Chapter 8
Similar Triangles

Similar Triangles:

Whenever we talk about two congruent figures then they have the 'same shape' and the 'same size'. There are figures that are of the 'same shape but not necessarily of the 'same size'. They are said to be similar. Congruent figures are similar but the converse is not true

All regular polygons of same number of sides are similar. They are equilateral triangles, squares etc. All circles are also similar.

Two polygons of the same number of sides are similar if their corresponding angles are sides are proportional.

Two triangles are similar if their corresponding are equal and corresponding sides are proportional.

Basic Proportionality Theorem or Thales Theorem.

Theorem-1

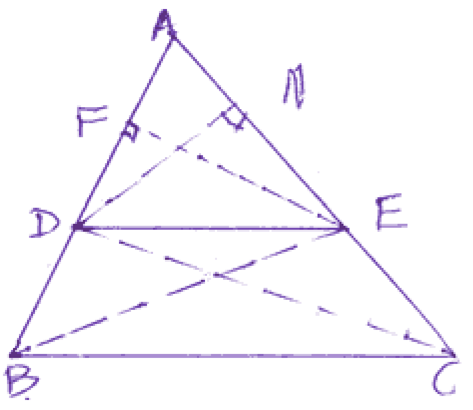
If a line is drawn parallel to one side of a triangle, to interest the other two sides indistinct points, the other two sides are divided in the same ratio.

Given: - In $\Delta ABC, DE \parallel BC$

To prove:- $\frac{AD}{DB} = \frac{AE}{EC}$

Construction:- BE and CD are joined. $EF \perp AB$ and $DN \perp AL$ are drawn.

Proof:-



$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EF$$

$$= \frac{1}{2} \times AE \times DN$$

$$ar(\Delta BDE) = \frac{1}{2} \times BD \times F$$

$$ar(\Delta CDE) = \frac{1}{2} \times EC \times DN$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} ----(1)$$

$$\frac{ar(\Delta ADE)}{ar(CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} ----(2)$$

But $ar(\Delta BDE) = ar(\Delta CDE) ---------(3)$

as they are on the same base DE and $DE \parallel BC$

· from (1), (2) and (3) we get $\frac{AD}{DB} = \frac{AE}{EC}$

Corollary: In ΔABC, $DE \parallel BC$ then

$$\frac{AD}{AB} = \frac{AE}{AC}$$

Proof:- We know

$$\frac{AD}{DB} = \frac{AE}{EC} -------- (1)$$

$$\therefore \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

$$\frac{AB}{DB} = \frac{AC}{EC}$$

$$\frac{DB}{AB} = \frac{EC}{AE} --------(2)$$

(Taking reciprocals)

Multiplying (1) and (2) we get

$$\frac{AD}{DB} \times \frac{DB}{AB} = \frac{AE}{EC} \times \frac{EC}{AC}$$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

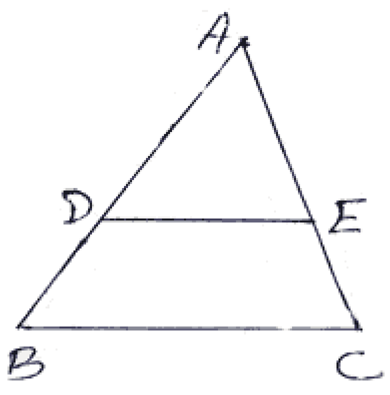
Property - 1. If a line divides any two sides of a triangle in the same ratio, the line is parallel to the third side.

Example 1. In $(\Delta ABC, DE \parallel BC$ and $AD = 2.4cm, AE = 3.2cm, EC = 4.8cm.$ Find AB.

Solution:-

$DE \parallel BC$ (given)

$\frac{AD}{DB} = \frac{AE}{EC}$ (Thales Theoram)



$$\frac{2.4}{BD} = \frac{3.2}{4.8}$$

$$\therefore BD = \frac{2.4 \times 4.8}{3.2}$$
$$= 3.6cm$$

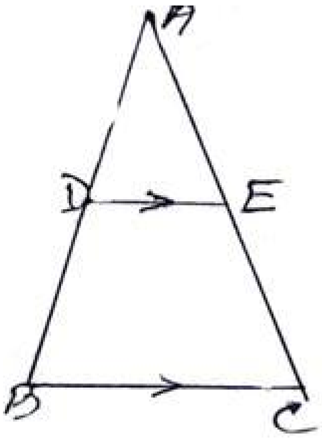
= 3.6cm

AB = AD + DB

= 2.4 + 3.6

= 6.0cm

Example 2. In the given figure, $DE \parallel BC$ and $AD = 4x - 3, AE = 8x - 7, BD = 3x - 1, CE = 5x - 3;$ Find x.



Solution:-

In ΔABC, $DE \parallel BC$ (given)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

Or, (4x - 3) (5x - 3) = (8x - 7) (3x - 1)

Or, $20x^2$ - 12x - 15x + 9 = $24x^2$ - 8x - 21x + 7

Or, $4x^2$ - 2x - 2 = 0

Or, $2x^2$ - x - 1 = 0

Or, $2x^2$ - 2x + x - 1 = 0

Or, 2x(x - 1) + 1 (x - 1) = 0

(x - 1) (2x + 1) = 0

x = 1, -1/2

But sides of a triangle cannot be negative

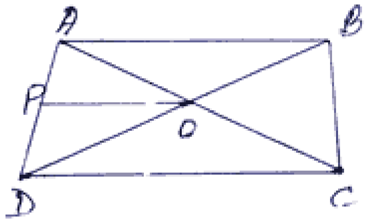
$\therefore x = 1$

Example 3. ABCD is a trapezium such that $AB \parallel CD.$ Its diagonals AC and BD intersest each other at o.

prove that $\frac{AO}{OC} = \frac{BO}{OD}$.

Solution:- Given $AB \parallel DC$ AC and BD intersectato

To Prove: $\frac{AO}{OC} = \frac{BO}{OD}$



Construction: OP || AB || CD is drawn

Proof:- In $\Delta ADC, OP \parallel CD$

$\therefore \frac{PA}{PD} = \frac{AO}{OC}$

In $\Delta DAB, OP \parallel AB$

$\therefore \frac{PA}{PD} = \frac{BO}{OD}$ (Corollary of Thales theo.)

From (i) and (ii) we get

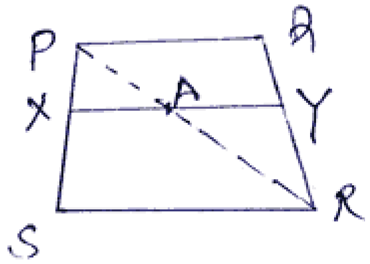
$\frac{AO}{OC} = \frac{BO}{OD}$

Example 4. Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally (i.e. in the same ratio)

Solution:-

Given:- PQRS is a trapezium in which PQ || SR || XY

To Prove:- $\frac{PX}{XS} = \frac{QY}{YR}$



Construction:- PR is joined which intersects XY at A.

Proof:- In $\Delta PSR, XA \parallel SR$

$\therefore \frac{PX}{XS} = \frac{PA}{AR} --------(1)$ [Thales Theorem]

$\Delta PRQ, AY \parallel PQ$

$\therefore \frac{RY}{YQ} = \frac{AR}{PA}$

$\frac{QY}{YR} = \frac{PA}{AR} -----(2)$ (Taking reciprocals)

From (i) and (ii) we get

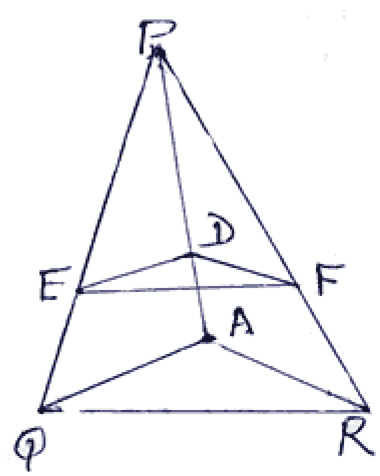
$\frac{PX}{XS} = \frac{QY}{YR}$

Example 5. In the given figure DE || AQ and DF || AR

Prove that EF || QR

Solution:-

In $\Delta APQ, DE \parallel AQ$



$\therefore \frac{PE}{EQ} = \frac{PD}{DA} - - - - - (1)$

$\Delta APR, DF \parallel AR$

$\therefore \frac{PD}{DA} = \frac{PF}{FR} - - - - - (2)$

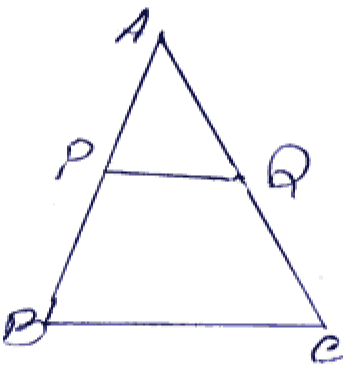
from (1) and (2) we get

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$

Exercise - 12

1. In the given figure, PQ || BC, AP = 2.4cm, AQ = 2cm, QC = 3cm and BC = 6cm. Find AB and PQ.



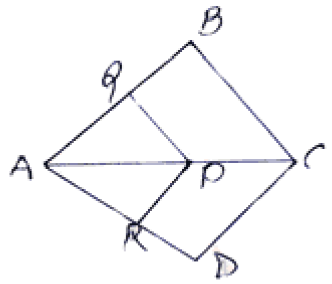
2. The diagonals AC and BD of a quadrilatereal ABCD intersect each other at O such that

$\frac{AO}{OC} = \frac{BO}{OD}$.

prove that the quadrilateral ABCD is traperzium.

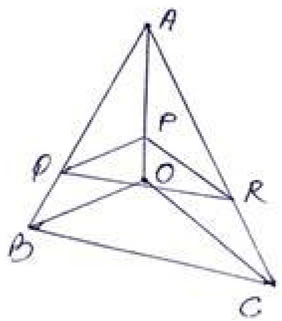
3. In $\Delta ABC, DE \parallel BC$ and $\frac{AD}{DB} = \frac{3}{5}$. if AC = 4.8cm, find AE

4. In the given figurer, PQ || BC and PR || CD, prove that $\frac{AR}{AD} = \frac{AQ}{AB}$.



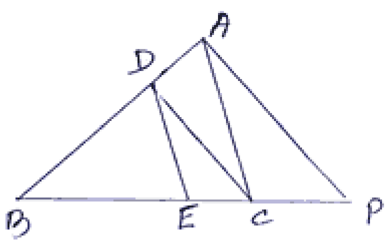
5. In ΔABC, DE is parallel to base BC, with D on AB and E on AC. If $\frac{AD}{DB} = \frac{2}{3}$, find $\frac{BC}{DE}$.

6. In the given figure, PQ || AB and PR || AC. prove that QR || BC.

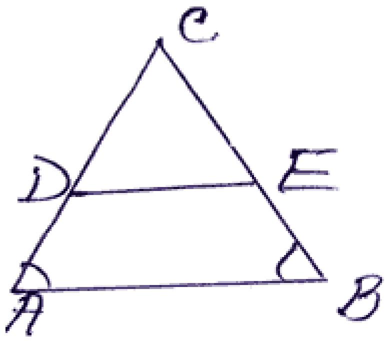


7. If three or more parallel lines, are intersected by two transversals, prove that the intercepts made by them on the trans versals are proportional.

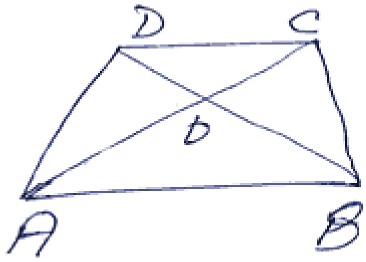
8. In the given figure, DE || AC and DC || AP, prove that $\frac{BE}{EC} = \frac{BC}{CP}$.



9. In the given figure, $\angle A = \angle B$ and DE || AB prove that AD = BE.



10. In the given figure AB || CD. If OA = 3x - 19, OB = x - 4, OC = x - 3 and OD = 4cm, determine x.



Answers

(10). (x = 11 cm or 8 cm)

Critieria for similarities of two triangles.

1. If in two triangles, the corresponding angles are equal, then their corresponading sides are proportional (i.e. in the same ratio) and hence the triangles are similar.

This property is referred to as the AAA similarily criterian

In the above property if only two angles are equal, then the third angle will be automatically equal

Hence AAA criteria is same as AA criteria.

2. If the coreponding sides of two trianlgles are proportional (i.e.in the same ratio), their corresponding angles are equal and hence the triabgles are similar.

This property is referredd to as SSS similarily criteria.

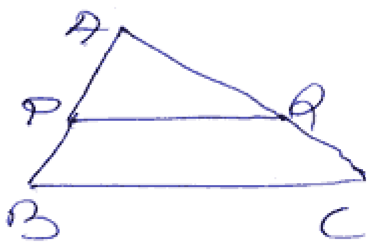
3. If one angles of a triangle is equal to one angle of the other and the sides including these angles are proportional, the triagngle are similar.

This proprerty is referred to as SAS critreria.

Example 6. P and Q are pointes on AB and AC respectively of ΔABC If AP = 1cm, PB = 2cm, AQ = 3cm and QC = 6cm. Show that BC = 3PQ.

Solution:-

Given:- ΔABC in which P and Q are points on AB and AC such that AP = 1cm, AQ = 3cm, PB = 2cm, QC = 6cm.



To Prove:- BC = 3PQ

Proof:- $\therefore \dfrac{AP}{PB} = \dfrac{1}{2}, \dfrac{AQ}{QC} = \dfrac{3}{6} = \dfrac{1}{2}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence PQ || BC

$$\therefore \angle P = \angle B \text{ and } \angle Q = \angle C$$

$$\therefore \Delta APQ \sim \Delta ABC \qquad (AA - Similarity)$$

$$\therefore \frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$$

But AB = AP + PB = 1 + 2 = 3cm

$$\therefore \frac{PQ}{BC} = \frac{AF}{AB} = \frac{1}{3}$$

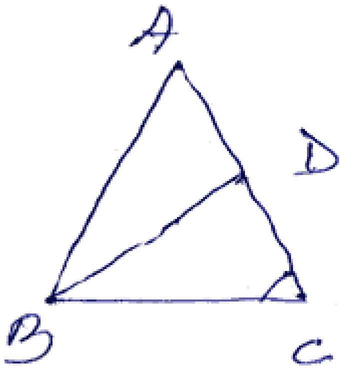
Hence BC = 3PQ.

Example 7. In a $\Delta ABC, AB = AC$ and D is a point on side AC, such that BC^2 = AC X CD

Prove that :- BD = BC

Solution:-

Given:- ΔABC in which AB = AC and D is a point on AC



Such that BC^2 = AC X CD

To Prove :- BD = BC

Proof:- BC^2 = AC X CD

Or, BC X BC = AC X CD

Or, $\frac{BC}{CD} = \frac{AC}{BC} --------(1)$

In $\Delta^s ABC$ and BCD, we have

$\frac{BC}{CD} = \frac{AC}{BC} \quad [by(1)]$

$\therefore \angle ACD = \angle DCB \ (common)$

$\therefore ABC \sim \Delta BCD \quad [SAS\ criteria]$

$\therefore \frac{BC}{CD} = \frac{AC}{BC} = \frac{AB}{BD}$

Or, $\frac{AC}{BC} = \frac{AB}{BD}$

Or, $\frac{AB}{BC} = \frac{AB}{BD} \quad [\because AB = AC]$

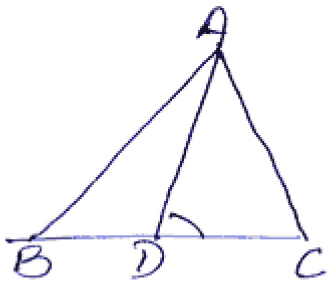
Or, $\frac{1}{BC} = \frac{1}{BD}$

$\therefore BD = BC$

Example 8. D is a point on the side BC of a ΔABC such that $\angle ADC$ and $\angle BAC$ are equal.

Prove that CA^2 = DC X CB

Solution:-



Given:- D is a point on the side BC of a ΔABC such that $\angle ADC = \angle BAC$

To Prove:- CA^2 = DC X CB

Proof:- In $\Delta' ADC$ and DAC

$\angle BAC = \angle ADC \qquad (given)$

$\angle ACB = \angle DCA \qquad (common)$

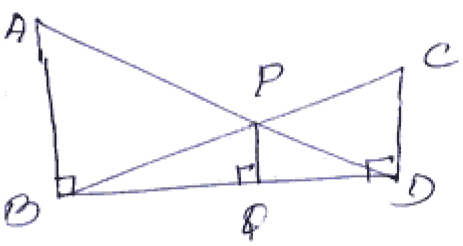
$\therefore \Delta ACB \sim \Delta DCA \qquad (AA\ similarity)$

$\therefore \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow AC^2 = DC \times CB$

Exercise - 13

1. In the adjoing figure, $\angle ABD = \angle CDB = \angle PQB = 90^0$. If



AB = x units CD = y units and PQ = Z units, Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$,

2. In a $\Delta ABC, P$ and Q are point on the side AB and AC respectively such that PQ is parallel to BC. Prove that median AD drawn from A to BC, bisect PQ.

3. Through the mid-point M of the side CD of a parallelogram AB CD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL.

4. ABC is a triangle right anlgled at C. If P is the length of perpendicular from C to AB and AB = c, BC = a and CA = b, show that pc = ab

5. Two right angles ABC and DBC are drawn on the same hypoeuuge BC and on the same side of BC. If AC and BD interscta at P, prove that AP X PC = BP X PD

6. The perimeter of two smilar triangles ABC and PQR are respectively 32cm and 24cm.If PQ = 12cm, find AB.

7. In a right triangles ABC, the perpendicular BD on the hypotenuse Ac is drown. Prove that AC X CD = BC^2

8. In $\Delta ABC, \angle A$ is aculte, BD and CE are perenducular on AC and AB respectively. Prove that AB X AE = AC X AD

9. Through the vertex D of a parallotogram ABCD, a line is drawn to intersect the sides AB and CB produced at E and F respectively prove that: $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$

10. Two sides and a mediam bisecting one of these sides of a triangle are respectively proportional to the two sides and the corresponding mediam of the other triangle. Prove that the triangles are similar.

11. If the angles of one triangles are respectively equal to the angles of another tranles. Prove that the ratio of their corresponding sides is the same as the ratio of their corresponding.

1. medians
2. altitudes
3. angle bisectors

12. E is a point on side AD produced of a parallelogram ABCD and BE intersects CD at F. prove that $\Delta ABE \sim \Delta CFB$.

13. If a perpecdicular is drawn from the vertex of the right angles of a right triangles to the hypoteuuse, the triangles on each side of the perpendicular are similar to the whole triangles and to each other.

Theorem 2. The ratio of the ares of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Given:- $\Delta ABC \sim \Delta PQR$

To prove:

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

Construction: $AD \perp BC$ and $PS \perp QR$ are drawn as in figure

proof:-

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{1/2 \times BC \times AD}{1/2 \times QR \times PS} \quad \left[area\ of \Delta = \frac{1}{2} \times base \times alt.\right]$$

Or,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC}{QR} \times \frac{AD}{PS} ------- (1)$$

Now in $\Delta^s\ ADB$ and PSQ,

$$\angle B = \angle Q \qquad [As\ \Delta ABC \sim \Delta PQR]$$

$$\angle ADB = \angle PSQ \qquad |Each\ 90^0|$$

$$\therefore\ \Delta ADB \sim \Delta PSQ \qquad [AA\ similarly]$$

$$\therefore \frac{AD}{PS} = \frac{AB}{PQ} ------------ (2)$$

$$But \quad \frac{AB}{PQ} = \frac{BC}{QR} \qquad [\sin ce \Delta ABC \sim \Delta PQR]$$

$$\therefore \frac{AD}{PS} = \frac{BC}{QR} \qquad [from (2)] ------- (3)$$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} \qquad [from (1)\ \&\ (3)]$$

$$= \frac{BC^2}{QR^2} ------------ (4)$$

$$Also \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} ------ (5) [as, \Delta ABC \sim \Delta PQR]$$

Hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad [from (4) and (5)]$$

Example 9. ABC and DEF are two similar triangles such that AB = 2DE and area of ΔABC is 56sq.cm, find the area of ΔDEF.

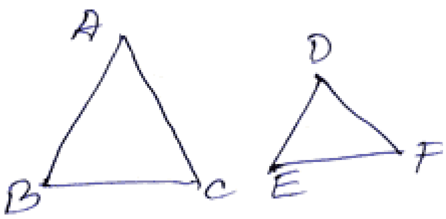
Solution:-

Given:- $ar(\Delta ABC) = 56 sq.cm., AB = 2DE$

To find: Area of ΔDEF.

Proof: $\Delta ABC \sim \Delta DEF \qquad (given)$

and $AB = 2DE \qquad (given)$



$$\therefore \frac{AB^2}{DE^2} = \frac{ar(\Delta ABC)}{ar(\Delta DEF)}$$

$$Or, \frac{(2DE)^2}{DE2} = \frac{56}{ar(\Delta DEF)}$$

$$Or, \frac{4DE^2}{DE^2} = \frac{56}{ar(\Delta DEF)}$$

$$Or, 4 \times ar\Delta DEF = 56$$

$$Or, ar\Delta DEF = \frac{56}{4} = 14 sq.cm$$

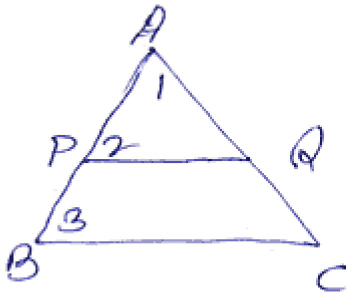
Example 10. ABC is a triangle, PQ is the line segruent intersecting AB in P and AC in Q such that PQ || BC and divides ΔABC, into two parts equal in area. Find BP : AB

Solution:

Given: ΔABC, in which PQ || BC, and PQ divides ΔABC, into two parts equal in area.

To find: BP : AB

Proof:- In $\Delta^s APQ$ and ABC



$\angle 1 = \angle 1 \qquad [common]$

$\angle 2 = \angle 3 \qquad [PQ \parallel BC]$

$\therefore\ \Delta APQ \sim \Delta ABC \qquad [AA\ corollary]$

$$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$Or,\ \frac{ar(\Delta APQ)}{2ar(APQ)} = \frac{AP^2}{AB^2}$$

$$Or,\ \frac{1}{2} = \frac{AP^2}{AB^2}$$

$$\therefore\ \frac{AP}{AB} = \frac{1}{\sqrt{2}}$$

$$\frac{AB - BP}{AB} = \frac{1}{\sqrt{2}}$$

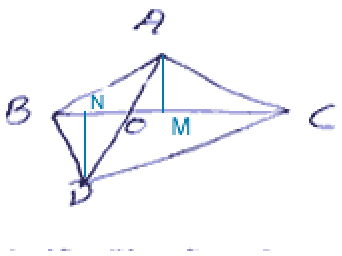
$$\frac{AB}{AB} - \frac{BP}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - \frac{1}{\sqrt{2}} = \frac{BP}{AB}$$

$$\frac{\sqrt{2}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{BP}{AB}$$

$$\therefore\ \frac{BP}{AB} = \frac{2-\sqrt{2}}{2}$$

Example 11. In the given figure, ABC and DBC are two triangles on the same base BC. IF AD intersects BC at O, Prove that $\frac{ar(\Delta ABC)}{ar(DBC)} = \frac{AO}{DO}$



Solution:

Given: ABC and DBC are two triangles on the same base BC. AD intersect BC at O.

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction:- $AM \perp BC, DN \perp BC$ are drawn.

Proof:- In $\Delta^s AMO$ and DNO

$\angle AMO = \angle DNO$ $\quad [each 90^0]$

$\angle AOM = \angle DON$ $\quad [vertically\ opposite\ angles]$

$\therefore\ \Delta AMO \sim \Delta DNO$ $\quad (AA\ similarity)$

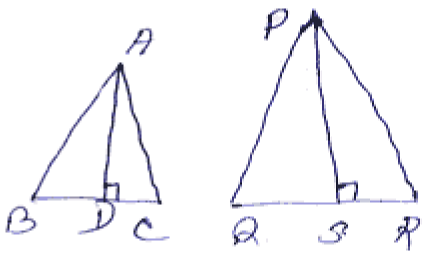
$$\therefore \frac{AO}{DO} = \frac{AM}{DN} ---------- (1)$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{1/2 \times BC \times AM}{1/2 \times BC \times DN} = \frac{AM}{DN}$$

$$= \frac{AO}{DO} \qquad [form(1)]$$

Example 12. In th e given fig ABC and PQR are isosceles triangles in which $\angle A = \angle P$.

$If\ \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{16},\ find\ \frac{AD}{PS}$



Solution:-

Given:- In $\Delta^s ABC$ and PQR, $\angle A = \angle P.\ AB = AC$ and PQ = PR

$$\frac{ar(\Delta ABC)}{ar(PQR)} = \frac{9}{16}$$

To find: $\frac{AD}{PS} = ?$

Proof:- $\Delta ABC,$ is isosceles with AB = AC

$$\therefore \frac{AB}{AC} = 1 \ldots\ldots\ldots\ldots(1)$$

ΔPQR is isosceles with PQ = PR

$$\therefore \frac{PQ}{PR} = 1 \ldots\ldots\ldots\ldots(2)$$

From (1) and (2) we get

$$\frac{AB}{AC} = \frac{PQ}{PR}$$

Or, $\frac{AB}{PQ} = \frac{AC}{PR}$

$\angle A = \angle P \qquad (given)$

$\therefore \Delta ABC \sim \Delta PQR \qquad (SAS)$

$In\ \Delta^s ADC\ and\ PSR$

$\angle ACD = \angle PRS \qquad (\because \Delta ABC \sim \Delta PQR)$

$\angle ADC = \angle PSR \qquad (= 90^0)$

$\therefore \Delta ADC \sim \Delta PSR \qquad (AA\ similarity)$

$$\frac{AD}{PS} = \frac{AC}{PR} \ldots\ldots\ldots\ldots\ldots(3) \quad (\because \Delta ABC \sim \Delta PQR)$$

$$But\ \frac{BC}{QR} = \frac{AC}{PR} \ldots\ldots\ldots\ldots\ldots(4)$$

from (3) and (4)

$$\therefore \frac{AD}{PS} = \frac{BC}{QR} \ldots\ldots\ldots\ldots(5)$$

We know $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$

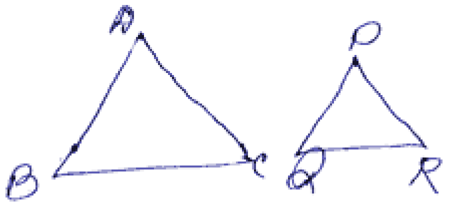
$$\therefore \frac{9}{16} = \frac{AD^2}{PS}$$

$$Or,\ \frac{AD}{PS} = \frac{3}{4}$$

Exercise - 14

1. Prove that the area of the equilateral triangles describe on the side of a square is half the are of the equilateral triangle describe on its diagonals.

2. In the given figure $\Delta ABC \sim \Delta PQR.$ Also $ar(\Delta ABC) = 4ar(\Delta PQR).$ If BC = 12cm, find QR.



3. ABC is a triangle right angled at A, AD is perpendicular to BC. IF BC = 13cm and AC = 5cm, find teh ratio of the areas of ΔABC and ΔADC.

4. The area of two similar triagles are 121cm^2 and 64cm^2 respectively. If the median of the first triangle is 12.1cm, find the correstponding median of the other.

5. In an equilateral triangle with side a, prove that the area of the triangles is $\frac{a^2\sqrt{3}}{4}$.

6. D and E are points on the sides AB and Ac respectively of ΔABC such that DE is parallel to BC and AD : DB = 4 : 5. CD and BE intersect each other at F. Find the ratio of the areas of ΔDEF and ΔBCF.

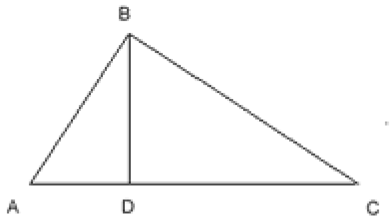
Answers

(2) 6cm **(3)** 169 : 25 **(4)** 8.8cm

(6) 16 : 81

Pythagoras Theorem. (B audhayan Theorem)

Theorem 8.3: - In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: - $\angle B$ is a right angle of ΔABC

To Prove:- $AC^2 = AB^2 + BC^2$

Construction:- $BD \perp AC$ is drawn

Proof:- In $\Delta^s ADB \text{ and } ABC$

$\angle ADB = \angle ABC \quad (= 90^0)$

$\angle BAD = \angle BAC \quad (Common)$

$\therefore \Delta ADB \sim \Delta ABC \quad (AA - Cor)$

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$

Or, AB^2 = AC X AD ---------------------(i)

Similarly $\Delta ADC \sim \Delta ABC$

$\therefore \frac{CD}{BC} = \frac{BC}{AC}$

Or, BC^2 = AC X CD ---------------------(ii)

Adding (i) and (ii) we get

$AB^2 + BC^2$ = AC X AD + AC X CD

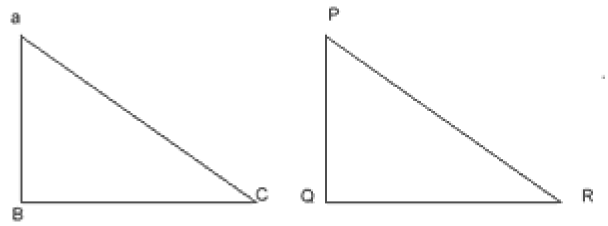
= AC X (AD + CD)

= AC X AC

= AC^2

Or, $AC^2 = AB^2 + BC^2$

Theorem 8.4 (Converse of Pythagoras Theorem): - In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



Given:- In $\Delta ABC, AB^2 + BC^2 = AC^2$

To prove:- $\angle ABC = 90^0$

Construction:- A triangle PQR is constructed such that PQ = AB, QR = BC and $\angle PQR = 90^0$

Proof:- In $\Delta PQR, \angle Q = 90^0$

$\therefore PR^2 = PQ^2 + QR^2$ [*Pythagoras Theorem*]

Or, $PR^2 = AB^2 + BC^2$---------------(i) [PQ = AB, QR = BC]

But $AC^2 = AB^2 + BC^2$ ------------(ii) (given)

$\therefore PR^2 = AC^2$ [*from (i) and (ii)*]

Or, PR = AC

Or, $Or, \Delta ABC \cong \Delta PQR$ (*SSS Congruency*)

$\therefore \angle B = \angle Q = 90^0$ (*CPCT*)

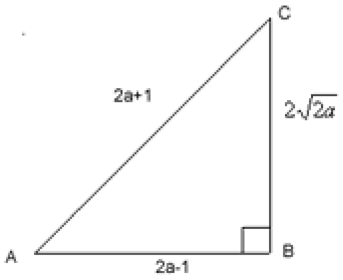
Hence $\angle ABC = 90^0$

Example 13. Determine whether the triangle having sides (2a – 1) cm, $2\sqrt{2a}\ cm$ and (2a + 1) cm is a right angled triangle.

Sol:- Let AB = (2a - 1) cm,

$BC = 2\sqrt{2a}\ cm$

AC = (2a + 1) cm



$AB^2 + BC^2 = (2a-1)^2 + (2\sqrt{2a})^2$

$= 4a^2 - 4a + 1 + 8a$

$= 4a^2 = 4a + 1$

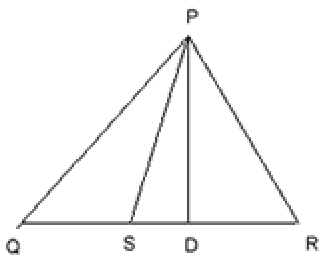
$= (2a+1)^2$

$= AC^2$

$\therefore\ \Delta ABC$ is a right angled triangle.

Example 14. In an equilateral triangle PQR, the side QR is trisected at S. prove that $9\ PS^2 = 7\ PQ^2$

Solution:-



Given:- In an equilateral $\Delta PQR,\ QR$ is trisected at S.

To Prove:- $9\ PS^2 = 7\ PQ^2$

Construction:- $PD \perp QR$ is drawn

Proof:- QD = DR = QR/2 ------------(i)

[$\perp$ Drawn on the base of an equilatera l Δ bisect it]

Side QR is trisected at S(given)

$$\left.\begin{aligned}\therefore QS &= \frac{1}{3}QR \\ SR &= \frac{2}{3}QR\end{aligned}\right\}---------(ii)$$

In $\Delta PSR, \angle R$ is acute

$$\therefore PS^2 = PR2 + SR2 - 2 \qquad RSRD$$

$$PS^2 = PR^2 + \left(\frac{2}{3}QR\right)^2 - 2.\frac{2}{3}QR \times \frac{QR}{2}$$

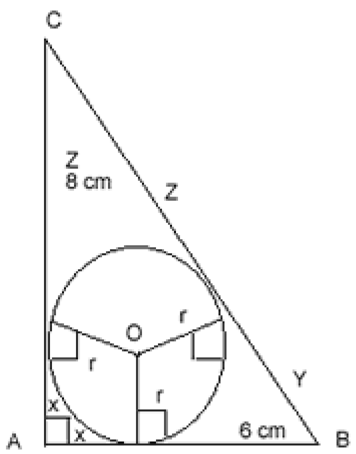
$$PS^2 = PR^2 + \frac{4}{9}QR^2 - \frac{2}{3}QR^2$$

$$PS^2 = PQ2 + \frac{4}{9}PQ^2 - \frac{2}{3}PR^2$$

$$PS^2 = \frac{9PQ^2 + 4PQ^2 - 6PQ^2}{9}$$

$$9PS^2 = 7PQ^2$$

Example 15. In the given figure, ABC is right angled triangle with the AB = 6cm and AC = 8cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.



Solution:- In right ΔCAB

$BC^2 = AB^2 + AC^2$ [By Pathagoras theorem]

$= 6^2 + 8^2$

$= 36 + 64$

$= 100$

$BC = \sqrt{100} = 10cm$

$S = \dfrac{6+8+10}{2} = 12cm$

$Area\ of\ \Delta ABC = \dfrac{1}{2} \times base \times altitude$

$= 1/2 \text{ X } 6 \text{ X } 8 = 24cm^2$

$\therefore$ radius, $r = \dfrac{Area\ of\ \Delta ABC}{S}$

= 24/12

= 2cm

Example 16. ABC is a right triangle, right angled at C. If p is the length of the perpendicular from C to AB and a, b, c have the usual meaning, then prove that

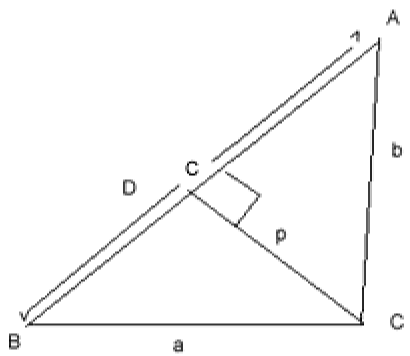
1. pc = ab
2. $Or, \quad \dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

Solution:- (i) Area of ΔABC, taking BC as base = 1/2 X BC X AC

= 1/2ab--------(i)

Area of ΔABC, taking AB as base = 1/2 X AB X CD

= 1/2 cp------(ii)



$\therefore$ from (i) and (ii) 1/2 ab = 1/2 cp

Or, pc = ab

Or, c = ab/p---------(iii)

(ii) In right $\Delta ABC,$,

$$C^2 = a^2 + b^2$$

$$\left(\frac{ab}{p}\right)^2 = a^2 + b^2$$

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} = \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2}$$

$$Or, \quad \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Exercise - 15

1. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

2. P and Q are points on the side CA and CB respectively of a ΔABC right angled at C.

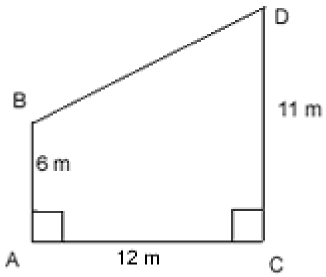
Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$.

3. In ΔABC , if AD is the median, Show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$

4. PQR is an isosceles right triangle, right angled at R. Prove that $PQ^2 = 2PR^2$.

5. In a $\Delta ABC, \angle B$ is an acute angle and $AD \perp BC.$ Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.

6. In the adjoining figure, find the length of BD, If $AB \perp AC \text{ and } CD \perp AC.$

1. 

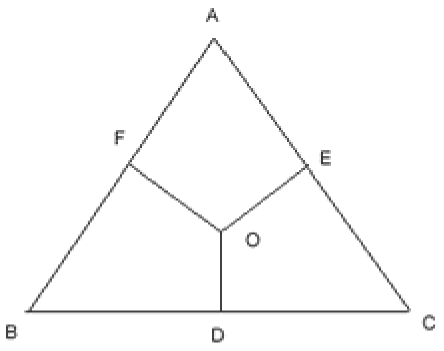
7. Prove that the altitude of an equilateral triangle of side $2a \text{ is } a\sqrt{3}.$

8. P and Q are the midpoint of the sides CA and CB respectively of ΔABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

9. In a triangle ABC, AD is perpendicular on BC. Prove that $AB^2 + CD^2 = AC^2 + BD^2$

10. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

11. In adjoining figure, OD, OE and OF are respectively perpendiculars to the sides BC, CA and AB from any point O in the interior of the triangle Prove that
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



12. O is any point in the insertor of a rectangle ABCD. Prove that interior $OB^2 + OD^2 = OC^2 + OA^2$

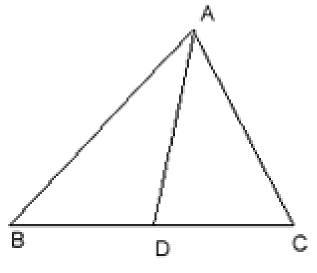
Answers

(6) 13m

Internal Bisector of an angle of a Triangle

1. The internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.
2. If a line-segment drawn from the vertex of a triangle to its opposite side and divides it in the ratio of the sides containing the angle, then the line segment bisect the angle of the vertex.

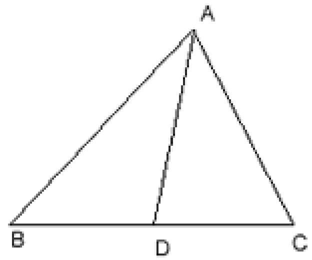
Example 17. In the adjoining fig AD is the bisector of $\angle A$. If BD = 4cm, DC = 3cm and AB = 6cm, determine AC.



Solution:- In $\Delta ABC, AD$ is the bisector of $\angle A$.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$
$$\therefore \frac{6}{AC} = \frac{4}{3}$$
$$\therefore AC = \frac{6 \times 3}{4}$$
$$= 4.5cm$$

Example 18. In the adjoining fig, AD is bisector of $\angle A$. If AB = 5.6cm, AC = 4cm, DC = 3cm, find BC.



Solution:- In $\Delta ABC, AD$ is the bisector of $\angle A$.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$
$$\frac{5.6}{4} = \frac{BD}{3}$$
$$\therefore BD = \frac{5.6 \times 3}{4} = 4.2cm$$
$$BC = BD + DC$$
$$= 4.2 + 3cm$$
$$= 7.2$$

Exercise - 16

1. In ΔABC. the bisector of $\angle B$. intersects the side AC at D. A line parallel to side AC intersects line segment AB, DB and CB at points P, R and Q respectively. Prove that

1. AB X CQ = BC X AP
2. PR X BQ = QR X BP

2. ABCD is a quadrilateral in which AB = AD. The bisector of $\angle BAC$ and $\angle CAD$ intersects the side BC and CD respectively at E and F. Prove that the segment EF is parallel to the diagonal BD.

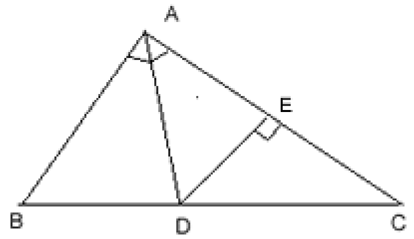
3. In $\Delta ABC, \angle B = 2\angle C$ and the bisector of $\angle B$. intersects AC at D. Prove that $\frac{BD}{DA} = \frac{BC}{BA}$

4. If the diagonal BD of a quadrilateral ABCD bisects both $\angle B \text{ and } \angle D,$ show that $\frac{AB}{BC} = \frac{AD}{CD}$

5. D is the midpoint of side BC of ΔABC. DE and DF are respectively bisectors of $\angle BDA \text{ and } \angle CDA$ such that E and F lie on AB and AC, respectively. Prove that EF || BC.

6. O is a point inside a ΔABC. The bisector of $\angle AOB, \angle BOC \text{ and } \angle COA$ meet the sides AB, BC and CA in points D, E and F respectively. Prove that AD. BE. CF = DB. EC. FA

7. In the adjoining figure, $\angle BAC = 90^0$, AD is bisector of $\angle BAC$. $DE \perp AC$, Prove that DE X (AB + AC) = AB X AC.



8. If the bisector of an angle of a triangle bisect the opposite side, prove that the triangle is isosceles.

9. BO and CO are respectively the bisectors of $\angle B \text{ and } \angle C \text{ of } \Delta ABC$. AO is produced to meets BC at P. Show that

1. $\frac{AB}{BP} = \frac{AO}{OP}$
2. $\frac{AC}{CP} = \frac{AO}{OP}$
3. $\frac{AB}{AC} = \frac{BP}{CP}$
4. AP is the bisector of $\angle BAC$