Class 10 Arithmetic progression CBSE Test Paper-3

1. How many terms of the AP: 9, 17, 25, …….. Must be taken to give a sum of 636?

Ans: a = 9, d = 8 and

S_n = 636

$\Rightarrow \frac{n}{2}$ [2a + (n-1)d] = 636 $\Rightarrow \frac{n}{2}$ [2 x 9 + (n - 1) x 8] = 636 $\Rightarrow$ 1272 = 18n + 8n^2 – 8n

$\Rightarrow$ 8n^2 + 10n – 1272 = 0 $\Rightarrow$ 4n^2 + 5n – 636 = 0

D = b^2 – 4ac = 25 – 4 x 4 x – 636 = 25 + 10176 = 10201

$\Rightarrow \sqrt{D} = \sqrt{10201}$ = 101

n = $\frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm 101}{2 \, x \, 4} \Rightarrow$ n = $\frac{106}{8} = \frac{53}{4}$ and $-\frac{96}{8} = -12$

As 12 is an integer so number of term will be 12

2. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans: a = 5, a_n = l = 45 and S_n = 400

S_n = $\frac{n}{2}$ [a + l] $\Rightarrow$ 400 = $\frac{n}{2}$ [5 + 45] $\Rightarrow \frac{800}{50}$ = n $\Rightarrow$ n = 16

Now, a_n = a + (n - 1) d $\Rightarrow$ 45 = 5 + (16 - 1)d $\Rightarrow$ d = $\frac{40}{15} = \frac{8}{3}$

3. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is the sum?

Ans: a = 17,d = 9, a_n = 350 $\Rightarrow$ 17 + (n - 1) x 9 = 350 $\Rightarrow$ n = 38

S_n = $\frac{n}{2}$ [a + l] = $\frac{38}{2}$ [17 + 350] $\Rightarrow$ 19 x 367 = 6973

4. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Ans: n = 22, d = 7

22nd term is 149 $\Rightarrow$ a + 21d

= 149 ⇒ a + 21 x 7 = 149

⇒ a = 149 – 147 = 2

$S_n = \frac{n}{2}$ [a + l] ⇒ $S_{22} = \frac{22}{2}$ [2 + 149] = 11[2 + 149] ⇒ 1661

5. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans: t_2 = 14 and t_3 = 18 ⇒ d = t_3 - t_2 = 18 – 14 = 4

a = t_2 – d = 14 – 4 = 10

The sum of first 51 terms = 25.5 [2 x 10 + 50 x 4] = 5610

6. The ratio of the sums of m and n terms of an AP is $m^2 : n^2$. show that the ratio of the m^{th} and n^{th} terms is (2m - 1) : (2n - 1)

Ans: Given: The ratio of the sums of m and n terms of an AP is $m^2 : n^2$

$$\frac{m^2}{n^2} = \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} \Rightarrow \frac{m}{n} = \frac{[2a+(m-1)d]}{[2a+(n-1)d]}$$

⇒ $m[2a+(n-1)d] = n[2a+(m-1)d]$ ⇒ 2am + $m(n-1)d$ = 2an + n $(m-1)d$

⇒ 2a (m – n) = d [n $(m-1)$ - m$(n-1)$] ⇒ 2a (m – n) = d [mn - n – mn + m]

⇒ 2a (m – n) = d [m - n] ⇒ d = 2a

Now, $\frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)x\,2a}{a+(n-1)\,x\,2a} = \frac{a+2am-2a}{a+2an-2a} = \frac{2am-a}{2an-a} = \frac{a(2m-1)}{a(2n-1)} = \frac{(2m-1)}{(2n-1)}$

Hence, ratio of m^{th} and n^{th} term is $2m - 1 : 2n - 1$.

7. In an AP, the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$, find its 25^{th} term

Ans: the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$

$S_1 = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow S_1 = a_1 = 4$ and $S_2 = \frac{3\,x\,2^2}{2} + \frac{5\,x\,2}{2} = 11 \Rightarrow a_2 = S_2 - S_1 = 11 - 4 = 7$

Now, d = a_2 - a_1 = 7 – 4 = 3 ;

25^{th} term of this AP = a + 24d = 4 + 24 x 3 = 4 + 72 = 76

8. Prove that the n^{th} of an AP cannot be $n^2 + 1$. Justify your answer.

Ans: a_n is the n^{th} term of an A.P. If $a_n + a_{n-1}$ = constant

If possible, $a_n = n^2 + 1$

$a_{n-1} - a_n = (n^2 + 1) - [(n - 1)^2 + 1]$

$= (n^2 + 1) - (n^2 - 2n + 2) = n^2 + 1 - n^2 + 2n - 2 = 2n - 1$

$\therefore$ $a_{n-1} - a_n \neq$ constant Thus, $a_n = n^2 + 1$ cannot be the n term of A.P

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans: the sum of first 7 terms of an AP is 49 $\Rightarrow$ 49 = 3.5[2a + 6d] $\Rightarrow$ a + 3d = 7 ------- (i)

the sum of first 17 terms of an AP is 289 $\Rightarrow$ 289 = 8.5[2a + 16d] $\Rightarrow$ a + 8d = 17 ------ (ii)

Solving them, a + 8d – a – 3d = 17 – 7 $\Rightarrow$ 5d = 10 $\Rightarrow$ d = 2

Putting this value in expression (i) , a + 3 x 2 = 7 $\Rightarrow$ a = 1

The sum of first n terms = $\frac{n}{2}\,[\,2a + (n-1)d = \frac{n}{2}\,[\,2\,x\,1 + (n-1)\,x\,2]$

$\Rightarrow \frac{n}{2}\,[\,2 + (n-1)\,x\,2] \Rightarrow n + (n-1)n = n + n^2 - n = n^2$

10. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth term.

Ans: the sum of the first n terms of an AP is $4n - n^2$

$a_1 = S_1 = 4 \times 1 - 1^2 = 3 \Rightarrow a = 3$

The sum of first two terms = $S_2 = 4 \times 2 - 2^2 = 4$

What is the second term = $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a_1 = 1 - 3 = -2$

$a_3 = a + 2d = 3 + 2 \times -2 = 3 - 4 = -1$

$a_{10} = a + 9d = 3 + 9 \times -2 = 3 - 18 = -15$

$a_n = a + (n-1)d = 3 + (n-1) \times -2 = 3 - 2n + 2 = 5 - 2n$