Paper: 04 Class-X-Math: Summative Assessment - I

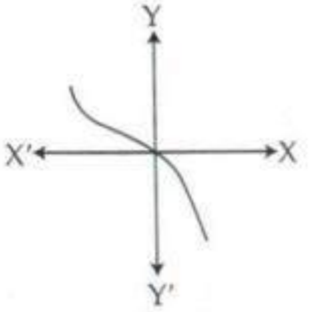
Total marks of the paper: 90 Total time of the paper: 3.5 hrs

Questions:

1] Δ ABC is right angled at A, the value of tan B $\times$ tan C is : [Marks:1]

A. None of the above
B. -1
C. 0
D. 1

2] The graph of a polynomial y = f(x) is shown in fig. The number of zeroes of f(x) is : [Marks:1]



A. 2
B. 0
C. $\frac{3}{2}$
D. 1

3] If mode = 80 and mean = 110 then the median is: [Marks:1]

A. 90
B. 120
C. 110
D. 100

4] The following pairs of linear equations 2x + 5y = 3 and 6x + 15y = 12 represent : [Marks:1]

A. None from a, b, c
B. Coincident lines
C. Intersecting
D. Parallel lines

5] $3\cos\theta = 1$, then the value of $\operatorname{cosec}\theta$ is : [Marks:1]

A. $\frac{4}{3}\sqrt{2}$
B. $2\frac{\sqrt{3}}{3}$
C. $2\sqrt{2}$
D. $\frac{3}{2\sqrt{2}}$

6] $\Delta ABC \sim \Delta PQR$, M is the mid-point of BC and B is the mid-point of QR. If the area of Δ ABC = 100sq. cm and the area of Δ PQR = 144 sq. cm If AM = 4 cm then PN is: [Marks:1]

A. 5.6 cm
B. 4 cm
C. 12 cm
D. 4.8 cm

7] If two positive integers a and b are written as $a = x^2 y^2$ and $b = xy^2$; x,y are prime numbers then HCF (a, b) is : [Marks:1]

A. x^2y^2
B. x^2y^3
C. xy
D. xy^2

8] For the decimal number $0.\overline{7}$, the rational numbers is: [Marks:1]

A. $\frac{1}{3}$
B. $\frac{111}{167}$
C. $\frac{33}{50}$
D. $\frac{7}{9}$

9] Find the zeroes of the quadratic polynomial $x^2 + 7x + 12$ and verify the relationship between the zeroes and its coefficients. [Marks:2]

10] Can the number 6^n, n being a natural number end with the digit 5? Given reasons. [Marks:2]

11] Find the median of the following data : [Marks:2]

Marks	0 - 10	10 - 30	30 - 60	60 - 80	80 - 100
Frequency	5	15	30	8	2

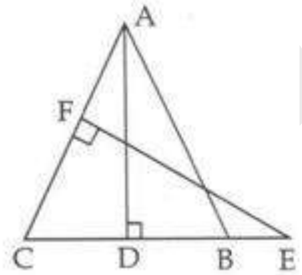
12] For what value of k, will the following system of linear equations have infinitely many solutions? $2x + 3y = 4$ and $(k + 2)x + 6y = 3k + 2$. [Marks:2]

13] Given that sin (A + B) = sin A cos B + cos A sin B, find the value of sin 75° [Marks:2]

OR

It cosec $\theta = \frac{13}{12}$, find the value of $\cot\theta + \tan\theta$.

14] In the given figure. E is a point on side CB produced of an isosceles $\triangle$ ABC with AB = BC. If AD $\perp$ BC and EF $\perp$ AC. Prove that $\triangle ABD \sim \triangle ECF$. [Marks:2]



15] Rekha's mother is five times as old as her daughter Rekha. Five years later, Rekha's mother will be three times as old as her daughter Rekha. Find the present age of Rekha and her mother's age. [Marks:3]

OR

Two numbers are in the ratio 5 :6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

16] If $\sin\theta = \frac{m}{n}$, find the value of $\frac{\tan\theta + 4}{4\cot\theta + 1}$ [Marks:3]

17] Find unknown entries a,b,c,d,e,f in the following distribution of heights of students in a class and the total number of students in the class in 50. [Marks:3]

Height in c.m	150 - 155	155 - 160	160 - 165	165 - 170	170 - 175	175 - 180
Frequency	12	b	10	d	e	2
Cumulative Frequency	a	25	c	43	48	f

18] Find the mean of the following frequency distribution. [Marks:3]

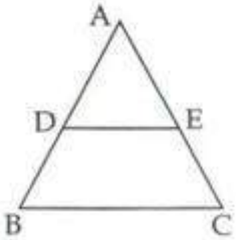
C.I.	0-100	100-200	200-300	300-400	400-500
f	2	3	5	2	3

19] Prove that $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta.\operatorname{cosec}\theta$ [Marks:3]

OR

Evaluate : $\frac{\sin 35^0}{\cos 55^0}+\frac{\cos 55^0.\operatorname{cosec}35^0}{\tan 5^0\ \tan 25^0\ \tan 45^0\ \tan 65^0\ \tan 85^0}$

20] In fig, DE || BC and AD : DB = 5:4, find $\frac{\text{area }\Delta DEF}{\text{area }\Delta CFB}$ [Marks:3]



21] If α, β, γ are zeroes of polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$. [Marks:3]

22] Show that $6 + \sqrt{2}$ is irrational. [Marks:3]

OR

Prove that $5 - \sqrt{3}$ is an irrational.

23] In Δ PQR, PD $\perp$ QR such that D lies on QR . If PQ = a, PR = b, QD = c and DR = d and a,b,c,d are positive units, prove that (a + b) (a- b)=(c+d)(c-d). [Marks:3]

24] Solve for x and y: [Marks:3]

$$\frac{5}{x-1}+\frac{1}{y-2}=2$$
$$\frac{6}{x-1}-\frac{3}{y-2}=1$$

25] Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q. [Marks:4]

26] State and Prove Basic proportionality theorem. [Marks:4]

OR

Prove that ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides.

27] Prove that: [Marks:4]

$$\frac{2}{\cos^2\theta}-\frac{1}{\cos^4\theta}-\frac{2}{\sin^2\theta}+\frac{1}{\sin^4\theta}=\cot^4\theta-\tan^4\theta$$

28] Solve the following equations graphically: [Marks:4]
x -y = 1 and 2x + y = 8. Shade the region between the two lines and y - axis.

29] Prove that: [Marks:4]

$$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{2}{2\sin^2\theta-1}$$

OR

Without using trigonometric tables evaluate :

$$\frac{3\tan 35^\circ \tan 40^\circ \tan 50^\circ \tan 55^\circ - \frac{1}{2}\tan^2 60^\circ}{4\left(\cos^2 39^\circ + \cos^2 51^\circ\right)}$$

30] Calculate the mode of the following frequency distribution table. [Marks:4]

Marks	No. of Students
above 25	52
above 35	47
above 45	37
above 55	17
above 65	8
above 75	2
above 85	0

31] If the remainder on division $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$. [Marks:4]

32] Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. [Marks:4]

33] Prove that $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$ [Marks:4]

34] For the data given below draw less than ogive graph. [Marks:4]

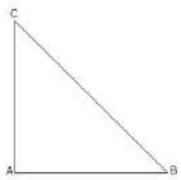
Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of students	7	10	23	51	6	3

Solutions paper - 4 :

1]

$\tan B = \frac{AC}{AB}$

$\tan C = \frac{AB}{AC}$



$\tan B \times \tan c = \frac{AC}{AB} \times \frac{AB}{AC} = 1$

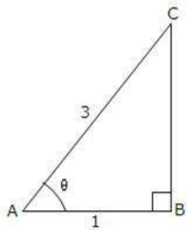
2] Number of zeros is one as the graph touches the x-axis at one point.

3] 3Median = Mode + 2 mean

Median = $\frac{80 + 220}{3} = \frac{300}{3} = 100$

4] Since $\frac{2}{6} = \frac{5}{15} \neq \frac{3}{12}$ Therefore, lines are parallel.

5] BC = $\sqrt{3^2 - 1} = \sqrt{8} = 2\sqrt{2}$



$\cos ec\theta = \frac{3}{2\sqrt{2}}$

6] $\frac{ar\,(\Delta ABC)}{ar\,(\Delta PQR)} = \frac{AM^2}{PN^2}$ $\Rightarrow \frac{100}{144} = \frac{4^2}{PN^2}$ Therefore, PN = 4.8 cm

7] $a = x^2y^2 = x \times x \times y \times y$

$b = xy^2 = x \times y \times y$

$HCF(a,b) = x \times y \times y = xy^2$

8] Let $x = 0.\bar{7}$

Then, x=0.7777…… … (1) Here ,the number of digits recurring is only 1,so we multiply both sides of the equation by 10.

$\therefore 10x = 7.777\ldots$ … (2)

Subtracting(1) from(2),we get

9x=7 $\Rightarrow$ x = /9

9] $x^2 + 7x + 12 = (x + 3)(x + 4)$

$\therefore$ -3 and -4 are zeroes of the polynomial

Sum of zeros = -3 - 4 = -7= $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeros = (-3) (-4) = $\frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

10] Let it possible 6n ends with digit 0

$\Rightarrow$ 6n = 10 $\times$ q

(2 $\times$ 3)n = 2 $\times$ 5 $\times$ q

2n $\times$ 3n = 2 $\times$ 5 $\times$ q $\Rightarrow$ is a prime factor of 2n $\times$ 3n

Which is not possible 2n $\times$ 3n can have only 2 and 3 are prime factors. Hence, it is not possible the number ends with digit 5.

11]

Marks	f	cf
0 - 10	5	5
10 - 30	15	20
30 - 60	30	50
60 - 80	8	58
80 - 100	2	6

N= $\sum f_i = 60$ Here, N=60 So, N/2=30

The cumulative frequency is just greater than N/2=30 is 50 and the corresponding class is 30-60.

Hence,30-60 is the median class.

Therefore, l=30,f=30 ,F=20,h=30

Now, Median= $l + \dfrac{\frac{N}{2} - F}{f} \times h = 30 + \dfrac{30 - 20}{30} \times 30$ Median = 40

12] Condition for infinitely many solution

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{a_1}{a_2}$ (½)

$\dfrac{2}{k+2} = \dfrac{3}{6} = \dfrac{4}{3k+2}$

$\dfrac{2}{k+2} = \dfrac{1}{2}$ similarly, $\dfrac{4}{3k+2} = \dfrac{1}{2}$

k + 2 = 4 3k + 2 = 8

k = 2 k = 2

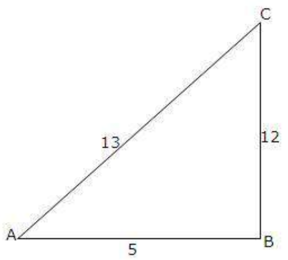
$\therefore$ k = 2 is the common solution.

13] Sin(45 + 30) = sin 45o cos 30o + cos 45o sin 30o

Sin 75o = $\dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2}$ $= \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$ $= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

OR

Cosec = $\frac{13}{12}$



AB = $\sqrt{13^2 - 12^2} = \sqrt{25} = 5$

Cot θ + tan θ = $\frac{5}{12} + \frac{12}{5}$ = $\frac{25 + 144}{60}$ = $\frac{169}{60}$

14] In Δ ABD and Δ ECF

$\angle$D = $\angle$F = 90o

$\angle$B = $\angle$C $(\because AC = AB)$

By AA similarity, Δ ABD $\sim$ Δ ECF

15] Let Rekha's Age be 'x'years

And her mother's age be 'y' years

y = 5x as per given data ... (1)

After 5 years

y + 5 = 3(x+5)

y - 3x = 10 ... (2)

Solving (1) and (2) equation.

Rekha's age = 5 years

Mother's age = 25 years

OR,

Let the two number be 5x, 6x

$$\frac{5x-8}{6x-8}=\frac{4}{5}$$
$$\Rightarrow 25x-40=24x-32$$
$$\Rightarrow 25x-24x=-32+40$$
$$\Rightarrow x=8$$

Two numbers are 40,48.

16] Sin $\theta = \frac{m}{n} \Rightarrow \tan\theta = \frac{m}{\sqrt{n^2-m^2}}$

$$\frac{\tan\theta+4}{4\cot\theta+1}=\frac{\frac{m}{\sqrt{n^2-m^2}}+4}{\frac{4\sqrt{n^2-m^2}}{m}+1}$$
$$=\frac{\frac{m+4\sqrt{n^2-m^2}}{\sqrt{n^2-m^2}}}{\frac{4\sqrt{n^2-m^2}+m}{m}}=\frac{m+4\sqrt{n^2-m^2}}{\sqrt{n^2-m^2}}\times\frac{m}{4\sqrt{n^2-m^2}+m}$$
$$=\frac{m}{\sqrt{n^2-m^2}}$$

17] a=12 a+b=25 $\Rightarrow$b=13 c=25+10=35 c+d=43 $\Rightarrow$d=43-35=8 43+e=48 $\Rightarrow$ e=5

f = 48 + 2 = 50

18] To calculate the mean, first obtain the column of mid value and then multiply the corresponding values of frequency and mid value.

C.I.	f	Mid value (x)	fx
0-100	2	50	100
100-200	3	150	450
200-300	5	250	1250
300-400	2	350	700
400-500	3	450	1350
	15		3850

Here $\sum f = 15$ and $\sum fx = 3850$, so the mean is given as

$$\bar{x}=\frac{\sum fx}{\sum f}=\frac{3850}{15}=256.67$$

19] $\frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)} = \frac{1-\tan^3\theta}{\tan\theta(1-\tan\theta)}$

$$= \frac{(1 - \tan\theta)(1 + \tan\theta + \tan^2\theta)}{\tan\theta(1 - \tan\theta)}$$

$$\frac{\sec^2\theta + \tan\theta}{\tan\theta} = \frac{\sec^2\theta}{\tan\theta} + 1 = \frac{\cos\theta}{\sin\theta\cos^2\theta} + 1$$

$$= \frac{1}{\sin\theta\cos\theta} + 1$$

Simplifying we get, 1 + sec θ . cosec θ

OR

$$\frac{\sin 35^o}{\sin(90 - 35^o)} + \frac{\cos 55 \times \frac{1}{\cos(90 - 35)}}{\tan 5\ \tan 25 \cot 25\ \cot 5}$$

$$= \frac{\sin 35^o}{\sin 35^o} + \frac{\cos 55\ \frac{1}{\cos 55}}{\tan 5\ \tan 25\ \cot 25\ \cot 5}$$

$$= 1 + \frac{1}{\tan 5 \tan 25 \frac{1}{\tan 5 \tan 25}}$$

$$= 1 + 1 = 2$$

20] Δ ADE ~ Δ ABC by AA similarity

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} \ldots(1)$$

$\Delta DFE \sim \Delta CFB$ by AA similarity

$$\frac{DE}{BC} = \frac{AD}{AB}$$

$$\frac{(ar\ \Delta\ DEF)}{(ar\ \Delta\ CFB)} = \frac{DE^2}{BC^2}$$

$$\frac{ar\ (\Delta DEF)}{ar\ (\Delta CFB)} = \frac{AD^2}{AB^2} = \frac{5^2}{9^2} = \frac{25}{81}$$

21] Given p(x) = 6x3 + 3x2 - 5x + 1

a = 6, b = 3, c = -5 d = 1

α, β ,r are zero. $\therefore \alpha + \beta + r = \frac{-b}{a} = \frac{-1}{2}$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-1}{6}$$

$\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \frac{-\frac{5}{6}}{-\frac{1}{6}} = 5$

22] Let $6 + \sqrt{2}$ be rational and equal to $\frac{a}{b}$

then $\frac{6 + \sqrt{2}}{1} = \frac{a}{b}$ where a and b are co primes, $b \neq 0$

$\therefore \sqrt{2} = \frac{a}{b} - 6$

$\sqrt{2} = \frac{a - 6b}{b}$ here a,b are integers $\frac{a - 6b}{b}$ is rational .Therefore, $\sqrt{2}$ is rational

$\therefore \sqrt{2}$ is rational which is a contradiction

$\therefore 6 + \sqrt{2}$ is an irrational number

OR

Let 5 - $\sqrt{3}$ be rational equal to $\frac{a}{b}$

Then 5 - $\sqrt{3} = \frac{a}{b} \Rightarrow \sqrt{3} = 5 - \frac{a}{b}$

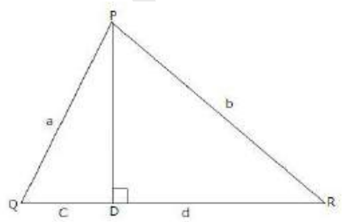
$\sqrt{3} = \frac{5b - a}{b}$

$\sqrt{3}$ is rational because $\frac{5b - a}{b}$ is rational a,b are integers

$\therefore \sqrt{3}$ is rational which is a contradiction

Hence 5 - $\sqrt{3}$ is an irrational number

23] In fig, Δ PQD, $\angle PDQ = 90^0$



Using Pythagoras thm.

PD2 = a2 - c2 ...(1)

Similarly in ΔPDR , $\angle PDR = 90^0$

PD2 = b2 - d2 ...(2)

From (1) and (2) a2 - c2 = b2 - d2

⇒a2 - b2 = c2 - d2

∴ (a + b) (a - b) = (c + d) (c - d)

24] $\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad ...(1)$

$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad ...(2)$

Multiply equation (1) by 3 and add in equation (2), we get

$$\frac{15}{x-1} + \frac{3}{y-2} + \frac{6}{x-1} - \frac{3}{y-2} = 6+1$$

$$\Rightarrow \frac{21}{x-1} = 7 \Rightarrow 7(x-1) = 21$$

$$\Rightarrow x - 1 = 3 \Rightarrow x = 4$$

Using equation (1),

$$\frac{5}{3} + \frac{1}{y-2} = 2 \Rightarrow \frac{1}{y-2} = 2 - \frac{5}{3} = \frac{1}{3}$$

⇒y - 2 = 3 ⇒y = 5

Hence x = 4, y = 5.

25] Let 5q + 2, 5q + 3 be any positive integers

(5q + 2)2 = 25q2 + 20q + 4

= 5q (5q + 4) + 4 is not of the form 5q + 2

Similarly for 2nd

(5q + 3)2 = 25q2 + 30q + 9

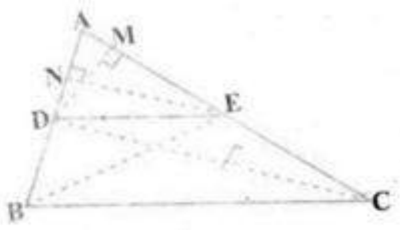
=5q(5q+6)+ 9 is not of the form 5q+3

So, the square of any positive integer cannot be of the form5q+2 or 5q+3

For any integer q

26] Statement If a line is drawn parallel of one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ration.

Given:A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively (see fig.)



To prove that $\frac{AD}{BD} = \frac{AE}{EC}$.

Construction:Let us join BE and CD and then draw DM $\perp$ AC and EN $\perp$ AB.

Proof:Now, area of $\Delta ADE \left(= \frac{1}{2} \text{base} \times \text{height}\right) = \frac{1}{2} AD \times EN.$

Letus denote the area of ΔADE is denoted as are (ADE).

So, $ar(ADE) = \frac{1}{2} AD \times EN$

Similarly, $ar(BDE) = \frac{1}{2} DB \times EN.$

$ar(ADE) = \frac{1}{2} AE \times DM$ and $ar(DEC) = \frac{1}{2} EC \times DM.$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$

and $\frac{ar(ADE)}{ar(DEG)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$

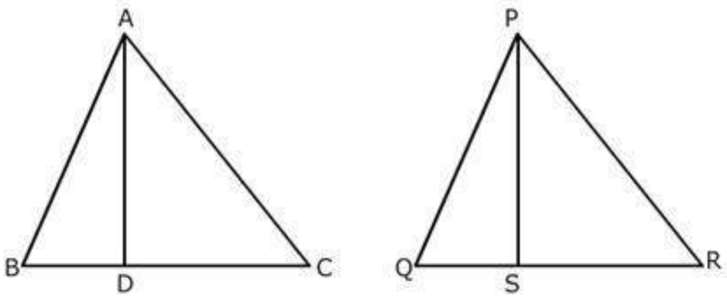
Note that Δ BDE and DEC are on the same base DE and between the same parallels BC and DE.

So, ar(BDE) = ar(DEG)

Therefore, from (1), (2) and (3), we have:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

OR



Statement: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: ?ABC ~ ?PQR To Prove: $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$ Construction: Draw AD?BC and PS?QR

Proof: $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS} = \frac{BC}{QR} \times \frac{AD}{PS}$

?ADB ~ ?PSQ (AA)

Therefore, $\frac{AD}{PS} = \frac{AB}{PQ}$ ... (iii)

But ?ABC ~ ?PQR

Therefore, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ... (iv)

Therefore, $\frac{AD}{PS} = \frac{BC}{QR}$

From (iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{BC^2}{QR^2}$

$\frac{ar(ABC)}{ar(PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$

27] 2 sec2 θ - sec4 θ - 2sec2 θ + cosec4 θ

= 2 (1 + tan2 θ) - (1 + tan2 θ)2 - 2(1 + cot2 θ) + (1 + cot2 θ)2

= (1 + tan2 θ) (1 - tan2 θ) - (1 + cot2 θ) (1 - cot2 θ) $= 1 - \tan^4\theta - (1 - \cot^4\theta)$

= 1 - tan4 θ -1+cot4 θ

= cot4 - tan4 θ

28] We have,

x-y=1

2x+y=8

Graph of the equation x-y=1:

We have,

x-y=1 =>y=x-1 and x=y+1

Putting x=0,we get y=-1

Putting y=0,we get x=1

Thus, we have the following table for the points on the line x-y=1:

x	0	1
y	-1	0

Plotting points A(0,-1),B(1,0) on the graph paper and drawing a line passing through them,we obtain the graph of the line represented by the equation x-y=1 as shown in fig.

Graph of eqn 2x+y=8:

We have,

2x+y=8 =>y=8-2x

Putting x=0,we get y=8

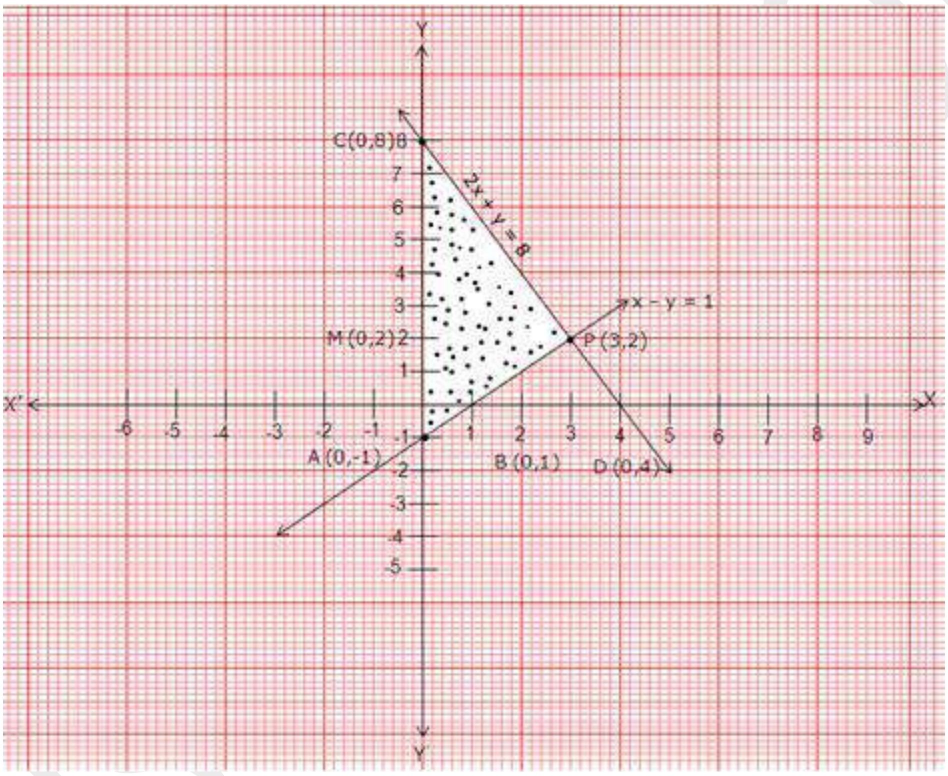
Putting y=0,we get x=4

Thus,we have the following table giving two points on the line represented by the equation 2x+y=8.

x	0	4
y	8	0

Plotting points C(0,8) and D(4,0) on the same graph paper and rawing a line passing through them, we obtain the graph of the line represented by the equation 2x+y=8 as shown in fig.

Clearly ,the 2 line intersect at P(3,2).The area bounded by these 2 lines and y-axis is shaded in the given fig.



29] $\dfrac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)}{\sin^2\theta - \cos^2\theta}$

$= \dfrac{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$

$= \dfrac{2\sin^2\theta + 2\cos^2\theta}{\sin^2\theta - (1 - \sin^2\theta)}$

$$= \frac{2\left(\sin^2\theta + \cos^2\theta\right)}{2\sin^2\theta - 1} = \frac{2}{2\sin^2\theta - 1}$$

OR

$$\frac{3\tan 35^{0}\cot(90^{0}-55^{0})\tan 40^{0}\cot(90^{0}-50^{0})-\frac{1}{2}(\sqrt{3})^{2}}{4(\cos^{2}39^{0})} = \frac{3-\frac{3}{2}}{4} = \frac{6-8}{8} = \frac{3}{8}$$

30]

Marks	Frequency
25 - 35	5
35 - 45	10
45 - 55	20
55 - 65	9
65 - 75	6
75 - 85	2
Total	52

Here the maximum frequency is 20 and the corresponding class is 45-55.So,45-55 is the modal class.

We have,l=45,h=10,f=20, $f_1 = 10, f_2 = 9$

Mode = $\ell + \left[\frac{f - f_1}{2f - f_1 - f_2}\right] \times h = 45 + \left[\frac{20-10}{40-10-9}\right] \times 10$ Mode=49.7

31] Let p(x) = x3 + 2x2 + kx + 3

Then using Remainder theorem

p(3) = 33 + 2 $\times$ 32 + 3k + 3 = 21

$\Rightarrow$ k = -9

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ x-3 & x^3 + 2x^2 - 9x + 3 \\ & x^3 - 3x^2 \\ & - \quad + \\ \hline & 5x^2 - 9x + 3 \\ & 5x^2 - 15x + 3 \\ & - \quad + \\ \hline & 6x + 3 \\ & 6x - 18 \\ & - \quad + \\ \hline & 21 \end{array}$$

Quotient of p(x) is x2 + 5x + 6

Hence, x3 + 2x2 - 9x + 3 = (x2 + 5x + 6) (x - 3) + 21

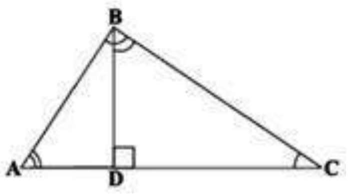
∴ x3 + 2x2 -9x - 18 = (x - 3) (x + 2) (x + 3)

All the zeros of p(x) are 3,-2,-3.

32] Given: ΔABC is a right angled triangle, ∠B = 900

To prove: AB2 + BC2 = AC2

Construction: Drop a perpendicular BD on the side AC.



Proof: From triangle ADB and triangle ABC, $\frac{AD}{AB} = \frac{AB}{AC}$

We can re-write as, AC × AD = AB2

Also, triangle BDC is similar to triangle ABC.

Equating the proportional sides of the similar triangles BDC and ABC,

$$\frac{CD}{BC} = \frac{BC}{AC}$$

⇒AC × CD = BC2

Now adding this to the equation that we had obtained,

AC × AD + AC × CD = AB2 + BC2

⇒ AC × (AD + CD) = AB2 + BC2

⇒ AC × AC = AB2 + BC2

⇒ AC2 = AB2 + BC2

33] LHS= $\frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{(\cos A - \sin A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

= Cos A + sin A = RHS

34] We first prepare the cumulative frequency distribution table by less than method as given below:

Marks no. of students marks less than cumulative frequency
0-10 7 10 7
10-20 10 20 17
20-30 23 30 40
30-40 51 40 91
40-50 6 50 97
50-60 2 60 100

Other than the given class intervals ,we assume a class-10-0 before the first class interval 0-10 with zero frequency.

Now, we mark the upper class limits along X-axis on a suitable scale and the cumulative frequencies along Y-axis on a suitable scale.

Thus, we plot the points(0,0),(10,7),(20,17),(30,40),(40,91),(50,97)and(60,100).

Now, we join the plotted points by a free hand curve to obtain the required ogive.