Paper: 05 Class-X-Math: Summative Assessment - I

Total marks of the paper: 90 Total time of the paper: 3.5 hrs

Questions:

1] The decimal expansion of the rational number $\frac{2^3}{2^2 x\, 5}$ will terminate after. [Marks:1]

- A. more than three decimal places
- B. three decimal places
- C. two decimal places
- D. one decimal place

2] n^2 - 1 is divisible by 8, if n is [Marks:1]

- A. an even integer
- B. a natural number
- C. an integer
- D. an odd integer

3] If one of the zeroes of the quadratic polynomial (K - 1) x^2 + 1 is -3, then the value of k is [Marks:1]

- A. $\frac{-4}{9}$
- B. $\frac{4}{9}$
- C. $\frac{-8}{9}$
- D. $\frac{8}{9}$

4] The lines representing the linear equations 2x - y = 3 and 4x - y = 5: [Marks:1]

- A. intersect at exactly two points
- B. are coincident
- C. are parallel
- D. intersect at a point

5] Construction of a cumulative frequency table is useful in determining The: [Marks:1]

- A. all the above three measures
- B. mode
- C. mean
- D. median

6] If x = 3 $\sec^2\theta$ - 1, y = $\tan^2\theta$ - 2 then x - 3y is equal to [Marks:1]

- A. 5
- B. 4
- C. 3
- D. 8

7] If $\cos\theta + \cos^2\theta = 1$, the value of $(\sin^2\theta + \sin^4\theta)$ is [Marks:1]

- A. 2
- B. -1
- C. 0

D. 1

8] If $\Delta ABC \sim \Delta RQP$, $\angle A = 80°$, $\angle B = 60°$, the value of $\angle P$ is [Marks:1]

A. 30°

B. 50°

C. 60°

D. 40°

9] Use Euclid's division algorithm to find H.C.F. of 870 and 225. [Marks:2]

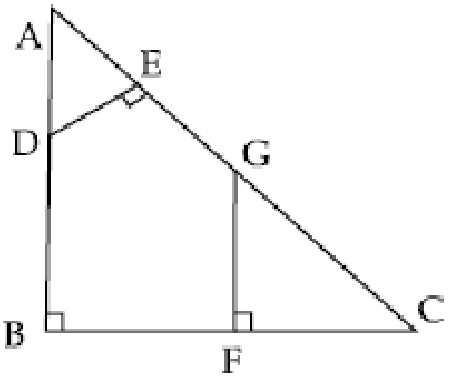
10] Solve 37x + 43y = 123, 43x + 37y = 117.

OR

$x + \frac{6}{y} = 6,\ 3x - \frac{8}{y} = 5.$ [Marks:2]

11] α, β are the roots of the quadratic polynomial p (x) = $x^2 - (k + 6)x + 2(2k - 1)$. Find the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$. [Marks:2]

12] In fig., AB $\perp$ BC, DE $\perp$ BC. Prove that $\Delta ADE \sim \Delta GCF$. [Marks:2]



13] If $\cot\theta = \frac{7}{8}$, find the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ [Marks:2]

14] Find the median class and the modal class for the following distribution. [Marks:2]

C.I	135 - 145	140-145	145 - 150	150 - 155	155 - 160	160 - 165
F	4	7	11	6	7	5

15] Show that 5 + $\sqrt{2}$ is an irrational number.

OR

Prove that $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number. [Marks:3]

16] If α and β are the zeroes of the quadratic polynomial f(x)= x^2-2x+1, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$. [Marks:3]

17] If in a rectangle, the length is increased and breadth is reduced each by 2 metres, the area is reduced by 28 sq mtrs.If the length is reduced by 1 metre and breadth is increased by 2 metres, the area is increased by 33 sq mtrs. Find the length and breadth of the rectangle. [Marks:3]

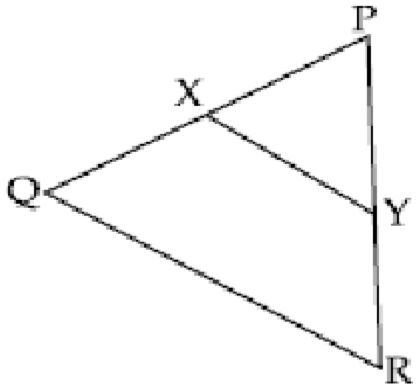
OR

A chemist has one solution which is 40% acid and a second which is 60% acid. How much of each should be mixed to make 10 litres of 50% acid solution.

18] For what values of a and b does the following pairs of linear equations have an infinite number of solutions: [Marks:3]

2x + 3y = 7; a (x + y) -b (x - y) = 3a + b - 2

19] In figure, Ay || QR, $\frac{PQ}{XQ} = \frac{7}{3}$ and Pr = 6.3 cm. Find YR. [Marks:3]



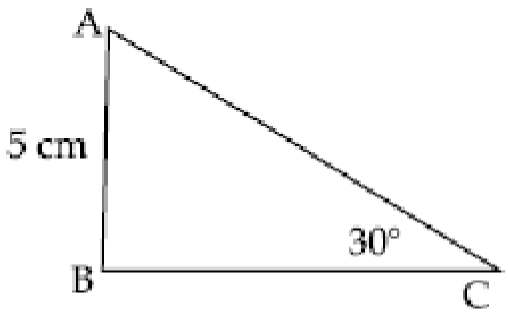
20] Prove that in any triangle the sum of the squares of any two sides is equal to twice the square of half of the third together with twice the square of the median, which bisect the third side. [Marks:3]

21] If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

OR [Marks:3]

Find the value of $\frac{\sec(90^\circ - \theta).\operatorname{cosec}\theta - \tan(90^\circ - \theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$

22] In figure, ABC is a triangle right angled at B, AB = 5 cm, ∠ACB = 30°. Find the length of BC and AC.



[Marks:3]

23] In the following distribution, if the mean of the distribution is 86 then the value of p is

Wages (in Rs.)	50-60	60-70	70-80	80-90	90-100	100-110
Number of workers	5	3	4	p	2	13

[Marks:3]

24] Find the modal age of 100 residents of a colony from the following data :

Age in yrs. (more than or equal to)	0	10	20	30	40	50	60	70
No. of Persons	100	90	75	50	28	15	5	0

[Marks:3]

25] A number of the form 15^n where $n \in N$ the set of natural numbers, can never end with a zero. Justify this statement. [Marks:4]

26] Solve the equations 2x − y +6 = 0 and 4x + 5y − 16 = 0 graphically. Also determine the coordinate of the vertices of the triangle formed by these lines and the x-axis. [Marks:4]

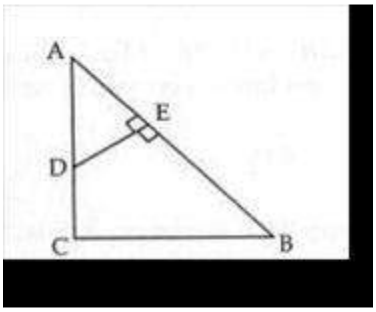
27] What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$. [Marks:4]

28] In triangle ABC, D is the mid-point of BC and AE ⊥ BC. If AC > AB,

Show that $AB^2 = AD^2 - BC \times DE + \frac{BC^2}{4}$ [Marks:4]

OR
In a right angled triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

29] In figure, Δ ABC is right angled at C. DE $\perp$ AB. If BC = 12 cm. AD = 3 cm and DC = 2 cm, then prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.



[Marks:4]

30] Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec} A + \cot A$

OR [Marks:4]

Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A

31] If sec θ + tan θ = p, show that $\frac{p^2-1}{p^2+1}$ = sin θ. [Marks:4]

32] $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\ \text{cos ec}\ \theta$ [Marks:4]

33]

Daily income (in Rs)	100 - 120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

The following distribution gives the daily income of 50 workers of a factory. Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive. [Marks:4]

34] In the distribution given below 50% of the observations is more than 14.4. Find the values of x and y, if the total frequency is 20. [Marks:4]

Class Interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1

Solutions paper- 4:

1] One decimal place

2] An odd integer

3] Since -3 is the root of quadratic polynomial, we have

(K - 1)(-3)2 + 1 = 0 $\Rightarrow 9(k-1) = -1 \Rightarrow k - 1 = \frac{-1}{9} \Rightarrow k = 1 - \frac{1}{9} = \frac{8}{9}$

4] $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = 1$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ intersect at point.

5] Median

6] $x = 3\sec^2\theta - 1,\ y = \tan^2\theta - 2$

$x - 3y = 3\sec^2\theta - 1 - 3\tan^2\theta + 6$

$= 3(\sec^2\theta - \tan^2\theta) + 5$

$= 3 + 5$

$= 8$

7] $\cos\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \cos\theta$

$\sin^2\theta + \sin^4\theta = \left(1 - \cos^2\theta\right) + (1 - \cos^2\theta)^2$

$= \cos\theta + \cos^2\theta$

$= 1$

8] $\Delta ABC \sim \Delta RQP$

$\angle A = \angle R = 80^0$

$\angle B = \angle Q = 60^0$

$\therefore \angle P = 180 - 140 = 40^0$

9] Since, $870 = 225 \times 3 + 195$

$225 = 195 \times 1 + 30$

$195 = 30 \times 6 + 15$

$30 = 15 \times 2 + 0$

$\therefore$ HCF (870,225) = 15

10]

$37x + 43y = 123$	$37x + 43y = 123$
$43x + 37y = 117$	$43x + 37y = 117$
(+) (+) (+)	(-) (-) (-)
$80x + 80y = 240$	$-6x + 6y = 6$
$\Rightarrow x + y = 3$	$-x + y = 1$

$\left.\begin{array}{l} x + y = 3 \\ -x + y = 1 \end{array}\right\}$ Solving $x = 1, y = 2$

OR

We have $\left(x + \frac{6}{y} = 6\right)3 \Rightarrow 3x + \frac{18}{y} = 18 \quad \ldots(1)$

Subtracting equation (1) from $3x - \frac{8}{y} = 5$, we get

$-\frac{26}{y} = -13 \Rightarrow y = 2$

From equation (1), x = 3

11] α, β are roots of $x^2 - (k + 6)x + 2(2k - 1)$

$\alpha + \beta = k + 6,\ \alpha\beta = 2(2k - 1)$

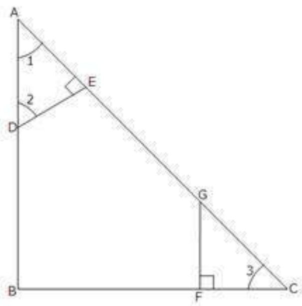
Now $\alpha + \beta = \frac{1}{2}\alpha\beta \Rightarrow k + 6 = \frac{1}{2} \times 2(2k - 1)$

$\Rightarrow k + 6 = 2k - 1$

$\Rightarrow k = 7$

12] From $\Delta ABC,\ \angle 1 + \angle 3 = 90^0$

From $\Delta ADE,\ \angle 1 + \angle 2 = 90^0$



$\angle 1 + \angle 3 = \angle 1 + \angle 2 \Rightarrow \angle 3 = \angle 2$

$\therefore$ In $\Delta ADE \sim \Delta GCF$ by AA rule as $\angle E = \angle F = 90^0$ and $\angle 2 = \angle 3$

13]

Cot $\theta = \frac{7}{8}$ (given)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

$$= \frac{49}{64}$$

14]

C.I	f	c.f.
135 - 140	4	4
140 - 145	7	11
145 - 150	11	22
150 - 155	6	28
155 - 160	7	35
160 - 165	5	40

Here, n = 40 $\Rightarrow \frac{n}{2} = 20$

Median class is 145 - 150

Also, since highest frequency is 11, Modal class is 145 - 150.

15]

To prove 5 + $\sqrt{2}$ is irrational, let us assume 5 + $\sqrt{2}$ is rational.

$\therefore$ We can find integers a and b where a, b are co-prime, b $\neq 0$

Such that, $5 + \sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{b} - 5$

Now a, b are integers, $\frac{a}{b} - 5$ is rational.

$\Rightarrow \sqrt{2}$ is rational.

Which is a contradiction. So 5 + $\sqrt{2}$ is irrational.

OR

Let us assume to the contrary, that $\sqrt{n-1} + \sqrt{n+1}$ is a rational number.

$\Rightarrow (\sqrt{n-1}+\sqrt{n+1})^2$ is rational.

$\Rightarrow$ (n-1)+(n+1)-2 $(\sqrt{n-1}\times\sqrt{n+1})$ is rational

$\Rightarrow$ 2n+2 $\sqrt{n^2-1}$ is rational

But we know that $\sqrt{n^2-1}$ is an irrational number

So 2n+2 $\sqrt{n^2-1}$ is also an irrational number

So our basic assumption that the given number is rational is wrong.

Hence, $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number.

16] f(x) = x2 - 2x + 1
Zeroes of f(x) are α, β
Sum of zeroes α + β = 2 and α.β = 1

$$\text{Now}\ \frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}=2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=2\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$$

$$=2\frac{((\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta}=\frac{2\times 2}{1}=4$$

$$\text{Also,}\ \frac{2\alpha}{\beta}\times\frac{2\beta}{\alpha}=4$$

Required polynomial = k(x2 - 4x + 4), where k is any integer.

17] Let the length and breadth of the rectangle be x and y respectively.
So the original area of the rectangle=xy
According to question,
(x+2)(y-2)=xy-28
i.e. xy-2x+2y-4=xy-28
2x-2y=24 ...(i)
Next, (x-1)(y+2)=xy+33
i.e. xy+2x-y-2=xy+33
2x-y=35 ..(ii)
Now we need to solve (i) and (ii)
From (ii) we get,
y=2x-35
substituting this value in (i) we get,
2x-4x+70=24
-2x=-46
x=23
substituting this value in (ii)
we get,
y=11
So the length and breadth of the rectangle are 23 metres and 11 metres respectively.

OR

Let 40 % acids in the solution be x litres
Let 60 % of other solution be y litres
Total Volume in the mixture = x + y
Given volume is 10 litres
x + y = 10 ---(i)

$$\text{Also,}\ \frac{40}{100}x+\frac{60}{100}y=\frac{50}{100}\times 10$$

So, 40 x + 60 y =500 or 2x+3y =25 ...(ii)
Solving (i) and (ii) we get x = y = 5 litres

18] The system has infinitely many solution

$$\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\Rightarrow \underset{(1)}{\frac{2}{a-b}}=\underset{(2)}{\frac{3}{a+b}}=\underset{(3)}{\frac{7}{3a+b-2}}$$

Equating (1) and (2), we get a = 5b
Equating (2) and (3), we get 2a - 4b = 6 On solving, we get b = 1 and a = 5.

19] By BPT

$$\frac{PX}{XQ}=\frac{PY}{YR}$$

$$\Rightarrow \frac{PX}{XQ}+1=\frac{PY}{YR}+1$$

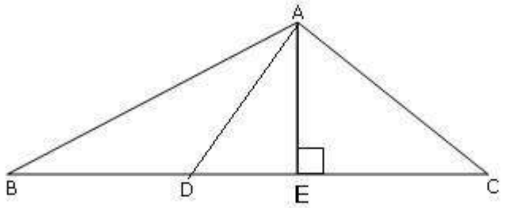
$$\Rightarrow \frac{PX + XQ}{XQ} = \frac{PY + YR}{YR}$$

$$\Rightarrow \frac{PQ}{XQ} = \frac{PR}{YR}$$

$$\Rightarrow \frac{7}{3} = \frac{6.3}{YR}$$

$$\Rightarrow YR = \frac{6.3 \times 3}{7} = 2.7 \text{ cm}$$

20]



To prove $AB^2 = AC^2 = 2AD^2 + 2\left(\frac{1}{2}BC\right)^2$

Draw $AE \perp BC$

In ?ABD since $\angle D > 90°$

$\therefore \therefore$ $AB^2 = AD^2 + BD^2 + 2BD \times DE$ ...(1) (using Obtuse angle property) ?ACD = since $\angle D < 90°$

$AC^2 = AD^2 + DC^2 - 2DC \times DE$ ...(2) (using acute angle property)

Adding (1) and (2)

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

$=2(AD^2 + \left(\frac{1}{2}BC\right)^2)$

Or $AB^2 + AC^2 = 2(AD^2 + BD^2)$

Hence proved.

21]

Given: $\frac{\cos\alpha}{\cos\beta} = m,\qquad \frac{\cos\alpha}{\sin\beta} = n$

$$\Rightarrow m^2 = \frac{\cos^2\alpha}{\cos^2\beta},\qquad n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

L.H.S. = $(m^2 + n^2)\cos^2\beta$

$$= \left[\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{\sin^2\beta + \cos^2\beta}{\sin^2\beta\cos^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta} = n^2 = \text{R.H.S.}$$

Therefore, $(m^2 + n^2)\cos^2\beta = n^2$

OR

Using $\sec(90^\circ-\theta)=\text{cosec}\,\theta,\ \tan(90^\circ-\theta)=\cot\theta$

and $\cos(90^\circ-\theta)=\sin\theta$

$$\frac{\sec(90^\circ-\theta).\text{cosec}\,\theta-\tan(90^\circ-\theta)\cot\theta+\cos^2 25^\circ+\cos^2 65^\circ}{3\tan 27^\circ\tan 63^\circ}$$

$$=\frac{\text{cosec}\,\theta.\text{cosec}\,\theta-\cot\theta.\cot\theta+\cos^2(90^\circ-65^\circ)+\cos^2 65^\circ}{3\tan(90^\circ-63^\circ)\tan 63^\circ}$$

$$=\frac{\text{cosec}^2\theta-\cot^2\theta+\sin^2 65^\circ+\cos^2 65^\circ}{3\cot 63^\circ\tan 63^\circ}$$

[Since, $\sin^2\theta+\cos^2\theta=1$ and $\text{cosec}^2\theta-\cot^2\theta=1$]

$$=\frac{1+1}{3}=\frac{2}{3}$$

22] In ΔABC, $\angle B=90^\circ$, we have

$$\frac{AB}{AC}=\sin 30^\circ=\frac{1}{2}\Rightarrow\frac{5}{AC}=\frac{1}{2}\Rightarrow AC=10\text{ cm}$$

And, $\frac{BC}{AC}=\cos 30^\circ=\frac{\sqrt{3}}{2}\quad\Rightarrow\frac{BC}{10}=\frac{\sqrt{3}}{2}\Rightarrow BC=5\sqrt{3}\text{ cm}$

23]

CI	50-60	60-70	70-80	80-90	90-100	100-110	Total
fi	5	3	4	p	2	13	27+p
xi	55	65	75	85	95	105	
fi xi	275	195	300	85p	190	1365	2325+85p

Mean = $\frac{\sum f_i x_i}{\sum f_i}$ Substituting the values we get

$\Rightarrow 86=\frac{2325+85p}{27+p}$

$\Rightarrow 86p+2322=2325+85p$

$\Rightarrow p=3$

24]

Since the maximum frequency is 25 and it lies in the class interval 20-30.

Age in yrs. (more than or equal to)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of persons (fi)	10	15	25	22	13	10	5

Therefore, modal class = 20 - 30

??= 20, h = 10, f0 = 15, f1 = 25, f2 = 22

mode = ??+ $\frac{f_1-f_0}{2f_1-f_0-f_2}\times h$

$= 20+\left(\frac{25-15}{2(25)-15-22}\right)\times 10$

= 20+7.69 = 27.69 years (approx.)

25] If the number 15n where n $\in$ N, were to end with a zero, then its prime factorisation must have 2 and 5 as its factors. But 15=5 $\times$ 3

15n = (5 $\times$ 3)n = 5n $\times$ 3n

So Prime factors of 15n includes only 5 but not 2

Also from the Fundamental theorem of Arithmetic, the prime factorisation of a number is unique.

Hence a number of the form 15n where n $\in$ N, will never end with a zero.

26] To solve the equations, make the table corresponding to each equation.

$2x - y + 6 = 0$

$\Rightarrow y = 2x + 6$

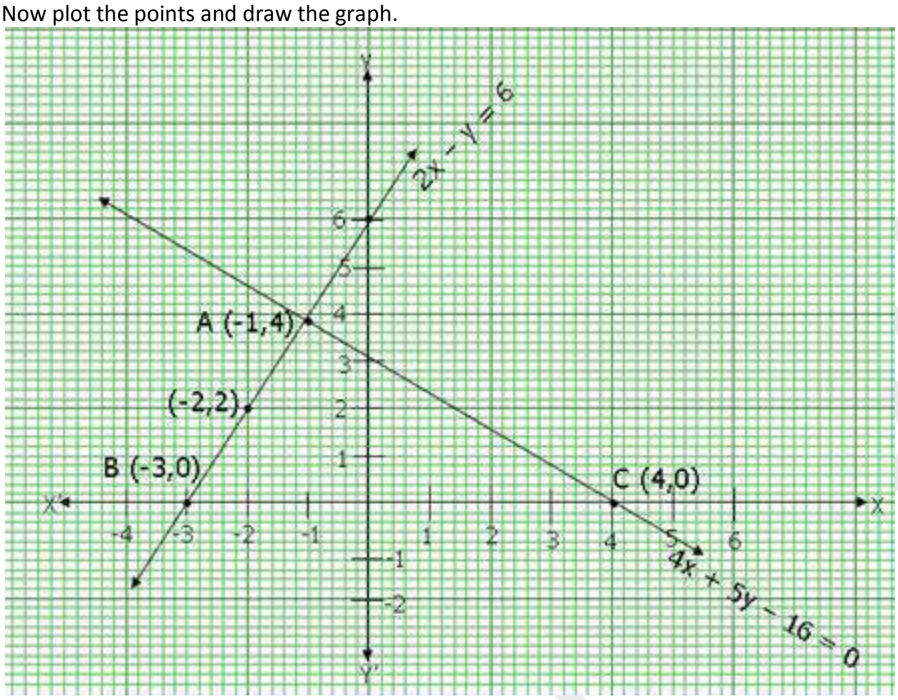
x	?1	?2	?3
y	4	2	0

$4x + 5y - 16 = 0$

$\Rightarrow y = \frac{16 - 4x}{5}$

x	4	?1
y	0	4

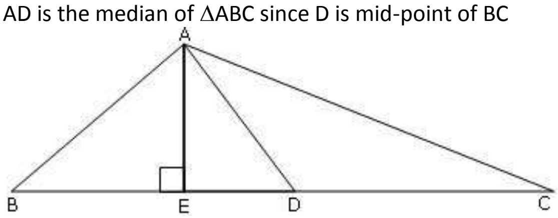
Now plot the points and draw the graph.



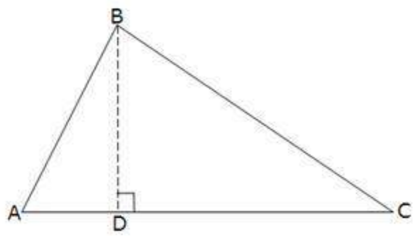
Since the lines intersect at the point (?1, 4), so x = ?1 and y = 4 be the solution.
Also by observation vertices of triangle formed by lines and x-axis are A (?1, 4), B (?3, 0) and C (4, 0).

27] Let p(x) = x3 - 6x2 - 15x + 80
Let say that we subtracted ax + b so that it is exactly divisible by x2 + x - 12
s(x) = x3 - 6x2 - 15x + 80 - (ax + b)
= x3 - 6x2 - (15 + a)x + (80 - b)
Dividend = Divisor x Quotient + Remainder
But remainder = 0
∴ ∴ Dividend = Divisor x Quotient
s(x) = (x2 + x -12) x quotient
s(x) = x3 - 6x2 - (15 + a)x + (80 - b)
x (x2 + x - 12) - 7(x2 + x - 12)
= x3 + x2 - 7x2 - 12x - 7x + 84
= x3 - 6x2 - 19x + 84
Hence, x3 - 6x2 - 19x + 84 = x3 - 6x2 - (15 + a)x + (80 - b)
-15 - a = - 19 $\Rightarrow$ a = +4
and 80 - b = 84 $\Rightarrow$ b = -4
Hence if in p(x) we subtracted 4x - 4 then it is exactly divisible by x2 + x -12.

28] AD is the median of ΔABC since D is mid-point of BC



$\Rightarrow$ BD = DC= $\frac{BC}{2}$ ....(i)
In right triangle AEB,
$AB^2=AE^2+BE^2$ ...Pythagoras theorem
=(AD^2-DE^2)+ $(BD-DE)^2$
Using Pythagoras theorem for right triangle AED and BE=BD-DE
= AD^2-DE^2 +($(\frac{BC}{2}-DE)^2$ ....from (i)
AB2= AD^2-DE^2 + $\frac{BC^2}{4}$ + DE^2 -2 $\left(\frac{BC \times DE}{2}\right)$
$\Rightarrow AB^2=AD^2$ -BC x DE + $\frac{BC^2}{4}$
Hence proved.
OR
Given: A right triangle ABC right angled at B.
To prove: that AC2 = AB2 + BC2
Construction:Let us draw BD $\perp$ AC (See fig.)



Proof :
Now, Δ ADB $\sim$ Δ ABC (Using Theorem:If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse ,then triangles on both sides of the perpendicular are similar to the whole triangle and to each other)

So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)
Or, AD.AC = AB2
Also, Δ BDC $\sim$ Δ ABC (Theorem)
So, $\frac{CD}{BC} = \frac{BC}{AC}$
Or, CD. AC = BC2
Adding (1) and (2),
AD. AC + CD. AC = AB2 + BC2
OR, AC (AD + CD) = AB2 + BC2
OR, AC.AC = AB2 + BC2
OR AC2 = AB2 + BC2
Hence Proved.

29] Δ ABC $\sim$ Δ ADE (by AA Similarity)
$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} \ldots(1)$
In right ΔABC, AB2 = BC2 + AC2 (by PT)
$\Rightarrow$ AB2 = 52 + 122 = 25 + 144 = 169
$\Rightarrow$ AB = 13
Subsisting AB = 13 cm, BC = 12 cm and AC = 5 cm in (1) and
Getting DE = $\frac{36}{13}$ and AE = $\frac{15}{13}$

30] To prove: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec}A + \cot A$

Using the identity cosec2 A = 1 + cot2A

L.H.S = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$= \dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}$

$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$

$= \dfrac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$

$= \dfrac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$

$= \dfrac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$

$= \dfrac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$

$= \dfrac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$

$= \dfrac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$

= cosecA + cotA

= R.H.S

OR

$LHS = \sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sqrt{\dfrac{1+\sin A}{1-\sin A} \times \dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \dfrac{1+\sin A}{\sqrt{\cos^2 A}}$

$= \dfrac{1+\sin A}{\cos A}$

$= \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$ =RHS

31] LHS = $\dfrac{P^2 - 1}{P^2 + 1} = \dfrac{(\sec\theta + \tan\theta)^2 + 1}{(\sec\theta + \tan\theta)^2 + 1}$

$= \dfrac{\sec^2\theta + \tan^2\theta + 2\sec\theta \,.\, \tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta \,.\, \tan\theta - 1}$

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1+\tan^2\theta) + 2\sec\theta\tan\theta} = \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta} = \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\tan\theta+\sec\theta)} = \frac{\tan\theta}{\sec\theta}$$

$= \sin\theta$ = RHS

32]

$$\text{LHS} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$

$$= \frac{\left(\sqrt{\sec\theta - 1}\right)^2 + \left(\sqrt{\sec\theta + 1}\right)^2}{\left(\sqrt{\sec\theta + 1}\right)\left(\sqrt{\sec\theta - 1}\right)}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$$

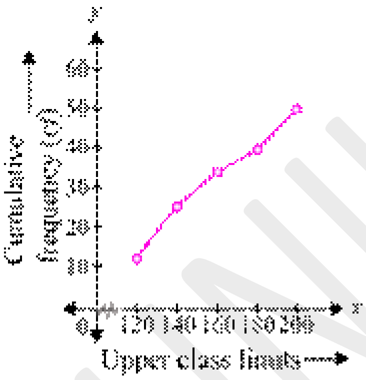
$$= \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta} = 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta = \text{RHS}$$

Hence, LHS = RHS.

33] We can find frequency distribution table of less than type as following -

Daily income (in Rs) (upper class limits)	Cumulative frequency
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now taking upper class limits of class intervals on x-axis and their respective frequencies on y-axis we can draw its ogive as following -



34]

Class Interval	0-6	6-12	12-18	18-24	24-30
Frequency	4	x	5	y	1
Cumulative frequency	4	4+x	9+x	9+x+y	10+x+y

It is given that total frequency N is 20

So, 10+x+y = 20 i.e. x + y = 10 ….(i)

Given 50% of the observations are greater than 14.4.

So median = 14.4, which lies in the class interval 12-18.

= 12, cf = 4 + x, h = 6, f = 5, N = 20

Median = $\ell + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$ 14.4 = 12 + $\left(\frac{10-(4+x)}{5}\right)$ x 6 $\Rightarrow$ 14.4 - 12 = $\frac{(6-x)}{5}$ x6 $\Rightarrow$ $\frac{2.4 \times 5}{6}$ = 6 - x

$\Rightarrow$x =4

Now using equation, 10+x+y = 20, we get y = 6. Hence x =4 and y = 6.