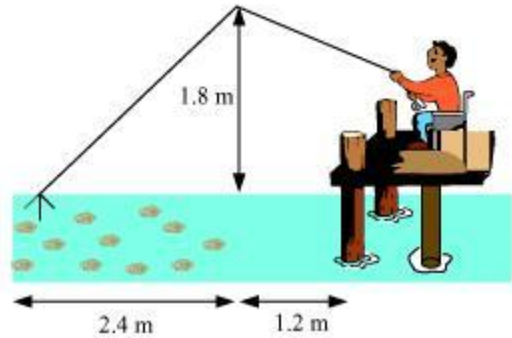
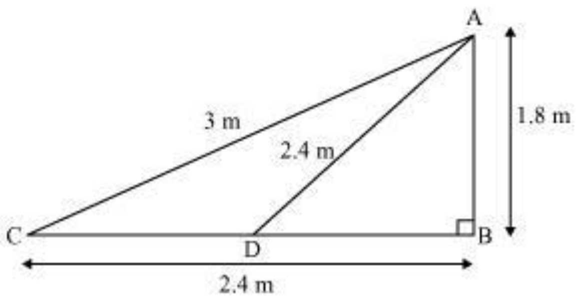
CBSE TEST PAPER SOLVED

CLASS - X Mathematics (Similar Triangle): Optional Exercise Solved

1. Q. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?





Δ ABC, $AC^2 = AB^2 + BC^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$

$AB^2 = (3.24 + 5.76)\text{ m}^2 \Rightarrow AB = \sqrt{9.00}\text{ m}^2$

AB = 3m

Thus, the length of the string out is 3 m.

String pulled in 12 seconds at the rate of 5 cm per second. = $12 \times 5 = 60$ cm = 0.6 m

String above water =AD = (AC – 0.6) m = 3-0.6= 2.4 m

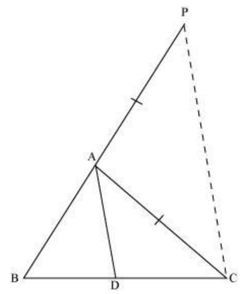
In ΔADB, $AB^2 + BD^2 = AD^2$

$(1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$ $\quad BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$ $\quad$ BD = 1.587 m

Horizontal distance of fly after 12 sec. = BD + 1.2 m = (1.587 + 1.2) m = 2.787 m = 2.79 m

2. Q. In the given figure, D is a point on side BC of ΔABC such that BD/CD = AB/AC

Prove that AD is the bisector of $\angle BAC$.



Let us extend BA to P such that AP = AC. Join PC.

It is given that, BD/CD = AB/AC $\Rightarrow$ BD/CD = AP/AC

By using the converse of basic proportionality theorem, we obtain

AD || PC

$\Rightarrow \angle BAD = \angle APC$ (Corresponding angles) … (1)

And, $\angle DAC = \angle ACP$ (Alternate interior angles) … (2)

By construction, we have AP = AC

$\Rightarrow \angle APC = \angle ACP$ … (3)

From (1), (2), and (3), we obtain

$\angle BAD = \angle APC$ $\Rightarrow$ AD is the bisector of the angle BAC.

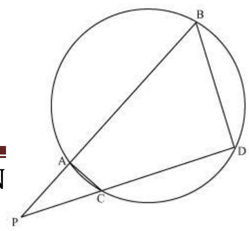
3. Q. In the given figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$ (ii) PA.PB = PC.PD

Sol: (i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ (Common)

$\angle PAC + < CAB = 180^0$ also $< CAB + \angle PDB = 180^0$

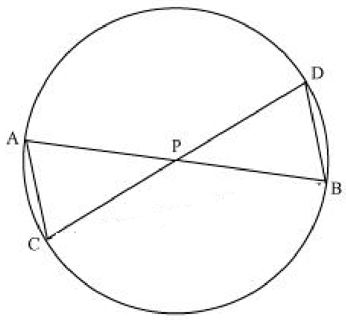


∠PAC = ∠PDB

∴ ΔPAC ~ ΔPDB (AA similarity)

(ii)We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB} \quad \Rightarrow \frac{PA}{PD} = \frac{PC}{PB} \qquad PA.PB = PC.PD$$



Q. In the given figure, two chords AB and CD intersect each other at the point P. prove that:

(i) ΔAPC ~ ΔDPB (ii) AP.BP = CP.DP

(i) In ΔAPC and ΔDPB,

∠APC = ∠DPB (Vertically opposite angles)

∠CAP = ∠BDP (Angles in the same segment)

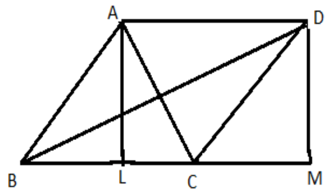
ΔAPC ~ ΔDPB (By AA similarity)

(ii) We have ΔAPC ~ ΔDPB

We know that the corresponding sides of similar triangles are proportional.

$$\therefore \frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD} \quad \Rightarrow \frac{AP}{DP} = \frac{PC}{PB} \qquad AP.PB = PC.DP$$

4. Q. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.



Sol: Let ABCD be a parallelogram.

Draw perpendicular DM on extended side BC and AL on side BC.

In Δ ABL and Δ DMC

<L = <N =90^0 AB=DM and AL=DM

Δ ABL ≅ Δ DMC (RHS) BL = CM (CPCT)

Applying Pythagoras theorem in ΔALC, we obtain $AC^2 = AL^2 + LC^2$ … (*i*)

Applying Pythagoras theorem in ΔDCM, we obtain $BD^2 = BM^2 + DM^2$ ----------(ii)

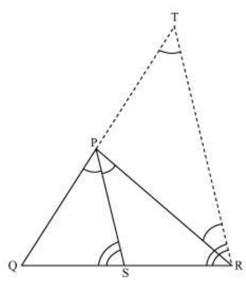
Adding (i) and (ii)

$AC^2 + BD^2 = AL^2 + LC^2 + BM^2 + DM^2$

= AB^2 + ~~BL^2~~ + $(BC-BL)^2$ + $(BC + CM)^2$ + DC^2 – ~~CM^2~~ [BL= CM]

= $AB^2 + BC^2 + BL^2$ - ~~2BC.BL~~ + BC^2 ~~+ CM^2~~ + ~~2BC .CM~~ + DC^2 – ~~CM^2~~

$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$



5. Q. In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that QS/SR = PQ/PR

Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠PTR also, ∠SPR = ∠PRT

But, ∠QPS = ∠SPR

∠PTR = ∠PRT ⇒ ∴ PT = PR

By using basic proportionality theorem for $\frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$ ΔQTR, PS || TR

6. Q. In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, BD⊥ AC Prove that:

(i) $DM^2 = DN.MC$ (ii) $DN^2 = DM.AN$

In Δ BDM and DMC

<M = <M

<DCM +<MDC = <BDM +<MDC =90^0

⇒<DCM = <BDM

Δ BDM ~ Δ DMC (AA similarity)

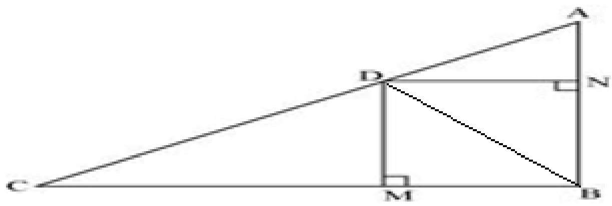
DM/BM = MC/DM

But, BM=DN

DM/ DN = MC/DM

$DM^2 = DN.MC$ proved.

Similarly we can prove $DN^2 = DM.AN$



7. Q. In the given figure, ABC is a triangle in which ∠ABC> 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

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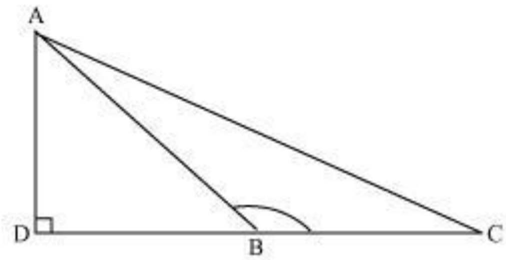
Sol: $AC^2 = AD^2 + DC^2$

$= AB^2 - DB^2 + (DB+BC)^2$

$= AB^2 - \cancel{DB^2} + \cancel{DB^2} + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$



Q. 8 : In the given figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that

$AC^2 = AB^2 + BC^2 - 2BC.BD$.

Sol: In ΔADC, $AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AB^2 - DB^2 + DC^2$

$\Rightarrow AC^2 = AB^2 - DB^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AB^2 - \cancel{DB^2} + BC^2 + \cancel{BD^2} - 2BC\ BD$

$AC^2 = AB^2 + BC^2 - 2BC.BD$ Proved