Class –IX Mathematics (Ex. 1.1)

Questions

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

2. Find six rational numbers between 3 and 4.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Class –IX Mathematics (Ex. 1.1)

Answers

1. Consider the definition of a rational number.
A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5} \ldots\ldots$.
So, we arrive at the conclusion that 0 can be written in the form of $\frac{p}{q}$, where q is any integer.
Therefore, zero is a rational number.

2. We know that there are infinite rational numbers between any two numbers.
A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
We know that the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 all lie between 3 and 4.
We need to rewrite the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.
So, after converting, we get $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$.
We can further convert the rational numbers $\frac{32}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$ into lowest fractions.
On converting the fractions into lowest fractions, we get $\frac{16}{5}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.
Therefore, six rational numbers between 3 and 4 are $\frac{31}{10}, \frac{16}{5}, \frac{33}{10}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

3. We know that there are infinite rational numbers between any two numbers.
A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
We know that the numbers $\frac{3}{5}$ and $\frac{4}{5}$ can also be written as 0.6 and 0.8.
We can conclude that the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 all lie between 0.6 and 0.8

We need to rewrite the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \frac{64}{100}$ and $\frac{65}{100}$.

We can further convert the rational numbers $\frac{62}{100}, \frac{64}{100}$ and $\frac{65}{100}$ into lowest fractions.

On converting the fractions, we get $\frac{31}{50}, \frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}, \frac{31}{50}, \frac{63}{100}, \frac{16}{25}$ and $\frac{13}{50}$.

4.

(i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5\ldots\ldots$.

We know that natural number series is $1, 2, 3, 4, 5\ldots\ldots$.

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where $q = 1$.

Now, considering the series of integers, we have $\ldots. -4, -3, -2, -1, 0, 1, 2, 3, 4\ldots\ldots$.

We know that whole number series is $0, 1, 2, 3, 4, 5\ldots\ldots$.

We can conclude that all the numbers of whole number series lie in the series of integers. But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$.

We know that whole number series is $0, 1, 2, 3, 4, 5\ldots\ldots$.

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}\ldots\ldots$

We conclude that every number of the whole number series is a rational number.

But, every rational number does not appear in the whole number series.

Therefore, we conclude that every rational number is not a whole number.

Class –IX Mathematics (Ex. 1.2)

Questions

1 State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.
(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
(iii) Every real number is an irrational number.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

3. Show how $\sqrt{5}$ can be represented on the number line.

Class –IX Mathematics (Ex. 1.2)

Answers

1.
(i) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, yes every irrational number is a real number.

(ii) Consider a number line. We know that on a number line, we can represent negative as well as positive numbers.

We know that we cannot get a negative number after taking square root of any number.

Therefore, we conclude that not every number point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, we conclude that, every real number is not a rational number.

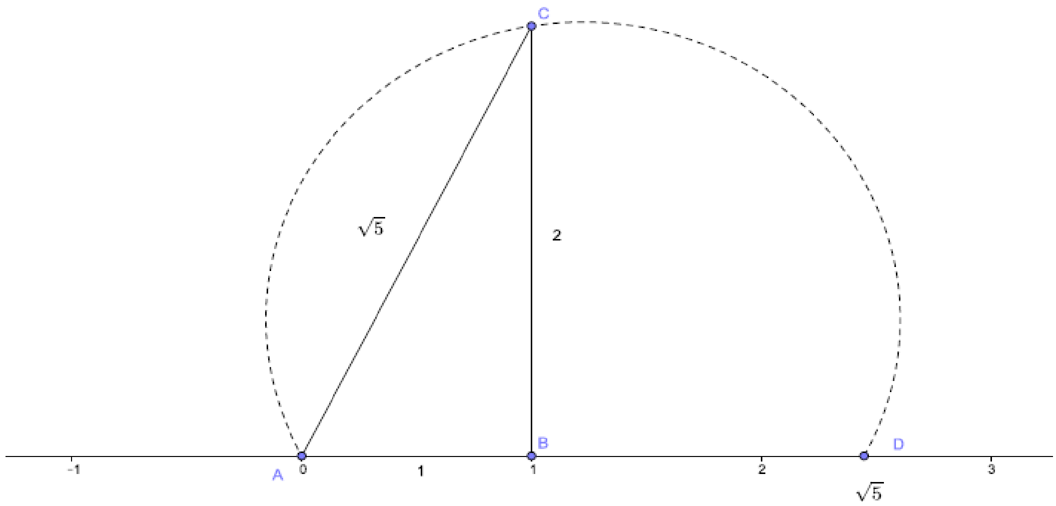
2. We know that square root of every positive integer will not yield an integer.
We know that $\sqrt{4}$ is 2, which is an integer. But, $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number.
Therefore, we conclude that square root of every positive integer is not an irrational number.

3. According to the Pythagoras theorem, we can conclude that

$\left(\sqrt{5}\right)^2 = (2)^2 + (1)^2$.

We need to draw a line segment AB of 1 unit on the number line. Then draw a straight line segment BC of 2 units. Then join the points C and A, to form a line segment BC.

Then draw the arc ACD, to get the number $\sqrt{5}$ on the number line.



Class –IX Mathematics (Ex. 1.3)

Questions

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$ (ii) $\frac{1}{11}$ (iii) $4\frac{1}{8}$ (iv) $\frac{3}{13}$ (v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

2. You know that $\frac{1}{7} = 0.142857......$ Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

[**Hint:** Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$ (ii) $0.4\overline{7}$ (iii) $0.\overline{001}$

4. Express $0.99999....$ in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

5. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

8. Find three different irrational numbers between the rational numbers $\frac{5}{11}$ and $\frac{9}{11}$.

9. Classify the following numbers as rational or irrational:

(i) 23 (ii) 225 (iii) 0.3796 (iv) 7.478478...

(v) 1.101001000100001...

Class –IX Mathematics (Ex. 1.3)

Answers

1.

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$\begin{array}{r|l} & 0.36 \\ 100 & 36 \\ & -0 \\ & 360 \\ & -300 \\ & 600 \\ & -600 \\ & 0 \end{array}$$

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$\begin{array}{r|l} & 0.0909.... \\ 11 & 1 \\ & -0 \\ & 10 \\ & -0 \\ & 100 \\ & -99 \\ & 10 \\ & -0 \\ & 100 \\ & -99 \\ & 1 \end{array}$$

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $\frac{1}{11} = 0.0909.....$ or $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating decimal and recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8}$

On dividing 33 by 8, we get

$$\begin{array}{r|l} & 4.125 \\ 8 & 33 \\ & -32 \\ & 10 \\ & -8 \\ & 20 \\ & -16 \\ & 40 \\ & -40 \\ & 0 \end{array}$$

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8} = \frac{33}{8} = 4.125$, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$\begin{array}{r|l} & 0.230769.... \\ 13 & 3 \\ & -0 \\ & 30 \\ & -26 \\ & 40 \\ & -39 \\ & 10 \\ & -0 \\ & 100 \\ & -91 \\ & 90 \\ & -78 \\ & 120 \\ & -117 \\ & 3 \end{array}$$

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13} = 0.230769.....$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating decimal and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$\begin{array}{r|l} & 0.1818.... \\ 11 & 2 \\ & -0 \\ & 20 \\ & -11 \\ & 90 \\ & -88 \\ & 20 \\ & -11 \\ & 90 \\ & -88 \\ & 2 \end{array}$$

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0.\overline{18}$, which is a non-terminating decimal and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$\begin{array}{r|l} & 0.8225 \\ 400 & 329 \\ & -0 \\ & 3290 \\ & -3200 \\ & 900 \\ & -800 \\ & 1000 \\ & -800 \\ & 2000 \\ & -2000 \\ & 0 \end{array}$$

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

2. We are given that $\frac{1}{7}=0.\overline{142857}$ or $\frac{1}{7}=0.142857......$.

We need to find the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that, $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2\times\frac{1}{7},3\times\frac{1}{7},4\times\frac{1}{7},5\times\frac{1}{7}$ and $6\times\frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as $0.142857.....$, we get

$2\times\frac{1}{7}=2\times0.142857.....=0.285714.........$

$3\times\frac{1}{7}=3\times0.142857.....=0.428571$

$4\times\frac{1}{7}=4\times0.142857...=0.571428$

$5\times\frac{1}{7}=5\times0.142857.....=0.714285$

$6\times\frac{1}{7}=6\times0.142857.....=0.857142$

Therefore, we conclude that, we can predict the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$\frac{2}{7}=0.\overline{285714},\frac{3}{7}=0.\overline{428571},\frac{4}{7}=0.\overline{571428},\frac{5}{7}=0.\overline{714285},$ and $\frac{6}{7}=0.\overline{857142}$

3. **Solution:**

(i) Let $x=0.\overline{6}$ $\Rightarrow$ $x=0.6666.....$ ....(a)

We need to multiply both sides by 10 to get

$10x=6.6666.....$ ....(b)

We need to subtract (a) from (b), to get

$$\begin{array}{r} 10x=6.6666.....\\ -\,x=0.6666.....\\ \hline 9x=6 \end{array}$$

We can also write $9x=6$ as $x=\frac{6}{9}$ or $x=\frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

(ii) Let $x=0.4\overline{7}$ $\Rightarrow$ $x=0.47777....$ ....(a)

We need to multiply both sides by 10 to get

$10x=4.7777.....$ ....(b)

We need to subtract (a) from (b), to get

$$10x = 4.7777.....$$
$$-x = 0.4777....$$
$$9x = 4.3$$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\bar{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) Let $x = 0.\overline{001}$ $\Rightarrow$ $x = 0.001001....$ ....(a)

We need to multiply both sides by 1000 to get

$1000x = 1.001001.....$ ....(b)

We need to subtract (a) from (b), to get

$$1000x = 1.001001.....$$
$$-x = 0.001001....$$
$$999x = 1$$

We can also write $999x = 1$ as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

4. Let $x = 0.99999....$ ....(a)

We need to multiply both sides by 10 to get

$10x = 9.9999....$ ....(b)

We need to subtract (a) from (b), to get

$$10x = 9.99999.....$$
$$-x = 0.99999....$$
$$9x = 9$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1.

Yes at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999……… goes on forever. SO there is not gap between 1 and 0.9999……. and hence they are equal.

5. We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

$$
\begin{array}{r}
0.0588235294117647.... \\
17\overline{)\quad 1} \\
\underline{-0} \\
10 \\
\underline{-0} \\
100 \\
\underline{-85} \\
150 \\
\underline{-136} \\
140 \\
\underline{-136} \\
40 \\
\underline{-34} \\
60 \\
\underline{-51} \\
90 \\
\underline{-85} \\
50 \\
\underline{-34} \\
160 \\
\underline{-153} \\
70 \\
\underline{-68} \\
20 \\
\underline{-17} \\
30 \\
\underline{-17} \\
130 \\
\underline{-119} \\
110 \\
\underline{-102} \\
80 \\
\underline{-68} \\
120 \\
\underline{-119} \\
\underline{1}
\end{array}
$$

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$, which is a non-terminating decimal and recurring decimal.

6. **Solution:**

Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which q must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2, 5 or both.

7. The three numbers that have their expansions as non terminating non recurring decimal are given below.

0.04004000400004....
0.07007000700007....
0.013001300013000013....

8. Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$\frac{5}{7} = 0.714285....$ and $\frac{9}{11} = 0.818181.....$

Three irrational numbers that lie between 0.714285.... and 0.818181.... are:
0.73073007300073....
0.74074007400074....
0.76076007600076....

9.

(i) $\sqrt{23}$

We know that on finding the square root of 23, we will not get an integer.
Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

We know that on finding the square root of 225, we get 15, which is an integer.
Therefore, we conclude that $\sqrt{225}$ is a rational number.

(iii) 0.3796

We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$0.3796 = \frac{3796}{10000}$.

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.
We can observe that 0.3796 can be converted into a rational number.
Therefore, we conclude that 0.3796 is a rational number.

(iv) 7.478478....

We know that 7.478478.... is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478.... into $\frac{p}{q}$ form, we get

$x = 7.478478....$ ....(a)

$1000x = 7478.478478....$ ....(b)

While, subtracting (a) from (b), we get

$$\begin{array}{r} 1000x = 7478.478478.... \\ -x = \quad 7.478478.... \\ \hline 999x = 7471 \end{array}$$

We know that $999x = 7471$ can also be written as $x = \frac{7471}{999}$.
Therefore, we conclude that 7.478478.... is a rational number.

(v) 1.101001000100001....

We can observe that the number 1.101001000100001.... is a non terminating non recurring decimal.

We know that non terminating and non recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.

Class –IX Mathematics (Ex. 1.4)

Questions

1. Visualize 3.765 on the number line using successive magnification.

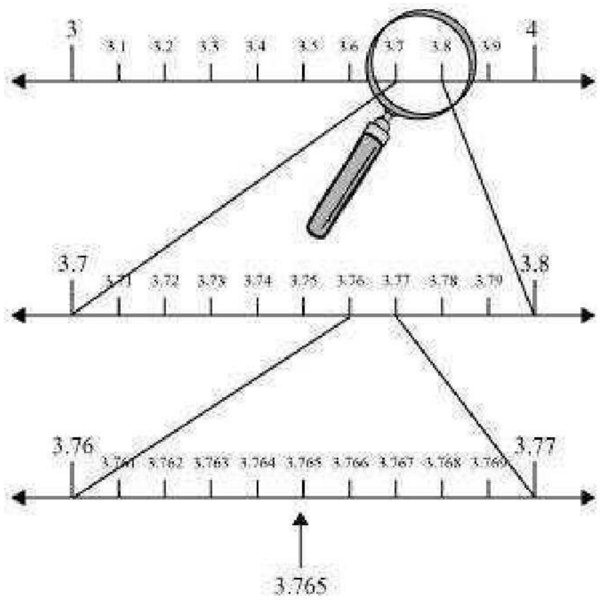
2. Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

Class –IX Mathematics (Ex. 1.4)

Answers

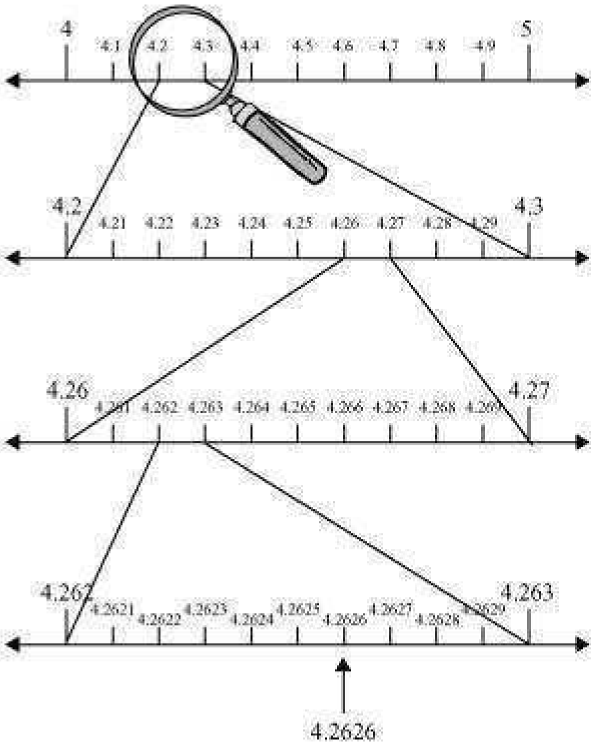
1. We know that the number 3.765 will lie between 3.764 and 3.766.
We know that the numbers 3.764 and 3.766 will lie between 3.76 and 3.77.
We know that the numbers 3.76 and 3.77 will lie between 3.7 and 3.8.
We know that the numbers 3.7 and 3.8 will lie between 3 and 4.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 3 and 4 on the number line.



2. We know that the number $4.\overline{26}$ can also be written as $4.262....$.
We know that the number $4.262....$ will lie between 4.261 and 4.263.
We know that the numbers 4.261 and 4.263 will lie between 4.26 and 4.27.
We know that the numbers 4.26 and 4.27 will lie between 4.2 and 4.3.
We know that the numbers 4.2 and 4.3 will lie between 4 and 5.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 4 and 5 on the number line.


4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
4.2
4.21
4.22
4.23
4.24
4.25
4.26
4.27
4.28
4.29
4.3
4.26
4.261
4.262
4.263
4.264
4.265
4.266
4.267
4.268
4.269
4.27
4.262
4.2621
4.2622
4.2623
4.2624
4.2625
4.2626
4.2627
4.2628
4.2629
4.263
4.2626

Class –IX Mathematics (Ex. 1.5)

Questions

1. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$ (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$ (iv) $\frac{1}{\sqrt{2}}$ (v) 2π

2. Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$ (iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

(iv) $\left(5-\sqrt{2}\right)\left(5+\sqrt{2}\right)$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

4. Represent 9.3 on the number line.

5. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$ (iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Class –IX Mathematics (Ex. 1.5)

Answers

1.

(i) $2-\sqrt{5}$

We know that $\sqrt{5}=2.236....,$ which is an irrational number.

$2-\sqrt{5}=2-2.236....$

$=-0.236....,$ which is also an irrational number.

Therefore, we conclude that $2-\sqrt{5}$ is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

$\left(3+\sqrt{23}\right)-\sqrt{23}=3+\sqrt{23}-\sqrt{23}$

$=3.$

Therefore, we conclude that $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

We can cancel $\sqrt{7}$ in the numerator and denominator, as $\sqrt{7}$ is the common number in numerator as well as denominator, to get

$\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

We know that $\sqrt{2}=1.414....,$ which is an irrational number.

We can conclude that, when 1 is divided by $\sqrt{2}$, we will get an irrational number.

Therefore, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

We know that $\pi=3.1415....,$ which is an irrational number.

We can conclude that 2π will also be an irrational number.

Therefore, we conclude that 2π is an irrational number.

2.

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$.

$$\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$$
$$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$, we get $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$.

$$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=3\left(3-\sqrt{3}\right)+\sqrt{3}\left(3-\sqrt{3}\right)$$
$$=9-3\sqrt{3}+3\sqrt{3}-3$$
$$=6$$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$, we get 6.

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

We need to apply the formula $\left(a+b\right)^2=a^2+2ab+b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$$
$$=5+2\sqrt{10}+2$$
$$=7+2\sqrt{10}.$$

Therefore, on simplifying $\left(\sqrt{5}+\sqrt{2}\right)^2$, we get $7+2\sqrt{10}$.

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

We need to apply the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$$
$$=5-2$$
$$=3.$$

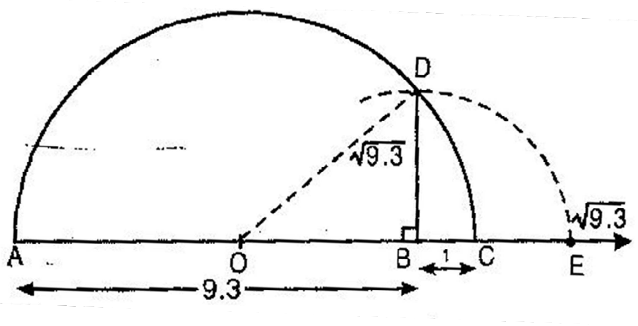
Therefore, on simplifying $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$, we get 3.

3. We know that when we measure the length of a line or a figure by using a scale or any device, we do not get an exact measurement. In fact, we get an approximate

rational value. So, we are not able to realize that either circumference (c) or diameter (d) of a circle is irrational.

Therefore, we can conclude that as such there is not any contradiction regarding the value of π and we realize that the value of π is irrational.

4. Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that AB = 9.3 units. From B mark a distance of 1 unit and call the new point as C. Find the mid-point of AC and call that point as O. Draw a semi-circle with centre O and radius OC = 5.15 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D. Then BD = $\sqrt{9.3}$.



5.

(i) $\frac{1}{\sqrt{7}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$, to get

$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$.

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}}$, we get $\frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$, to get

$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}$.

We need to apply the formula $\left(a-b\right)\left(a+b\right) = a^2 - b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$=\sqrt{7}+\sqrt{6}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$, we get $\sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$, to get

$$\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}.$$

We need to apply the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$, we get $\frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$, to get

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

We need to apply the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-\left(2\right)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}$$

$$=\frac{\sqrt{7}+2}{3}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$, we get $\frac{\sqrt{7}+2}{3}$.

Class –IX Mathematics (Ex. 1.6)

Questions

1. Find: (i) $64^{\frac{1}{5}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

2. Find: (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

3. Simplify: (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $\left(3^{\frac{1}{3}}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Class –IX Mathematics (Ex. 1.6)

Answers

1.

(i) $64^{\frac{1}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $64^{\frac{1}{2}}$ can also be written as $\sqrt[2]{64} = \sqrt[2]{8\times 8}$

$\sqrt[2]{64} = \sqrt[2]{8\times 8} = 8.$

Therefore the value of $64^{\frac{1}{2}}$ will be 8.

(ii) $32^{\frac{1}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $32^{\frac{1}{5}}$ can also be written as $\sqrt[5]{32} = \sqrt[2]{2\times 2\times 2\times 2\times 2}$

$\sqrt[5]{32} = \sqrt[2]{2\times 2\times 2\times 2\times 2} = 2$

Therefore the value of $32^{\frac{1}{5}}$ will be 2.

(iii) $125^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $125^{\frac{1}{3}}$ can also be written as $\sqrt[3]{125} = \sqrt[3]{5\times 5\times 5}$

$\sqrt[3]{125} = \sqrt[3]{5\times 5\times 5} = 5$

Therefore the value of $125^{\frac{1}{3}}$ will be 5.

2.

(i) $9^{\frac{3}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $9^{\frac{3}{2}}$ can also be written as $\sqrt[2]{(9)^3} = \sqrt[2]{9\times 9\times 9} = \sqrt[2]{3\times 3\times 3\times 3\times 3\times 3}$

$\sqrt[2]{(9)^3} = \sqrt[2]{3\times 3\times 3\times 3\times 3\times 3}$

$= 3\times 3\times 3$

$= 27.$

Therefore the value of $9^{\frac{3}{2}}$ will be 27.

(ii) $32^{\frac{2}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $32^{\frac{2}{5}}$ can also be written as

$$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)(2\times2\times2\times2\times2)}$$
$$= 2\times2$$
$$= 4.$$

Therefore the value of $32^{\frac{2}{5}}$ will be 4.

(iii) $16^{\frac{3}{4}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $16^{\frac{3}{4}}$ can also be written as

$$\sqrt[4]{(16)^3} = \sqrt[4]{(2\times2\times2\times2)(2\times2\times2\times2)(2\times2\times2\times2)}$$
$$= 2\times2\times2$$
$$= 8.$$

Therefore the value of $16^{\frac{3}{4}}$ will be 8.

(iv) $125^{\frac{-1}{3}}$

We know that $a^{-n} = \frac{1}{a^n}$

We conclude that $125^{\frac{-1}{3}}$ can also be written as $\frac{1}{125^{\frac{1}{3}}}$, or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$.

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We know that $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ can also be written as

$$\sqrt[3]{\left(\frac{1}{125}\right)} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$$
$$= \frac{1}{5}.$$

Therefore the value of $125^{\frac{-1}{3}}$ will be $\frac{1}{5}$.

3.

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

We know that $a^m \cdot a^n = a^{(m+n)}$.

We can conclude that $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$.

$$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{10+3}{15}} = (2)^{\frac{13}{15}}$$

Therefore the value of $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ will be $(2)^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

We know that $a^m \times a^n = a^{m+n}$

We conclude that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $\left(3^{\frac{7}{3}}\right)$..

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

We know that $\frac{a^m}{a^n} = a^{m-n}$

We conclude that $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$.

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}}$$

$$= 11^{\frac{1}{4}}$$

Therefore the value of $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

We know that $a^m \cdot b^m = (a \times b)^m$.

We can conclude that $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$.

$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$$

Therefore the value of $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$.