Gravitation Class 09 Term-1

EXAMPLE Calculate the gravitational force between two bodies A and B having mass 50 kg each and sitting at a distance of 1 m from each other.

SOLUTION

Here,

Mass of A (m_A) = Mass of B (m_B) = 50 kg (Given)

Distance (d) = 1 m (Given)

According to Newton's Universal Law of Gravitation, the force of gravitation is calculated by using the formula

$$\boxed{F = G\frac{m_1 m_2}{d^2}}$$

Here, $G = 6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

Now, putting these values in the above formula, we get

$$F = \frac{6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2 \times 50\ \text{kg} \times 50\ \text{kg}}{(1\ \text{m})^2}$$

$$F = 1.67 \times 10^{-7}\ \text{N}$$

Now, let us suppose you are A and B is your friend. You can see that the mutual force of attraction between you and your friend is so small that any one of you will not experience this force at all, thereby illustrating that the gravitational force (between two small objects having small masses) is very weak in nature.

EXAMPLE Calculate gravitational force between the earth and the moon. The mass of earth is 6×10^{24} kg and that of moon is 7.4×10^{22} kg and the distance between them is 3.84×10^5 km.

SOLUTION

Here,

Mass of the earth (m_1) = 6×10^{24} kg

Mass of the moon (m_2) = 7.4×10^{22} kg

Distance between the earth and the moon (d)

$= 3.84 \times 10^5$ km

$= 3.84 \times 10^5 \times 1000$ m

$= 3.84 \times 10^8$ m

$G = 6.7 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$

According to Newton's Universal Law of Gravitation,

$$\boxed{F = G\frac{m_1 m_2}{d^2}}$$

Now, putting these values in the above formula, we get

$$F = \frac{6.7 \times 10^{-11}\ \text{N m}^2/\text{kg}^2 \times 6 \times 10^{24}\ \text{kg} \times 7.4 \times 10^{22}\ \text{kg}}{(3.84 \times 10^8\ \text{m})^2}$$

$$F = 2.01 \times 10^{20}\ \text{N}$$

You can see that the mutual force of attraction between the earth and the moon is very large (2.01×10^{20} N). It is due to this large force that it binds them, such that it makes the moon to move around the earth with uniform speed. The above example proves that the gravitational force between big objects having very large masses is very strong in nature.

EXAMPLE How does the force of gravitation between two objects change, when the distance between them is reduced to half?

SOLUTION Let the masses of two objects be m_1 and m_2 and the distance between them be d. The force of gravitation is calculated by using formula for Newton's Universal Law of Gravitation, i.e.

$$\boxed{F = G\frac{m_1 m_2}{d^2}}$$

When the distance between the two objects is reduced to half i.e., from d to $d/2$, then

$$F' = G\frac{m_1 m_2}{\left(\frac{1}{2}d\right)^2} = G\frac{m_1 m_2}{\frac{1}{4}d^2} = \frac{4Gm_1 m_2}{d^2}$$

$$\text{So}\quad \frac{F'}{F} = \frac{\dfrac{4\,G\,m_1 m_2}{d^2}}{G\dfrac{m_1 m_2}{d^2}} = \frac{4\,G\,m_1 m_2}{d^2} \times \frac{d^2}{G\,m_1 m_2} = 4$$

$\therefore\quad F' = 4F$

Therefore, when the distance between the two objects is reduced to half, the force of gravitation becomes 4 times the initial value.

EXAMPLE A ball is thrown vertically upwards with a velocity of 49 m/s. Calculate

a. the maximum height to which it rises.

b. the total time it takes to return to the surface of the earth.

SOLUTION

a. Initial velocity of the ball, $u = 49$ m/s

Final velocity of the ball, $v = 0$ m/s (it stops)

Acceleration due to gravity, $g = -9.8$ m/s^2 (ball is thrown vertically upwards)]

Maximum height reached by the ball, $h = ?$ (to be calculated)

We know, for a freely falling body

$$\boxed{v^2 = u^2 + 2gh}$$

Now, putting all the known values in the above formula, we get

$$(0)^2 = (49 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(h)$$

or $0 - (49 \text{ m/s})^2 = -2 \times 9.8 \text{ m/s}^2 (h)$

$$h = \frac{49 \text{ m/s} \times 49 \text{ m/s}}{2 \times 9.8 \text{ m/s}^2}$$

$$h = 122.5 \text{ m}$$

So, the maximum height reached by the ball is 122.5 m.

b. Time taken to reach the highest point $(t) = ?$ (to be calculated)

We know, $\boxed{g = \frac{v-u}{t}}$ or $t = \frac{v-u}{g}$

Now, putting the known values in the above formula, we get

$$t = \frac{0 \text{ m/s} - 49 \text{ m/s}}{-9.8 \text{ m/s}^2} = \frac{-49 \text{ m/s}}{-9.8 \text{ m/s}^2} = 5 \text{ s}$$

We know, time taken by a body to reach to the highest point = time taken by the body to fall from the same height.

Therefore,

Total time taken by the ball to return to the surface of the earth = 5 s + 5 s = 10 s.

EXAMPLE A stone is released from the top of a tower of height 19.6 m. Calculate its final velocity just before touching the ground.

SOLUTION Initial velocity of the stone, $u = 0$ m/s [since it is dropped from a height]

Height of the tower, $h = 19.6$ m

Final velocity of the stone, $v = ?$ (to be calculated)

Acceleration due to gravity, $g = 9.8$ m/s^2

Now, for a freely falling body

$$\boxed{v^2 - u^2 = 2gh}$$

Putting the known values in the above equation, we get

$$v^2 - (0 \text{ m/s})^2 = 2 \times 9.8 \text{ m/s}^2 \times 19.6 \text{ m}$$

$$v^2 = 19.6 \text{ m/s}^2 \times 19.6 \text{ m}$$

$$v^2 = (19.6 \text{ m/s})^2$$

$$v = 19.6 \text{ m/s}$$

So, the final velocity of the stone is 19.6 m/s.

EXAMPLE A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 m/s. Calculate when and where the two stones meet. (Take $g = 10$ m/s^2)

SOLUTION Let us assume that the two stones meet after t seconds and at the distance x metres from the ground.

1. For the falling stone

Vertical distance travelled by the stone, $h = (100 - x)$ m

Initial velocity of the stone, $u = 0$ m/s

Acceleration due to gravity, $g = 10$ m/s^2

We know, $\boxed{h = ut + \frac{1}{2}gt^2}$

Putting the known values in the above equation, we get

$$(100 - x) = (0 \text{ m/s}) \times t + \frac{1}{2} \times 10 \text{ m/s}^2 \times t^2$$

$$(100 - x) = 0 + 5t^2$$

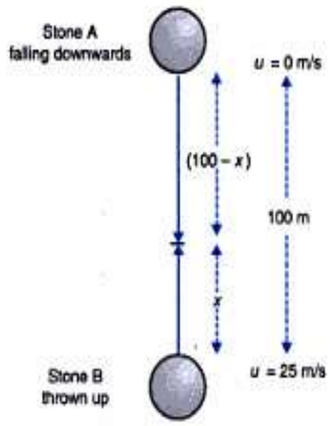
$$(100 - x) = 5t^2 \quad \text{... (i)}$$

2. For the stone thrown vertically upwards

Vertical distance travelled by the stone, $h = x$ m

Initial velocity of the stone, $u = 25$ m/s

Acceleration due to gravity, $g = -10$ m/s^2



(stone is thrown vertically upwards)

We know, $h = ut + \frac{1}{2}gt^2$

Putting the known values in the above equation, we get

$$x = (25\ \text{m/s})(t) + \frac{1}{2}(-10\ \text{m/s}^2)t^2$$

$$x = 25t - 5t^2 \quad \text{... (ii)}$$

Adding equations (i) and (ii), we get

$$100 - x + x = 5t^2 + 25t - 5t^2$$

or $$100 = 25t$$

or $$t = \frac{100}{25}$$

or $$t = 4\ \text{s}$$

Thus, the two stones meet 4 seconds after the start. Substituting the value of t in equation (ii), we get

$$x = 25(4) - 5(4)^2$$
$$x = 100 - 5(16)$$
$$x = 100 - 80$$
$$x = 20\ \text{m}$$

Thus, the two stones meet after 4 seconds from the start and at a height of 20 m above the ground.

EXAMPLE A ball thrown up vertically returns to the thrower after 6 s. Find

a. the velocity with which it was thrown up,

b. the maximum height it reaches, and

c. its position after 4 s.

SOLUTION Total time taken by the ball to reach the thrower = 6 s

We know, time taken by the ball to reach the highest point when thrown in the upward direction = Time taken by the ball to fall from the highest point to the ground.

∴ time taken by the ball to reach the highest point,

$$t = \frac{6}{2} = 3\ \text{s}$$

a. Final velocity of the ball, $v = 0$

(it stops at the highest point)

Acceleration due to gravity, $g = -9.8\ \text{m/s}^2$

(ball is thrown upwards)

We know, the equation of motion for a freely falling body

$$v = u + gt$$

Putting the known values in the above equation, we get

$$0 = u + (-9.8\ \text{m/s}^2)(3\ \text{s})$$

or $$-u = -29.4\ \text{m/s}$$

$$u = 29.4\ \text{m/s}$$

So, the initial velocity with which the ball is thrown up is 29.4 m/s.

b. Height, the ball can reach, h = ?

(to be calculated)

We know, the equation of motion for a freely falling body

$$h = ut + \frac{1}{2}gt^2$$

Putting the known values in the above equation, we get

$$h = (29.4\ \text{m/s})(3\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(3\ \text{s})^2$$

$$h = 88.2\ \text{m} - 44.1\ \text{m}$$

$$h = 44.1\ \text{m}$$

Thus, the maximum height the ball can reach is 44.1 m.

c. Position of the ball after 4 s, h = ?

(to be calculated)

We know, the equation of motion for a freely falling body

$$h = ut + \frac{1}{2}gt^2$$

Putting the known values in the above equation, we get

$$h = (29.4\ \text{m/s})(4\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(4\ \text{s})^2$$

$$h = 117.6\ \text{m} - 78.4\ \text{m}$$

$$h = 39.2\ \text{m}$$

After 4 s the ball will reach a height of 39.2 m from the bottom.

Thus, the position of the ball after 4 s from the top,

$$= 44.1\ \text{m} - 39.2\ \text{m} = 4.9\ \text{m}$$

EXAMPLE . A body thrown vertically upwards, rises to a height of 10 m. Calculate **a.** the velocity with which the body was thrown upwards and **b.** the time taken by the body to reach the highest point.

SOLUTION a. Initial velocity of the body, u = ?

Maximum height reached by the body, h = 10 m

Final velocity, v = 0 m/s

(it stops after reaching the maximum height)

Acceleration due to gravity, $g = -9.8\ \text{m/s}^2$

(body is thrown upwards)

We know, the equation of the motion for a freely falling body

$$\boxed{v^2 = u^2 + 2gh}$$

Putting the known values in the above equation, we get

$$(0\ \text{m/s})^2 = u^2 + 2(-9.8\ \text{m/s}^2)(10\ \text{m})$$
$$-u^2 = -196\ \text{m}^2/\text{s}^2$$
$$u^2 = 196\ (\text{m/s})^2$$
$$u = \sqrt{196\ (\text{m/s})^2}$$
$$u = 14\ \text{m/s}$$

Thus, the initial velocity with which the body was thrown upwards is 14 m/s.

b. Time taken by the body to reach the highest point, t = ?

We know, the equation of the motion for a freely falling body

$$\boxed{v = u + gt}$$

Putting the known values in the above given equation, we get

$$0 = 14\ \text{m/s} + (-9\ \text{m/s}^2)(t)$$
$$-14\ \text{m/s} = -9\ \text{m/s}^2(t)$$
$$t = \frac{14\ \text{m/s}}{9\ \text{m/s}^2}$$
$$t = 1.43\ \text{s}$$

Thus, the time taken by the body to reach the highest point is 1.43 s.