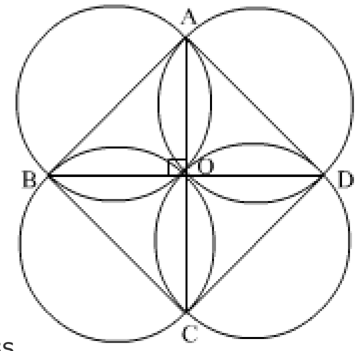
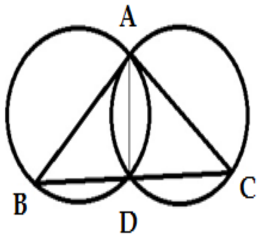
# Class IX Chapter- Circle Solved problems

Q. Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of its diagonals.




Let ABCD be a rhombus whose diagonals AC and BD intersect at O.
We know that, **diagonals of a rhombus intersect each other at right angle.**
∴ ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°
Now, circles with AB, BC, CD and DA as diameter passes through O.
(**Angle in a semi-circle is 90°**)
Thus, the circles described on the four sides of a rhombus as diameter, pass through the point of intersection of its diagonals.

Q. If the circles are drawn taking two sides of a traingle as diameters, prove that the point of intersection of these circles lie on the third side



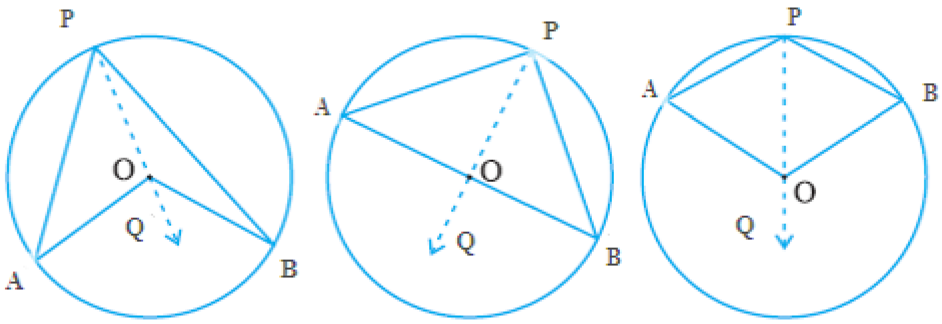

Let the side AB and AC are the diameters and AD be the common chord

Prove that D lies on BC

Proof: ∠ ADB = 90° (Angle in a semicircle) and ∠ ADC = 90° (Angle in a semicircle)
So, ∠ ADB + ∠ ADC = 90° + 90° = 180°
Therefore, BDC is a line. Hence the point of intersection of two circles lie on the third side.

Q. Prove that angles subtended by an arc at the centre is double the angle subtended by it at any other point on the circle

In the given condition, clearly we have three cases:
Case (1) $\widehat{AB}$ is a minor arc
Case (2) $\widehat{AB}$ is a semicircle
Case (3) $\widehat{AB}$ is a major arc
Construction: Join PO and extend it to a point Q.




(i) (ii) (iii)

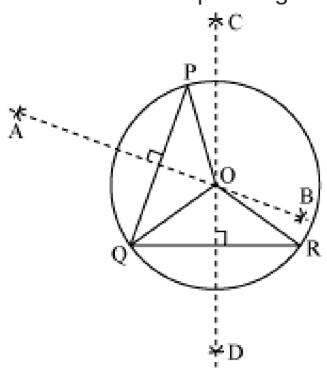
To prove: ∠AOB = 2∠APB

Proof:
We know that, an exterior angle of a triangle is equal to the sum of the interior opposite angles.
In ΔOPB,
∠QOB = ∠OPB + ∠OBP ...(1)
OB = OP (Radius of the circle)
⇒ ∠OPB = ∠OBP (In a triangle, equal sides have equal angle opposite to them)

∴ ∠QOB = ∠OPB + ∠ OPB
⇒ ∠ QOB = 2∠OPB ...(2)
In ΔOPA
∠QOA = ∠ OPA + ∠ OAP ...(3)
OA = OP (Radius of the circle)
⇒ ∠OPA = ∠OAP (In a triangle, equal sides have equal angle opposite to them)
∴ ∠QOA = ∠OPA + ∠OPA
⇒ ∠QOA = 2∠OPA ...(4)
Adding (2) and (4), we have
∠QOA + ∠QOB = 2∠OPA + ∠OPB
∴ ∠AOB = 2(∠OPA + ∠OPB)
⇒ ∠AOB = 2∠APB
For the case 3, where AB is the major arc, ∠AOB is replaced by reflex ∠AOB.
∴ reflex ∠AOB = 2∠APB

**Q. Prove that there is one and only one circle passing through three given non-collinear points.**

Given: Three non collinear points P, Q and R
To prove: There is one and only one circle passing through the points P, Q and R.
Construction: Join PQ and QR.
Draw perpendicular bisectors AB of PQ and CD of QR. Let the perpendicular bisectors intersect at the point O.
Now join OP, OQ and OR.
A circle is obtained passing through the points P, Q and R.




**Proof**: We know that, each and every point on the perpendicular bisector of a line segment is equidistant from its ends points.
Thus, OP = OQ [Since, O lies on the perpendicular bisector of PQ]
and OQ = OR. [Since, O lies on the perpendicular bisector of QR]
So, OP = OQ = OR.
Let OP = OQ = OR = r.
Now, draw a circle C(O, r) with O as centre and r as radius.
Then, circle C(O, r) passes through the points P, Q and R.
Next, we show: this circle is the only circle passing through the points P, Q and R.
If possible, suppose there is a another circle C(O′, t) which passes through the points P, Q, R.
Then, O′ will lie on the perpendicular bisectors AB and CD.
But O was the intersection point of the perpendicular bisectors AB and CD.
So, O ′ must coincide with the point O. [Since, two lines can not intersect at more than one point]
As, O′P = t and OP = r; and O ′ coincides with O, we get t = r
Therefore, C(O, r) and C(O, t) are congruent.
Thus, there is one and only one circle passing through three the given non-collinear points.
We can draw circles from more than 3 non-collinear points as a circle consists of infinite number of points.

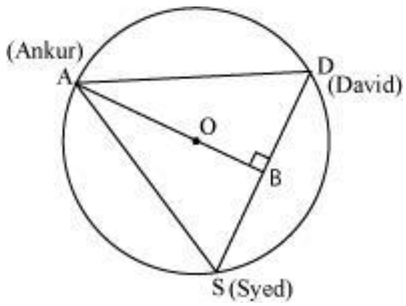
**Q. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David re sitting at equal distance on its boundary each having a toy telephone in their hand to talk each other. Find the length of the string of each phone.**

It is given that AS = SD = DA
Therefore, ΔASD is an equilateral triangle. OA = $r$ = 20 m
Medians of equilateral triangle pass through the circumcentre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2: 1. As AB is the median of equilateral triangle ASD, we can write

$$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$$

$$\Rightarrow \frac{20\text{ m}}{OB} = \frac{2}{1}$$

$$\Rightarrow OB = \left(\frac{20}{2}\right)\text{m} = 10\text{ m}$$




∴ AB = OA + OB = (20 + 10) m = 30 m

In ΔABD,

$AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$

$$\Rightarrow AD^2 = 900 + \frac{1}{4}AD^2$$

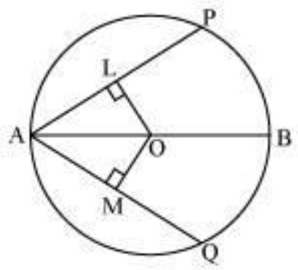
$$\Rightarrow \frac{3}{4}AD^2 = 900$$

$$\Rightarrow AD^2 = 1200$$

$$\Rightarrow AD = 20\sqrt{3}$$

Therefore, the length of the string of each phone will be $20\sqrt{3}$ m.

Q. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection; prove that the chords are equal.




**Given that:** AB is the diameter of the circle with centre O. AP and AQ are two intersecting chords of the circle such that ∠PAB = ∠QAB.

**To prove:** AP = AQ

**Construction:** Draw OL⊥AB and OM⊥AC.

**Proof**: In ΔAOL and ΔAOM

∠OLA = ∠OMB (each 90°), OA = OA (Common line)

∠OAL = ∠OAM (∠PAB = ∠QAB)

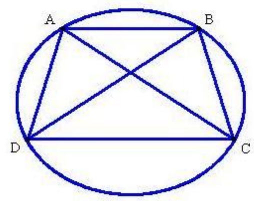
∴ ΔAOL ≅ ΔAOM (AAS congruence criterion)

⇒ OL = OM (C.P.C.T)

⇒ Chords AP and AQ are equidistant from centre O

⇒ AP = AQ (Chords which are equidistant from the centre are equal)

Q. If two sides of a cyclic quadrilateral are parallel , prove tha the remaining two sides are equal and the diagonals are also equal




ABCD is the cyclic quadrilateral.

It is given that AB is parallel to CD.

Prove that: AD and BC are equal and that AC and BD are equal.

Since ABCD is a cyclic quadrilateral, <DAB +<DCB= <CDA + <CBA =180

Since AB is parallel to CD,<DAB + <CDA = <DCB + < CBA = 180

Comparing the above two equations, it can be said that < CDA = < DCB

This is a property of an isosceles trapezium. Thus, AD=BC.

In Δ DAC and ΔCBD ; AD = BC , <CDA = < DCB , CD = DC Thus, Δ DAC ≅ ΔCBD

Thus, AC=BD. Thus, it has been proven that AD=BC and AC=BD