

## ARITHMETIC PROGRESSIONS

**One of the endlessly alluring aspects of mathematics is that its thorniest paradoxes have a way of blooming into beautiful theories**

1. The fourth term of an AP is 0. Prove that its 25<sup>th</sup> term is triple its 11<sup>th</sup> term.

**Ans:**  $a_4 = 0$   
 $\Rightarrow a + 3d = 0$   
 T.P  $a_{25} = 3(a_{11})$   
 $\Rightarrow a + 24d = 3(a + 10d)$   
 $\Rightarrow a + 24d = 3a + 30d$   
 RHS sub  $a = -3d$   
 $-3d + 24d = 21d$   
 LHS  $3a + 30d$   
 $-9d + 30d = 21d$   
 LHS = RHS  
 Hence proved

2. Find the 20<sup>th</sup> term from the end of the AP 3, 8, 13.....253.

**Ans:** 3, 8, 13 ..... 253  
 Last term = 253  
 $a_{20}$  from end  
 $= l - (n-1)d$   
 $253 - (20-1)5$   
 $253 - 95$   
 $= 158$

3. If the p<sup>th</sup>, q<sup>th</sup> & r<sup>th</sup> term of an AP is x, y and z respectively, show that  $x(q-r) + y(r-p) + z(p-q) = 0$

**Ans:** p<sup>th</sup> term  $\Rightarrow x = A + (p-1)D$   
 q<sup>th</sup> term  $\Rightarrow y = A + (q-1)D$   
 r<sup>th</sup> term  $\Rightarrow z = A + (r-1)D$   
 T.P  $x(q-r) + y(r-p) + z(p-q) = 0$   
 $= \{A+(p-1)D\}(q-r) + \{A+(q-1)D\}(r-p)$   
 $+ \{A+(r-1)D\}(p-q)$   
 $A \{(q-r) + (r-p) + (p-q)\} + D \{(p-1)(q-r)$   
 $+ (q-1)(r-p) + (r-1)(p-q)\}$   
 $\Rightarrow A.0 + D\{p(q-r) + q(r-p) + r(p-q)$   
 $- (q-r) - (r-p) - (p-q)\}$   
 $= A.0 + D.0 = 0.$   
 Hence proved

4. Find the sum of first 40 positive integers divisible by 6 also find the sum of first 20 positive integers divisible by 5 or 6.

**Ans:** No's which are divisible by 6 are  
6, 12 ..... 240.

$$S_{40} = \frac{40}{2} [a + 240]$$

$$= 20 \times 246$$

$$= 4920$$

No's div by 5 or 6

30, 60 ..... 600

$$\frac{20}{2} [a + 600] = 10 \times 630$$

$$= 6300$$

5. A man arranges to pay a debt of Rs.3600 in 40 monthly instalments which are in a AP. When 30 instalments are paid he dies leaving one third of the debt unpaid. Find the value of the first instalment.

**Ans:** Let the value of I instalment be x  $S_{40} = 3600$ .

$$\Rightarrow \frac{40}{2} [a + 39d] = 3600$$

$$\Rightarrow 2a + 39d = 180 \quad - \quad 1$$

$$S_{30} = \frac{30}{2} [a + 29d] = 2400$$

$$\Rightarrow 30a + 435d = 2400$$

$$\Rightarrow 2a + 29d = 160 \quad - \quad 2$$

Solve 1 & 2 to get

$$d = 2 \quad a = 51.$$

$\therefore$  I instalment = Rs.51.

6. Find the sum of all 3 digit numbers which leave remainder 3 when divided by 5.

**Ans:** 103, 108.....998

$$a + (n-1)d = 998$$

$$\Rightarrow 103 + (n-1)5 = 998$$

$$\Rightarrow n = 180$$

$$S_{180} = \frac{180}{2} [103 + 998]$$

$$= 90 \times 1101$$

$$S_{180} = 99090$$

7. Find the value of x if  $2x + 1$ ,  $x^2 + x + 1$ ,  $3x^2 - 3x + 3$  are consecutive terms of an AP.

**Ans:**  $a_2 - a_1 = a_3 - a_2$

$$\Rightarrow x^2 + x + 1 - 2x - 1 = 3x^2 - 3x + 3 - x^2 - x - 1$$

$$x^2 - x = 2x^2 - 4x + 2$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow (x-1)(x-2) = 0$$

$$\Rightarrow x = 1 \text{ or } x = 2$$

8. Raghav buys a shop for Rs.1,20,000. He pays half the balance of the amount in cash and agrees to pay the balance in 12 annual instalments of Rs.5000 each. If the rate of interest is 12% and he pays with the instalment the interest due for the unpaid amount. Find the total cost of the shop.

**Ans:** Balance = Rs.60,000 in 12 instalment of Rs.5000 each.

$$\text{Amount of I instalment} = 5000 + \frac{12}{100} 60,000$$

$$\begin{aligned} \text{II instalment} &= 5000 + (\text{Interest on unpaid amount}) \\ &= 5000 + 6600 \left[ \frac{12}{100} \times 55000 \right] \\ &= 11600 \end{aligned}$$

$$\text{III instalment} = 5000 + (\text{Interest on unpaid amount of Rs.50,000})$$

$\therefore$  AP is 12200, 11600, 11000

$$D = \text{is } 600$$

Cost of shop = 60000 + [sum of 12 instalment]

$$= 60,000 + \frac{12}{2} [24,400 - 6600]$$

$$= 1,66,800$$

9. Prove that  $a_{m+n} + a_{m-n} = 2a_m$

**Ans:**  $a_{m+n} = a_1 + (m+n-1)d$

$$a_{m-n} = a_1 + (m-n-1)d$$

$$a_m = a_1 + (m-1)d$$

Add 1 & 2

$$\begin{aligned} a_{m+n} + a_{m-n} &= a_1 + (m+n-1)d + a_1 + (m-n-1)d \\ &= 2a_1 + (m+n+m-n-1-1)d \\ &= 2a_1 + 2(m-1)d \\ &= 2[a_1 + (m-1)d] \\ &= 2[a_1 + (m-1)d] \\ &= 2a_m. \quad \text{Hence proved.} \end{aligned}$$

10. If the roots of the equation  $(b-c)x^2 + (c-a)x + (a-b) = 0$  are equal show that a, b, c are in AP.

**Ans:** Refer sum No.12 of Q.E.

If  $(b-c)x^2 + (c-a)x + (a-b)$  have equal root.

$$B^2 - 4AC = 0.$$

Proceed as in sum No.13 of Q.E to get  $c + a = 2b$

$$\Rightarrow b - a = c - b$$

$$\Rightarrow a, b, c \text{ are in AP}$$

11. Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second two balls and so on. If 669 more balls are added, then all the balls can be arranged in the shape of a square and each of its sides then contains 8 balls less than each side of the triangle. find the initial number of balls.

**Ans:** Let there be  $n$  balls in each side of the triangle

$$\therefore \text{No. of ball (in } \Delta) = 1 + 2 + 3 + \dots = \frac{n(n+1)}{2}$$

No. of balls in each side square =  $n-8$

No. of balls in square =  $(n-8)^2$

$$\text{APQ } \frac{n(n+1)}{2} + 660 = (n-8)^2$$

On solving

$$n^2 + n + 1320 = 2(n^2 - 16n + 64)$$

$$n^2 - 33n - 1210 = 0$$

$$\Rightarrow (n-55)(n+22) = 0$$

$$n = -22 \text{ (N.P)}$$

$$n = 55$$

$$\therefore \text{No. of balls} = \frac{n(n+1)}{2} = \frac{55 \times 56}{2}$$

$$= 1540$$

12. Find the sum of  $(1 - \frac{1}{n}) + (1 - \frac{2}{n}) + (1 - \frac{3}{n}) + \dots$  upto  $n$  terms.

**Ans:**  $(1 - \frac{1}{n}) + (1 - \frac{2}{n}) -$  upto  $n$  terms

$$\Rightarrow [1+1+\dots+n \text{ terms}] - [\frac{1}{n} + \frac{2}{n} + \dots + n \text{ terms}]$$

$n - [S_n \text{ up to } n \text{ terms}]$

$$S_n = \frac{n}{2} [2a + (n-1)d] \quad (d = \frac{1}{n}, a = \frac{1}{n})$$

$$= \frac{n}{2} \left[ \frac{2}{n} + (n-1)\frac{1}{n} \right]$$

$$= \frac{n+1}{2} \quad (\text{on simplifying})$$

$$n - \frac{n+1}{2} =$$

$$= \frac{n-1}{2} \text{ Ans}$$

13. If the following terms form a AP. Find the common difference & write the next 3 terms  $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

**Ans:**  $d = \sqrt{2}$  next three terms  $3 + 4\sqrt{2}$ ,  $3 + 5\sqrt{2}$ ,  $3 + 6\sqrt{2}$  .....

14. Find the sum of  $a+b$ ,  $a-b$ ,  $a-3b$ , ..... to 22 terms.

**Ans:**  $a + b$ ,  $a - b$ ,  $a - 3b$ , up to 22 terms

$$d = a - b - a - b = 2b$$

$$S_{22} = \frac{22}{2} [2(a+b) + 21(-2b)]$$

$$11[2a + 2b - 42b]$$

$$= 22a - 440b \text{ Ans.}$$

15. Write the next two terms  $\sqrt{12}$ ,  $\sqrt{27}$ ,  $\sqrt{48}$ ,  $\sqrt{75}$  .....

**Ans:** next two terms  $\sqrt{108}$ ,  $\sqrt{147}$  AP is  $2\sqrt{3}$ ,  $3\sqrt{3}$ ,  $4\sqrt{3}$ ,  $5\sqrt{3}$ ,  $6\sqrt{3}$ ,  $7\sqrt{3}$  .....

16. If the  $p^{\text{th}}$  term of an AP is  $q$  and the  $q^{\text{th}}$  term is  $p$ . P.T its  $n^{\text{th}}$  term is  $(p+q-n)$ .

**Ans:** APQ

$$a_p = q$$

$$a_q = p$$

$$a_n = ?$$

$$a + (p-1)d = q$$

$$a + (q-1)d = p$$

$$d[p - q] = q - p \quad \text{Sub } d = -1 \text{ to get } \Rightarrow = -1 \Rightarrow a = q + p - 1$$

$$a_n = a + (n-1)d$$

$$= a + (n-1)d$$

$$= (q + p - 1) + (n-1) - 1$$

$$a_n = (q + p - n)$$

17. If  $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$  are in AP find x.

**Ans:**  $\frac{1}{x+2}, \frac{1}{x+3}, \frac{1}{x+5}$  are in AP find x.

$$\frac{1}{x+3} - \frac{1}{x+2} = \frac{1}{x+5} - \frac{1}{x+3}$$

$$\Rightarrow \frac{1}{x^2 + 5x + 6} = \frac{2}{x^2 + 8x + 15}$$

On solving we get  $x = 1$

18. Find the middle term of the AP 1, 8, 15....505.

**Ans:** Middle terms

$$a + (n-1)d = 505$$

$$a + (n-1)7 = 505$$

$$n - 1 = \frac{504}{7}$$

$$n = 73$$

$\therefore 37^{\text{th}}$  term is middle term

$$a_{37} = a + 36d$$

$$= 1 + 36(7)$$

$$= 1 + 252$$

$$= 253$$

19. Find the common difference of an AP whose first term is 100 and sum of whose first 6 terms is 5 times the sum of next 6 terms.

**Ans:**  $a = 100$

$$\text{APQ } a_1 + a_2 + \dots + a_6 = 5(a_7 + \dots + a_{12})$$

$$6 \left( \frac{a_1 + a_6}{2} \right) = 5 \times 6 \left( \frac{a_7 + a_{12}}{2} \right)$$

$$\Rightarrow a + a + 5d = 5[a + 6d + a + 11d]$$

$$\Rightarrow 8a + 80d = 0 \quad (a = 100)$$

$$\Rightarrow d = -10.$$

20. Find the sum of all natural no. between 101 & 304 which are divisible by 3 or 5. Find their sum.

**Ans:** No let 101 and 304, which are divisible by 3.

102, 105.....303 (68 terms)

No. which are divisible by 5 are 105, 110.....300 (40 terms)

No. which are divisible by 15 (3 & 5) 105, 120..... (14 terms)

$\therefore$  There are 94 terms between 101 & 304 divisible by 3 or 5.  $(68 + 40 - 14)$

$$\therefore S_{68} + S_{40} - S_{14}$$

$$= 19035$$

21. The ratio of the sum of first  $n$  terms of two AP's is  $7n+1:4n+27$ . Find the ratio of their  $11^{\text{th}}$  terms.

**Ans:** Let  $a_1, a_2, \dots$  and  $d_1, d_2$  be the I terms and Cd's of two AP's.

$$\frac{S_n \text{ of one AP}}{S_n \text{ of II AP}} = \frac{7n+1}{4n+27}$$

$S_n$  of II AP

$$\frac{\frac{n}{2} [a_1 + (n-1)d_1]}{\frac{n}{2} [a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27}$$

We have sub.  $n = 21$ .

$$\frac{2a_1 + 20d_1}{2a_2 + 20d_2} = \frac{7 \times 21 + 1}{4(21) + 27}$$

$$\Rightarrow \frac{a_1 + 10d_1}{a_2 + 10d_2} = \frac{148}{111}$$

$$= \frac{4}{3}$$

$\therefore$  ratio of their  $11^{\text{th}}$  terms =  $4:3$ .

22. If there are  $(2n+1)$  terms in an AP, prove that the ratio of the sum of odd terms and the sum of even terms is  $(n+1):n$

**Ans:** Let  $a, d$  be the I term & Cd of the AP.

$$\therefore a_k = a + (k-1)d$$

$s_1 =$  sum to odd terms

$$s_1 = a_1 + a_3 + \dots + a_{2n+1}$$

$$s_1 = \frac{n+1}{2} [a_1 + a_{2n+1}]$$

$$= \frac{n+1}{2} [a_1 + 2nd]$$

$$s_1 = (n+1)(a + nd)$$

$s_2 =$  sum to even terms

$$s_2 = a_2 + a_4 + \dots + a_{2n}$$

$$s_2 = \frac{n}{2} [a_2 + a_{2n}]$$

$$= \frac{n}{2} [a + d + a + (2n-1)d]$$

$$= n[a + nd]$$

$$\therefore s_1 : s_2 = \frac{(n+1)(a + nd)}{n(a + nd)}$$

$$= \frac{n+1}{n}$$

23. Find the sum of all natural numbers amongst first one thousand numbers which are neither divisible 2 or by 5

**Ans:** Sum of all natural numbers in first 1000 integers which are not divisible by 2 i.e. sum of odd integers.

$$1 + 3 + 5 + \dots + 999$$

$$n = 500$$

$$S_{500} = \frac{500}{2} [1 + 999]$$

$$= 2,50,000$$

No's which are divisible by 5

$$5 + 15 + 25 \dots + 995$$

$$n = 100$$

$$S_n = \frac{100}{2} [5 + 995]$$

$$= 50 \times 1000 = 50000$$

$$\therefore \text{Required sum} = 250000 - 50,000$$

$$= 200000$$