder = r

CSA of cylinder = $2\pi rh$

= $2\pi r\,(2r)$

= $4\pi r^2$

(iii) $\text{Required ratio} = \dfrac{\text{Surface area of sphere}}{\text{CSA of cylinder}}$

$= \dfrac{4\pi r^2}{4\pi r^2}$

$= \dfrac{1}{1}$

Therefore, the ratio between these two surface areas is 1:1.

Exercise 13.5

Question 1:

A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Answer:

Matchbox is a cuboid having its length (l), breadth (b), height (h) as 4 cm, 2.5 cm, and 1.5 cm.

Volume of 1 match box = $l \times b \times h$

= $(4 \times 2.5 \times 1.5)$ cm^3 = 15 cm^3

Volume of 12 such matchboxes = (15×12) cm^3

= 180 cm^3

Therefore, the volume of 12 match boxes is 180 cm^3.

Question 2:

A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold? (1 m^3 = 1000l)

Answer:

The given cuboidal water tank has its length (l) as 6 m, breadth (b) as 5 m, and height (h) as 4.5 m.

Volume of tank = $l \times b \times h$

= $(6 \times 5 \times 4.5)$ m^3 = 135 m^3

Amount of water that 1 m^3 volume can hold = 1000 litres

Amount of water that 135 m^3 volume can hold = (135×1000) litres

= 135000 litres

Therefore, such tank can hold up to 135000 litres of water.

Question 3:

A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Answer:

Let the height of the cuboidal vessel be h.

Length (l) of vessel = 10 m

Width (b) of vessel = 8 m

Volume of vessel = 380 m^3

$\therefore l \times b \times h = 380$

[(10) (8) h] m^2= 380 m^3

$h = 4.75$ m

Therefore, the height of the vessel should be 4.75 m.

Question 4:

Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3.

Answer:

The given cuboidal pit has its length (l) as 8 m, width (b) as 6 m, and depth (h)as 3 m.

Volume of pit = $l \times b \times h$

= $(8 \times 6 \times 3)$ m^3 = 144 m^3

Cost of digging per m^3 volume = Rs 30

Cost of digging 144 m^3 volume = Rs (144×30) = Rs 4320

Question 5:

The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Answer:

Let the breadth of the tank be b m.

Length (l) and depth (h) of tank is 2.5 m and 10 m respectively.

Volume of tank = $l \times b \times h$

= $(2.5 \times b \times 10)$ m^3

= $25b$ m^3

Capacity of tank = $25b$ m^3 = 25000 b litres

$\therefore 25000\, b = 50000$

$\Rightarrow b = 2$

Therefore, the breadth of the tank is 2 m.

Question 6:

A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Answer:

The given tank is cuboidal in shape having its length (l) as 20 m, breadth (b) as 15 m, and height (h) as 6 m.

Capacity of tank = $l \times b \times h$

= $(20 \times 15 \times 6)$ m^3 = 1800 m^3 = 1800000 litres

Water consumed by the people of the village in 1 day = (4000 × 150) litres

= 600000 litres

Let water in this tank last for n days.

Water consumed by all people of village in n days = Capacity of tank

$n \times 600000 = 1800000$

$n = 3$

Therefore, the water of this tank will last for 3 days.

Question 7:

A godown measures 40 m × 25 m × 10 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Answer:

The godown has its length (l_1) as 40 m, breadth (b_1) as 25 m, height (h_1) as 10 m, while the wooden crate has its length (l_2) as 1.5 m, breadth (b_2) as 1.25 m, and height (h_2) as 0.5 m.

Therefore, volume of godown = $l_1 \times b_1 \times h_1$

= $(40 \times 25 \times 10)$ m^3

= 10000 m^3

Volume of 1 wooden crate = $l_2 \times b_2 \times h_2$

= $(1.5 \times 1.25 \times 0.5)$ m^3

= 0.9375 m^3

Let n wooden crates can be stored in the godown.

Therefore, volume of n wooden crates = Volume of godown

$0.9375 \times n = 10000$

$$n = \frac{10000}{0.9375} = 10666.66$$

Therefore, 10666 wooden crates can be stored in the godown.

Question 8:

A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Answer:

Side (a) of cube = 12 cm

Volume of cube = $(a)^3 = (12 \text{ cm})^3 = 1728 \text{ cm}^3$

Let the side of the smaller cube be a_1.

Volume of 1 smaller cube $= \left(\frac{1728}{8}\right) \text{cm}^3 = 216 \text{ cm}^3$

$$(a_1)^3 = 216 \text{ cm}^3$$

$\Rightarrow a_1 = 6$ cm

Therefore, the side of the smaller cubes will be 6 cm.

Ratio between surface areas of cubes $= \frac{\text{Surface area of bigger cube}}{\text{Surface area of smaller cube}}$

$$= \frac{6a^2}{6a_1^2} = \frac{(12)^2}{(6)^2}$$

$$= \frac{4}{1}$$

Therefore, the ratio between the surface areas of these cubes is 4:1.

Question 9:

A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Answer:

Rate of water flow = 2 km per hour

$$= \left(\frac{2000}{60}\right) \text{ m/min}$$

$$= \left(\frac{100}{3}\right) \text{ m/min}$$

Depth (h) of river = 3 m

Width (b) of river = 40 m

Volume of water flowed in 1 min $= \left(\frac{100}{3} \times 40 \times 3\right) \text{ m}^3$ = 4000 m^3

Therefore, in 1 minute, 4000 m^3 water will fall in the sea.

Exercise 13.6

Question 1:

The circumference of the base of cylindrical vessel is 132 cm and its height is 25 cm.

How many litres of water can it hold? ($1000\ cm^3 = 1l$) $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the cylindrical vessel be r.

Height (h) of vessel = 25 cm

Circumference of vessel = 132 cm

$2\pi r = 132$ cm

$$r = \left(\frac{132\times 7}{2\times 22}\right)\text{ cm} = 21\text{ cm}$$

Volume of cylindrical vessel = $\pi r^2 h$

$$= \left[\frac{22}{7}\times(21)^2\times 25\right]\text{ cm}^3$$

= 34650 cm^3

$$= \left(\frac{34650}{1000}\right)\text{ litres} \qquad [\because 1\text{ litre} = 1000\text{ cm}^3]$$

= 34.65 litres

Therefore, such vessel can hold 34.65 litres of water.

Question 2:

The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of cylindrical pipe = $\left(\frac{24}{2}\right)\text{ cm} = 12\text{ cm}$

Outer radius (r_2) of cylindrical pipe = $\left(\frac{28}{2}\right)$ cm $= 14$ cm

Height (h) of pipe = Length of pipe = 35 cm

Volume of pipe = $\pi\left(r_2^2 - r_1^2\right)h$

$= \left[\frac{22}{7} \times \left(14^2 - 12^2\right) \times 35\right] \text{cm}^3$

$= 110 \times 52 \text{ cm}^3$

$= 5720 \text{ cm}^3$

Mass of 1 cm^3 wood = 0.6 g

Mass of 5720 cm^3 wood = (5720 × 0.6) g

= 3432 g

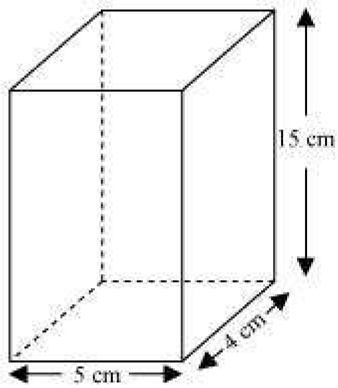
= 3.432 kg

Question 3:

A soft drink is available in two packs − (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

The tin can will be cuboidal in shape while the plastic cylinder will be cylindrical in shape.



Length (l) of tin can = 5 cm

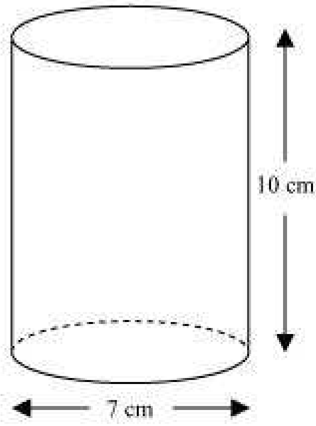
Breadth (b) of tin can = 4 cm

Height (h) of tin can = 15 cm

Capacity of tin can = $l \times b \times h$

= $(5 \times 4 \times 15)$ cm^3

= 300 cm^3



Radius (r) of circular end of plastic cylinder = $\left(\frac{7}{2}\right)$ cm $= 3.5$ cm

Height (H) of plastic cylinder = 10 cm

Capacity of plastic cylinder = $\pi r^2 H$

$= \left[\frac{22}{7} \times (3.5)^2 \times 10\right] \text{ cm}^3$

$= (11 \times 35) \text{ cm}^3$

$= 385 \text{ cm}^3$

Therefore, plastic cylinder has the greater capacity.

Difference in capacity = (385 − 300) cm^3 = 85 cm^3

Question 4:

If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find (i) radius of its base (ii) its volume. [Use π = 3.14]

Answer:

(i) Height (h) of cylinder = 5 cm

Let radius of cylinder be r.

CSA of cylinder = 94.2 cm^2

$2\pi rh$ = 94.2 cm^2

(2 × 3.14 × r × 5) cm = 94.2 cm^2

r = 3 cm

(ii) Volume of cylinder = $\pi r^2 h$

= (3.14 × $(3)^2$ × 5) cm^3

= 141.3 cm^3

Question 5:

It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) Inner curved surface area of the vessel

(ii) Radius of the base

(iii) Capacity of the vessel

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Rs 20 is the cost of painting 1 m^2 area.

Rs 2200 is the cost of painting = $\left(\frac{1}{20}\times 2200\right)$ m^2 area

= 110 m^2 area

Therefore, the inner surface area of the vessel is 110 m^2.

(ii) Let the radius of the base of the vessel be r.

Height (h) of vessel = 10 m

Surface area = $2\pi rh$ = 110 m^2

$$\Rightarrow \left(2\times\frac{22}{7}\times r\times 10\right)\text{ m} = 110\text{ m}^2$$

$$\Rightarrow r = \left(\frac{7}{4}\right)\text{ m} = 1.75\text{ m}$$

(iii) Volume of vessel = $\pi r^2 h$

$$= \left[\frac{22}{7}\times(1.75)^2\times 10\right]\text{ m}^3$$

= 96.25 m^3

Therefore, the capacity of the vessel is 96.25 m^3 or 96250 litres.

Question 6:

The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let the radius of the circular end be r.

Height (h) of cylindrical vessel = 1 m

Volume of cylindrical vessel = 15.4 litres = 0.0154 m^3

$$\pi r^2 h = 0.0154\text{ m}^3$$

$$\left(\frac{22}{7}\times r^2\times 1\right)\text{ m} = 0.0154\text{ m}^3$$

$\Rightarrow r = 0.07$ m

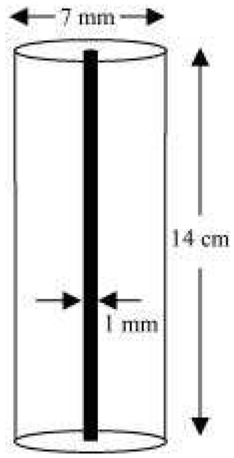
$$\text{Total surface area of vessel} = 2\pi r\left(r+h\right)$$
$$= \left[2\times\frac{22}{7}\times 0.07\left(0.07+1\right)\right] \text{ m}^2$$
$$= 0.44\times 1.07 \text{ m}^2$$
$$= 0.4708 \text{ m}^2$$

Therefore, 0.4708 m^2 of the metal sheet would be required to make the cylindrical vessel.

Question 7:

A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:



Radius (r_1) of pencil = $\left(\frac{7}{2}\right) \text{ mm} = \left(\frac{0.7}{2}\right) \text{ cm}$ = 0.35 cm

Radius (r_2) of graphite = $\left(\frac{1}{2}\right)$ mm $= \left(\frac{0.1}{2}\right)$ cm = 0.05 cm

Height (h) of pencil = 14 cm

Volume of wood in pencil = $\pi\left(r_1^2 - r_2^2\right)h$

$$=\left[\frac{22}{7}\left\{(0.35)^2-(0.05)^2\times 14\right\}\right] \text{cm}^3$$

$$=\left[\frac{22}{7}(0.1225-0.0025)\times 14\right] \text{cm}^3$$

$$=(44\times 0.12)\ \text{cm}^3$$

$$=5.28\ \text{cm}^3$$

$$\text{Volume of graphite } = \pi r_2^2 h = \left[\frac{22}{7}\times(0.05)^2\times 14\right] cm^3$$

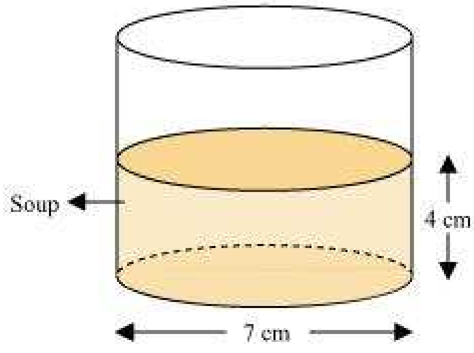
$$=(44\times 0.0025)\ cm^3$$

$$=0.11\ cm^3$$

Question 8:

A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:



Radius (r) of cylindrical bowl = $\left(\frac{7}{2}\right)$ cm $= 3.5$ cm

Height (h) of bowl, up to which bowl is filled with soup = 4 cm

Volume of soup in 1 bowl = $\pi r^2 h$

$= \left(\frac{22}{7} \times (3.5)^2 \times 4\right)$ cm^3

= (11 × 3.5 × 4) cm^3

= 154 cm^3

Volume of soup given to 250 patients = (250 × 154) cm^3

= 38500 cm^3

= 38.5 litres.

Exercise 13.7

Question 1:

Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm

(ii) radius 3.5 cm, height 12 cm

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius (r) of cone = 6 cm

Height (h) of cone = 7 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7\right] \text{ cm}^3$$

$$= (12 \times 22) \text{ cm}^3$$

$$= 264 \text{ cm}^3$$

Therefore, the volume of the cone is 264 cm^3.

(ii) Radius (r) of cone = 3.5 cm

Height (h) of cone = 12 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\right] \text{ cm}^3$$

$$= \left(\frac{1}{3} \times 22 \times \frac{1}{2} \times 3.5 \times 12\right) \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

Therefore, the volume of the cone is 154 cm^3.

Question 2:

Find the capacity in litres of a conical vessel with

(i) radius 7 cm, slant height 25 cm

(ii) height 12 cm, slant height 12 cm

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Radius (r) of cone = 7 cm

Slant height (l) of cone = 25 cm

Height (h) of cone $= \sqrt{l^2 - r^2}$

$= \left(\sqrt{25^2 - 7^2}\right)$ cm

$= 24$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left(\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right)$ cm^3

$= (154 \times 8)$ cm^3

$= 1232$ cm^3

Therefore, capacity of the conical vessel

$= \left(\frac{1232}{1000}\right)$ litres $(1 \text{ litre} = 1000 \text{ cm}^3)$

= 1.232 litres

(ii) Height (h) of cone = 12 cm

Slant height (l) of cone = 13 cm

Radius (r) of cone $= \sqrt{l^2 - h^2}$

$= \left(\sqrt{13^2 - 12^2}\right)$ cm

$= 5$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12\right] \text{ cm}^3$$

$$= \left(4 \times \frac{22}{7} \times 25\right) \text{ cm}^3$$

$$= \left(\frac{2200}{7}\right) \text{ cm}^3$$

Therefore, capacity of the conical vessel

$$= \left(\frac{2200}{7000}\right) \text{ litres } \left(1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$= \frac{11}{35} \text{ litres}$$

Question 3:

The height of a cone is 15 cm. If its volume is 1570 cm^3, find the diameter of its base. [Use π = 3.14]

Answer:

Height (h) of cone = 15 cm

Let the radius of the cone be r.

Volume of cone = 1570 cm^3

$$\frac{1}{3}\pi r^2 h = 1570 \text{ cm}^3$$

$$\Rightarrow \left(\frac{1}{3} \times 3.14 \times r^2 \times 15\right) \text{ cm} = 1570 \text{ cm}^3$$

$$\Rightarrow r^2 = 100 \text{ cm}^2$$

$\Rightarrow r = 10$ cm

Therefore, the radius of the base of cone is 10 cm.

Question 4:

If the volume of a right circular cone of height 9 cm is 48π cm^3, find the diameter of its base.

Answer:

Height (h) of cone = 9 cm

Let the radius of the cone be r.

Volume of cone = 48π cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 48\pi \text{ cm}^3$$

$$\Rightarrow \left(\frac{1}{3}\pi r^2 \times 9\right) \text{ cm} = 48\pi \text{ cm}^3$$

$$\Rightarrow r^2 = 16 \text{ cm}^2$$

$$\Rightarrow r = 4 \text{ cm}$$

Diameter of base = 2r = 8 cm

Question 5:

A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of pit $= \left(\frac{3.5}{2}\right) \text{ m} = 1.75 \text{ m}$

Height (h) of pit = Depth of pit = 12 m

Volume of pit $= \frac{1}{3}\pi r^2 h$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times (1.75)^2 \times 12\right] \text{ cm}^3$$

= 38.5 m^3

Thus, capacity of the pit = (38.5 × 1) kilolitres = 38.5 kilolitres

Question 6:

The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius of cone = $\left(\frac{28}{2}\right) \text{ cm} = 14 \text{ cm}$

Let the height of the cone be *h*.

Volume of cone = 9856 cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h = 9856 \text{ cm}^3$$

$$\Rightarrow \left[\frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h\right] \text{cm}^2 = 9856 \text{ cm}^3$$

h = 48 cm

Therefore, the height of the cone is 48 cm.

(ii) Slant height (*l*) of cone $= \sqrt{r^2 + h^2}$

$$= \left[\sqrt{(14)^2 + (48)^2}\right] \text{ cm}$$

$$= \left[\sqrt{196 + 2304}\right] \text{ cm}$$

$$= 50 \text{ cm}$$

Therefore, the slant height of the cone is 50 cm.

(iii) CSA of cone = π*rl*

$$= \left(\frac{22}{7} \times 14 \times 50\right) \text{ cm}^2$$

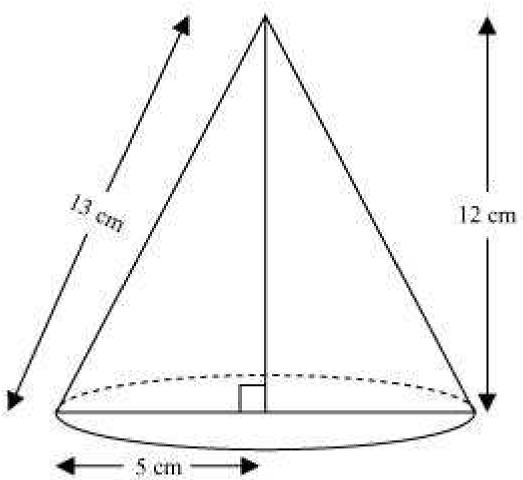
= 2200 cm^2

Therefore, the curved surface area of the cone is 2200 cm^2.

Question 7:

A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Answer:



When right-angled ΔABC is revolved about its side 12 cm, a cone with height (*h*) as 12 cm, radius (*r*) as 5 cm, and slant height (*l*) 13 cm will be formed.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \pi \times (5)^2 \times 12\right]\ \text{cm}^3$

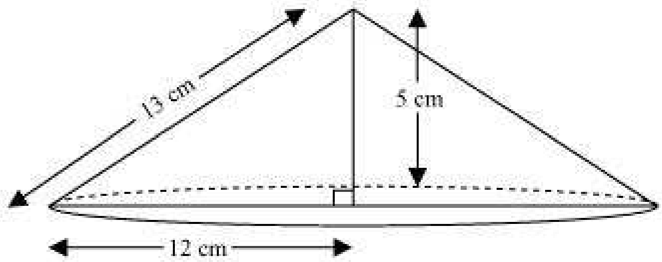
= 100π cm^3

Therefore, the volume of the cone so formed is 100π cm^3.

Question 8:

If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Answer:



When right-angled ΔABC is revolved about its side 5 cm, a cone will be formed having radius (*r*) as 12 cm, height (*h*) as 5 cm, and slant height (*l*) as 13 cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \left[\frac{1}{3} \times \pi \times (12)^2 \times 5\right] \text{ cm}^3$

$= 240\pi \text{ cm}^3$

Therefore, the volume of the cone so formed is 240π cm^3.

Required ratio $= \frac{100\pi}{240\pi}$

$= \frac{5}{12} = 5:12$

Question 9:

A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (*r*) of heap $= \left(\frac{10.5}{2}\right) \text{ m} = 5.25 \text{ m}$

Height (*h*) of heap = 3 m

Volume of heap $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3}\times\frac{22}{7}\times(5.25)^2\times 3\right)\ \text{m}^3$$

$$= 86.625\ \text{m}^3$$

Therefore, the volume of the heap of wheat is 86.625 m^3.

Area of canvas required = CSA of cone

$$= \pi r l = \pi r\sqrt{r^2 + h^2}$$

$$= \left[\frac{22}{7}\times 5.25\times\sqrt{(5.25)^2 + (3)^2}\right]\ \text{m}^2$$

$$= \left(\frac{22}{7}\times 5.25\times 6.05\right)\ \text{m}^2$$

$$= 99.825\ \text{m}^2$$

Therefore, 99.825 m^2 canvas will be required to protect the heap from rain.

Exercise 13.8

Question 1:

Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

$$\left[\text{Assume } \pi = \frac{22}{7}\right]$$

Answer:

(i) Radius of sphere = 7 cm

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times (7)^3\right] \text{ cm}^3$$

$$= \left(\frac{4312}{3}\right) \text{ cm}^3$$

$$= 1437\frac{1}{3} \text{ cm}^3$$

Therefore, the volume of the sphere is $1437\frac{1}{3}$ cm^3.

(ii) Radius of sphere = 0.63 m

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \left[\frac{4}{3} \times \frac{22}{7} \times (0.63)^3\right] \text{ m}^3$$

$$= 1.0478 \text{ m}^3$$

Therefore, the volume of the sphere is 1.05 m^3 (approximately).

Question 2:

Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

$\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Radius (r) of ball = $\left(\frac{28}{2}\right) \text{ cm} = 14 \text{ cm}$

Volume of ball = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (14)^3\right] \text{ cm}^3$

$= 11498\frac{2}{3} \text{ cm}^3$

Therefore, the volume of the sphere is $11498\frac{2}{3}$ cm^3.

(ii)Radius (r) of ball = $\left(\frac{0.21}{2}\right) \text{ m}$ = 0.105 m

Volume of ball = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (0.105)^3\right] \text{ m}^3$

$= 0.004851 \text{ m}^3$

Therefore, the volume of the sphere is 0.004851 m^3.

Question 3:

The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of metallic ball = $\left(\frac{4.2}{2}\right) \text{ cm} = 2.1 \text{ cm}$

Volume of metallic ball = $\frac{4}{3}\pi r^3$

$=\left[\frac{4}{3}\times\frac{22}{7}\times(2.1)^3\right]\text{ cm}^3$

$=38.808\text{ cm}^3$

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$

Mass = Density × Volume

= (8.9 × 38.808) g

= 345.3912 g

Hence, the mass of the ball is 345.39 g (approximately).

Question 4:

The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Answer:

Let the diameter of earth be *d*. Therefore, the radius of earth will be $\frac{d}{2}$.

Diameter of moon will be $\frac{d}{4}$ and the radius of moon will be $\frac{d}{8}$.

Volume of moon = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{8}\right)^3 = \frac{1}{512}\times\frac{4}{3}\pi d^3$

Volume of earth = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{1}{8}\times\frac{4}{3}\pi d^3$

$$\frac{\text{Volume of moon}}{\text{Volume of earth}} = \frac{\frac{1}{512} \times \frac{4}{3}\pi d^3}{\frac{1}{8} \times \frac{4}{3}\pi d^3}$$

$$= \frac{1}{64}$$

$$\Rightarrow \text{Volume of moon} = \frac{1}{64}\text{Volume of earth}$$

Therefore, the volume of moon is $\frac{1}{64}$ of the volume of earth.

Question 5:

How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of hemispherical bowl **=** $\left(\frac{10.5}{2}\right)$ cm = 5.25 cm

Volume of hemispherical bowl = $\frac{2}{3}\pi r^3$

$$= \left[\frac{2}{3} \times \frac{22}{7} \times (5.25)^3\right] \text{cm}^3$$

= 303.1875 cm^3

Capacity of the bowl = $\left(\frac{303.1875}{1000}\right)$ litre

=0.3031875 litre=0.303 litre (approximately)

Therefore, the volume of the hemispherical bowl is 0.303 litre.

Question 6:

A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Inner radius (r_1) of hemispherical tank = 1 m

Thickness of hemispherical tank = 1 cm = 0.01 m

Outer radius (r_2) of hemispherical tank = (1 + 0.01) m = 1.01 m

$$\text{Volume of iron used to make such a tank} = \frac{2}{3}\pi\left(r_2^3 - r_1^3\right)$$

$$= \left[\frac{2}{3} \times \frac{22}{7} \times \left\{(1.01)^3 - (1)^3\right\}\right] \text{m}^3$$

$$= \left[\frac{44}{21} \times (1.030301 - 1)\right] \text{m}^3$$

$$= 0.06348 \text{ m}^3 \quad \text{(approximately)}$$

Question 7:

Find the volume of a sphere whose surface area is 154 cm^2. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Let radius of sphere be r.

Surface area of sphere = 154 cm^2

$\Rightarrow 4\pi r^2 = 154 \text{ cm}^2$

$$\Rightarrow r^2 = \left(\frac{154 \times 7}{4 \times 22}\right) \text{cm}^2$$

$$\Rightarrow r = \left(\frac{7}{2}\right) \text{cm} = 3.5 \text{ cm}$$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (3.5)^3\right] \text{cm}^3$

$= 179\frac{2}{3} \text{ cm}^3$

Therefore, the volume of the sphere is $179\frac{2}{3}$ cm^3.

Question 8:

A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of Rs 498.96. If the cost of white-washing is Rs 2.00 per square meter, find the

(i) inside surface area of the dome,

(ii) volume of the air inside the dome. $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

(i) Cost of white-washing the dome from inside = Rs 498.96

Cost of white-washing 1 m^2 area = Rs 2

Therefore, CSA of the inner side of dome = $\left(\frac{498.96}{2}\right) \text{ m}^2$

= 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.

CSA of inner side of dome = 249.48 m^2

$2\pi r^2 = 249.48 \text{ m}^2$

$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 249.48 \text{ m}^2$

$\Rightarrow r^2 = \left(\frac{249.48 \times 7}{2 \times 22}\right) \text{m}^2 = 39.69 \text{ m}^2$

$\Rightarrow r = 6.3$ m

Volume of air inside the dome = Volume of hemispherical dome

$= \frac{2}{3}\pi r^3$

$= \left[\frac{2}{3}\times\frac{22}{7}\times(6.3)^3\right]$ m^3

= 523.908 m^3

= 523.9 m^3 (approximately)

Therefore, the volume of air inside the dome is 523.9 m^3.

Question 9:

Twenty seven solid iron spheres, each of radius *r* and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius *r*' of the new sphere, (ii) ratio of S and S'.

Answer:

(i)Radius of 1 solid iron sphere = *r*

Volume of 1 solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid iron spheres $= 27\times\frac{4}{3}\pi r^3$

27 solid iron spheres are melted to form 1 iron sphere. Therefore, the volume of this iron sphere will be equal to the volume of 27 solid iron spheres. Let the radius of this new sphere be *r*'.

Volume of new solid iron sphere $= \frac{4}{3}\pi r'^3$

$\frac{4}{3}\pi r'^3 = 27\times\frac{4}{3}\pi r^3$

$r'^3 = 27r^3$

$r' = 3r$

(ii) Surface area of 1 solid iron sphere of radius *r* = $4\pi r^2$

Surface area of iron sphere of radius *r*' = $4\pi (r')^2$

$= 4\pi (3r)^2 = 36\pi r^2$

$$\frac{S}{S'} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9} = 1:9$$

Question 10:

A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule? $\left[\text{Assume } \pi = \frac{22}{7}\right]$

Answer:

Radius (r) of capsule $= \left(\frac{3.5}{2}\right)$ mm $= 1.75$ mm

Volume of spherical capsule $= \frac{4}{3}\pi r^3$

$= \left[\frac{4}{3} \times \frac{22}{7} \times (1.75)^3\right]$ mm^3

= 22.458 mm^3

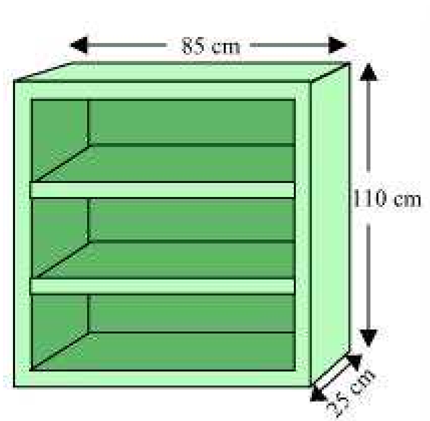
= 22.46 mm^3 (approximately)

Therefore, the volume of the spherical capsule is 22.46 mm^3.

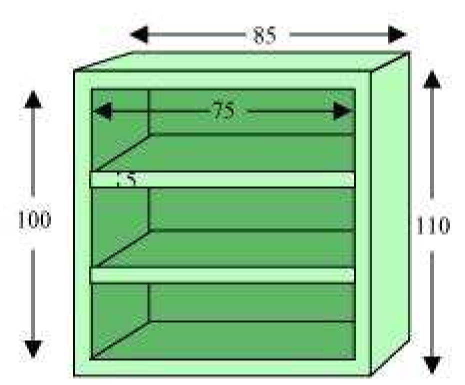
Exercise 13.9

Question 1:

A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see the given figure). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.



Answer:



External height (l) of book self = 85 cm

External breadth (b) of book self = 25 cm

External height (h) of book self = 110 cm

External surface area of shelf while leaving out the front face of the shelf

$= lh + 2(lb + bh)$

$= [85 \times 110 + 2(85 \times 25 + 25 \times 110)]\ \text{cm}^2$

$= (9350 + 9750)\ \text{cm}^2$

$= 19100\ \text{cm}^2$

Area of front face $= [85 \times 110 - 75 \times 100 + 2(75 \times 5)]\ \text{cm}^2$

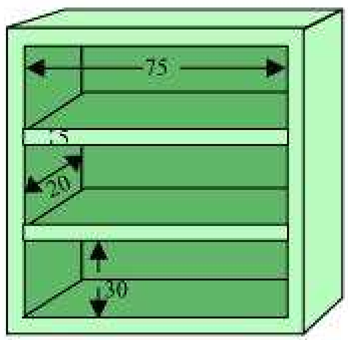
$= 1850 + 750\ \text{cm}^2$

$= 2600\ \text{cm}^2$

Area to be polished $= (19100 + 2600)\ \text{cm}^2 = 21700\ \text{cm}^2$

Cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs (21700×0.20) = Rs 4340



It can be observed that length (l), breadth (b), and height (h) of each row of the book shelf is 75 cm, 20 cm, and 30 cm respectively.

Area to be painted in 1 row $= 2(l + h)b + lh$

$= [2(75 + 30) \times 20 + 75 \times 30]\ \text{cm}^2$

$= (4200 + 2250)\ \text{cm}^2$

$= 6450\ \text{cm}^2$

Area to be painted in 3 rows $= (3 \times 6450)\ \text{cm}^2 = 19350\ \text{cm}^2$

Cost of painting 1 cm^2 area = Rs 0.10

Cost of painting 19350 cm^2 area = Rs (19350×0.1)

= Rs 1935

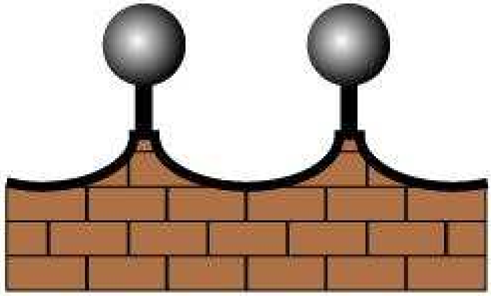
Total expense required for polishing and painting = Rs (4340 + 1935)

= Rs 6275

Therefore, it will cost Rs 6275 for polishing and painting the surface of the bookshelf.

Question 2:

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the given figure. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Answer:

Radius (r) of wooden sphere = $\left(\frac{21}{2}\right)$ cm $= 10.5$ cm

Surface area of wooden sphere = $4\pi r^2$

$= \left[4 \times \frac{22}{7} \times (10.5)^2\right] \text{cm}^2 = 1386 \text{ cm}^2$

Radius (r_1) of the circular end of cylindrical support = 1.5 cm

Height (h) of cylindrical support = 7 cm

CSA of cylindrical support = $2\pi rh$

$= \left[2 \times \frac{22}{7} \times (1.5) \times 7\right] \text{cm}^2 = 66 \text{ cm}^2$

Area of the circular end of cylindrical support = πr^2 $= \left[\frac{22}{7} \times (1.5)^2\right] \text{cm}^2$

= 7.07 cm^2

Area to be painted silver = [8 × (1386 − 7.07)] cm^2

= (8 × 1378.93) cm^2 = 11031.44 cm^2

Cost for painting with silver colour = Rs (11031.44 × 0.25) = Rs 2757.86

Area to be painted black = (8 × 66) cm^2 = 528 cm^2

Cost for painting with black colour = Rs (528 × 0.05) = Rs 26.40

Total cost in painting = Rs (2757.86 + 26.40)

= Rs 2784.26

Therefore, it will cost Rs 2784.26 in painting in such a way.

Question 3:

The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Answer:

Let the diameter of the sphere be *d*.

Radius (r_1) of sphere $= \frac{d}{2}$

New radius (r_2) of sphere $= \frac{d}{2}\left(1 - \frac{25}{100}\right) = \frac{3}{8}d$

CSA (S_1) of sphere = $4\pi r_1^2$

$$= 4\pi\left(\frac{d}{2}\right)^2 = \pi d^2$$

CSA (S_2) of sphere when radius is decreased = $4\pi r_2^2$

$$= 4\pi\left(\frac{3d}{8}\right)^2 = \frac{9}{16}\pi d^2$$

Decrease in surface area of sphere = $S_1 - S_2$

$$= \pi d^2 - \frac{9}{16}\pi d^2$$

$$= \frac{7}{16}\pi d^2$$

Percentage decrease in surface area of sphere $= \frac{S_1 - S_2}{S_1} \times 100$

$$= \frac{7\pi d^2}{16\pi d^2} \times 100 = \frac{700}{16} = 43.75\%$$