LESSON
RAY OPTICS

Introduction

Electromagnetic radiation belonging to the region of the electromagnetic spectrum (wavelength of about 400 nm to 750 nm) is called light. Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within this small range of the electromagnetic spectrum.
It is mainly through light and the sense of vision that we know and interpret the world around us.
There are two things that is easily identifiable with light
1. Light travels with enormous speed and
2. Light travels in a straight line.

Its presently accepted value in vacuum is $c = 2.99792458 \times 10^8 ms^{-1}$. For many purposes, it suffices to take $c = 3 \times 10^8 ms^{-1}$.

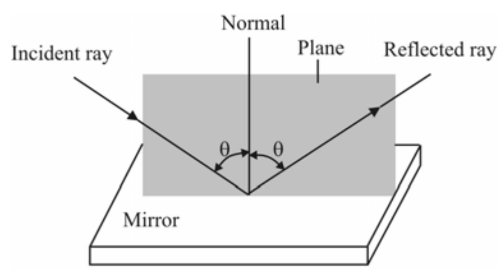
Note:
(i). The speed of light in vacuum is the highest speed attainable in nature.
(ii). The wavelength of light is very small compared to the size of ordinary objects that we encounter commonly, hence, it can be taken to be moving in a straight line

Ray of light- A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light,
Beam of light- A bundle of such rays constitutes a beam of light.

In this chapter, we consider the phenomena of reflection, refraction and dispersion of light, using the ray picture of light. Using the basic laws of reflection and refraction, we shall study the image formation by plane and spherical reflecting and refracting surfaces. We then go on to describe the construction and working of some important optical instruments, including the human eye.

Reflection of Light by Spherical Mirrors

Law of reflection - The law of reflection by a plane mirror states that:
(i) the angle of incidence (angle between incident ray and the normal to the mirror) equals the angle of reflection (angle between reflected ray and the normal). This law is also applied at every point on the surface of a spherical mirror.
(ii) the incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.



The incident ray, reflected ray and the normal to the reflecting surface lie in the same plane.

Note:
- These laws are valid at each point on any reflecting surface whether plane or curved.
- The normal in this case is to be taken as normal to the tangent to surface at the point of incidence. That is, the normal is along the radius, the line joining the centre of curvature of the mirror to the point of incidence.

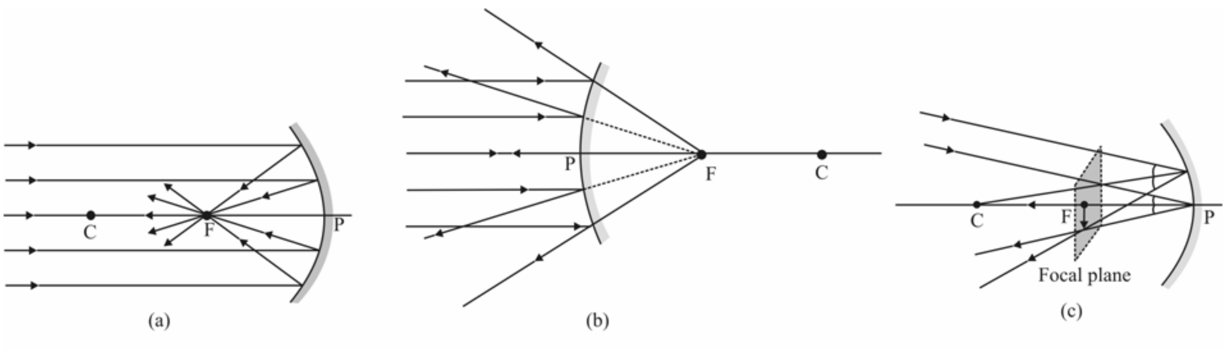
Terms associated with spherical mirrors

Pole- The geometric centre of a spherical mirror is called its **pole**.
Principal axis- The line joining the pole and the centre of curvature of the spherical mirror is known as the **principal axis**.
Paraxial Rays- The rays that are incident at points close to the pole P of the mirror and make small angles with the principal axis are called paraxial rays.
<u>**Focus**</u>
For Concave mirror
When a parallel beam of light is incident on a concave mirror, at points close to the pole of the mirror, P, the

reflected rays converge at a point F(principal focus of the mirror) on the axis for a concave mirror.

For Convex mirror

When a parallel beam of light is incident on a convex mirror, the reflected rays appear to diverge from a point F (principal focus of the mirror).The point F is called the **principal focus of the mirror**.



Focus of a concave and convex mirror.

Focal Length

The distance between the focus F and the pole P of the mirror is called the **focal length of the mirror.**

Focal Plane

If the parallel paraxial (close to the principal axis) beam were incident making some angle with the axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the axis. This is called the **focal plane of the mirror**.

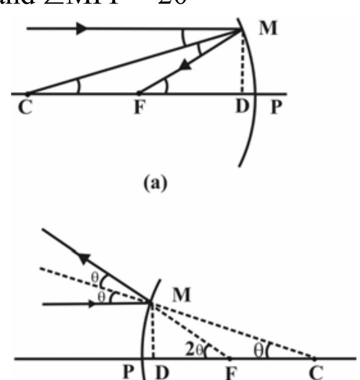
Derivation for focal length

The distance between the focus F and the pole P of the mirror is called the focal length of the mirror, denoted by f. We now show that f = R/2, where R is the radius of curvature of the mirror.

The geometry of reflection of an incident ray is shown in figure.

At the point of incidence M, applying the laws of reflection $\angle MCP = \theta$ and $\angle MFP = 2\theta$

Now $\tan\theta = \frac{MD}{CD}$, $\tan 2\theta = \frac{MD}{FD}$ (for small θ, $\tan\theta = \theta$ and $\tan 2\theta = 2\theta$);



thus we have $\frac{MD}{FD} = 2\frac{MD}{CD}$

or $FD = \frac{CD}{2}$ (1)

Also for small θ the point D is very close to the point P. Therefore,

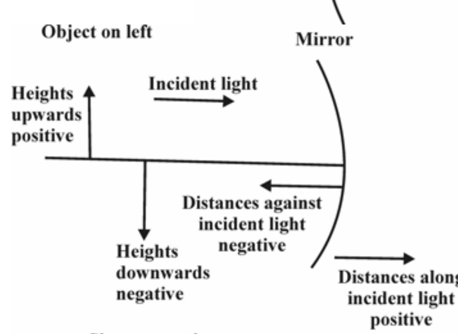
FD = f and CD = R; Equation (1) then gives F = R/2

Note: In this section V is the pole of the mirror

Sign Convention

New Cartesian sign convention. (see figure)

- The distances measured in the same direction as the incident light are taken as positive
- The distances measured in the direction opposite to the direction of incident light are taken as negative.
- The heights measured upwards (above x-axis) and normal to the principal axis of the mirror/lens are taken as positive.
- The heights measured downwards are taken as negative.



Sign convention

The Mirror Equation

Image of a point

If the rays starting from a point meet at another point after reflection and or refraction, the point is called the **image of the first point**.

Real Image – The image is real if the rays actually converges at the point

Virtual image - The image is virtual if the rays do not actually meet but appear to diverge from the point when produced backwards.

Image of an object

Take any two rays coming from an object, trace their paths, find their point of intersection and thus, obtain the image of the point. For convenience in application of geometry the following rays could be considered

(i) The ray which is parallel to the principal axis. The reflected ray goes through the focus of the mirror.

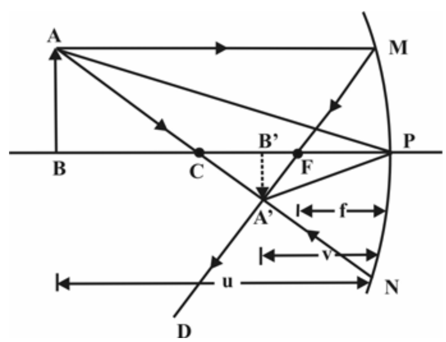
(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror. The reflected ray simply retraces the path.

(iii) The ray passing through the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror. The reflected ray is parallel to the principal axis.

Figure gives the ray diagram showing the image (in this case, real) of an object formed by a concave mirror.

Derivation of Mirror equation

Mirror equation is the relation between the object distance (u), image distance (v) and the focal length (f).



Using geometry the two right-angled triangles A'B'F and MPF are similar. (For paraxial rays, MP can be considered to be a straight line perpendicular to CP).

Therefore, $\frac{A'B'}{MP} = \frac{B'F}{PF}$

or $\frac{A'B'}{AB} = \frac{B'F}{PF}$ (1)

The right angled triangles A′B′P and ABP are also similar. Therefore,

$\frac{A'B'}{AB} = \frac{B'P}{BP}$ (2)

Comparing Eqs. (1) and (2), we get $\frac{B'P - PF}{PF} = \frac{B'P}{BP}$ (3)

Equation (3) is a relation involving magnitudes of the distance.

Applying sign conventions:

B′V = -v, VF = -f, BV = -u (4)

we get $\frac{-v + f}{-f} = \frac{-v}{-u}$ (5)

or $\frac{v - f}{f} = \frac{v}{u}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (6)

This relation is known as the **mirror equation**

Magnification (m) –

In triangles $A'B'P$ & ABP, we have

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

with sign convention, this becomes

$$-\frac{h'}{h} = \frac{-v}{-u} \Rightarrow \frac{h'}{h} = -\frac{v}{u}$$

Image formation for different cases

In this section we have derived the mirror equation, and the magnification formula, for the case of real, inverted image formed by a concave mirror only.

It is to be noted here that with the proper use of sign convention, these equations are valid for all the cases of reflection by a spherical mirror (concave or convex) whether the image formed is real or virtual.

Ray diagrams for image formations

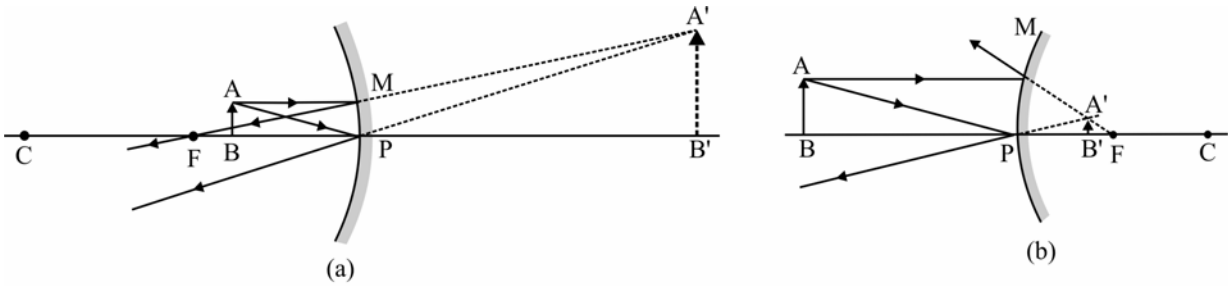


Image formation by (a) a concave mirror with object between P and F, and (b) a convex mirror.

The figures here show the ray diagrams for virtual image formed by a concave and convex mirror. It can be easily verified that the equations derived above are valid for these cases also

Refraction

When light travels from one medium to another, it changes the direction of its path at the interface of the two media. This is called **refraction of light**.

Laws of refraction

The following are the laws of refraction

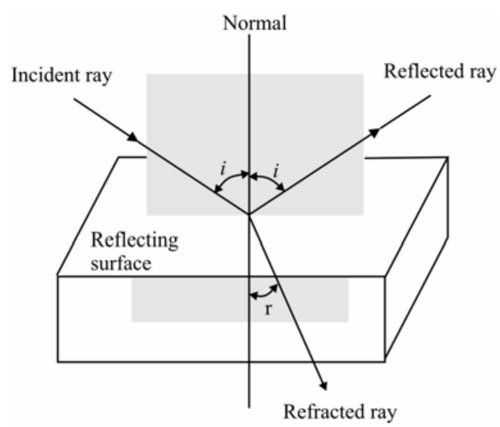
(i). The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

(ii). The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant.

Note: the angles of incidence (i) and refraction (r) are the angles that the incident and refracted rays make with the normal.

$$\frac{\sin i}{\sin r} = n_{21}$$

(where n_{21} is a constant, called the **refractive index** of the second medium with respect to the first medium and is independent of the angle of incidence.)



Refraction and Reflection of light.

Snell's law of refraction

Equation $\frac{\sin i}{\sin r} = n_{21}$, is the Snell's law of refraction

If $n_{21} > 1$, $r < i$, that is, the refracted ray bends towards the normal. The medium 2 is said to be optically denser than medium 1. On the other hand, if $n_{21} < 1$, $r > i$, the refracted ray bends away from the normal

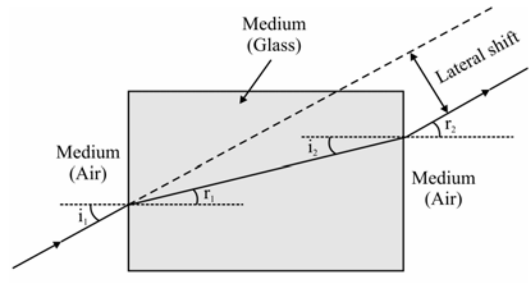
We have $n_{21} = \frac{n_2}{n_1}$

The equation shows that $n_{21} = \frac{1}{n_{12}}$ and $n_{32} = n_{31} \times n_{12}$

Note: Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media). For example in case of turpentine and water, the mass density of turpentine is less than that of water but its optical density is higher.

Refraction for a rectangular glass slab

Some elementary results based on the laws of refraction follow immediately. For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). It is easily seen from the figure that $r_2 = i_1$, i.e., the emergent ray is parallel to the incident ray i.e. there is no deviation, but it does suffer lateral displacement/shift with respect to the incident ray.

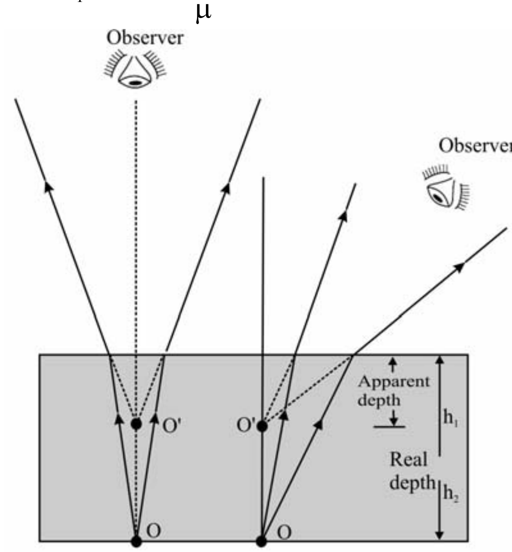


Lateral shift of a ray refracted through a parallel-sided slab.

Refraction for a water tank

Another familiar observation is that the bottom of a tank filled with water appears to be raised (as shown in the figure). For viewing near the normal direction, it can be shown that the apparent depth (h_1) is real depth (h_2) divided by the refractive index of the medium (water).

Refraction for a water tank formula - $h_1 = \frac{h_2(\text{real depth})}{\mu}$

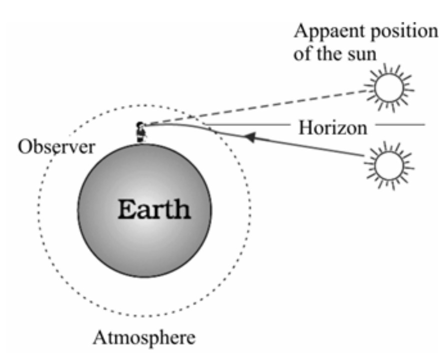


Apparent depth for (a) normal, and (b) oblique viewing.

Refraction of light through the atmosphere

It is because of the phenomenon of refraction that the sun is visible a little before the actual sunrise and until a little after the actual sunset due to refraction of light through the atmosphere.

- The refractive index of air with respect to vacuum is 1.00029.
- The apparent shift in the direction of the sun is by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes.
- The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to the same phenomenon.



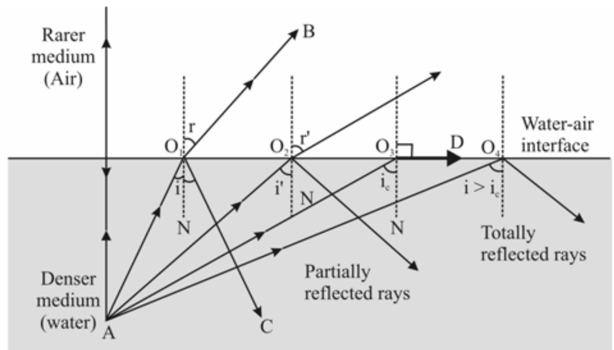
Advance sunrise and delayed sunset due to atmospheric refraction.

Note:

1. Actual sunrise means the actual crossing of the horizon by the sun.
2. The figure is highly exaggerated to show the effect.

Total Internal Reflection

When light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the internal reflection.



Refraction and internal reflectionof rays from a point A in the denser medium (water) incident at different angles at the interface with a rarer medium (air).

Let us consider the following when the light goes from the denser to rarer medium (as shown in the figure):

1. When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal, for example, the ray AO_1B in the figure.
2. The incident ray AO_1 is partially reflected (O_1C) and partially transmitted (O_1B) or refracted, the angle of refraction (r) being larger than the angle of incidence (i).
3. As the angle of incidence increases, so does the angle of refraction, till for the ray AO_3, the angle of refraction is $\frac{\pi}{2}$.
4. The refracted ray is bent so much away from the normal that it grazes the surface at the interface between the two media. This is shown by the ray AO_3D in the figure.
5. If the angle of incidence is increased still further (e.g., the ray AO_4), refraction is not possible, and the incident ray is totally reflected. This is called **total internal reflection.**

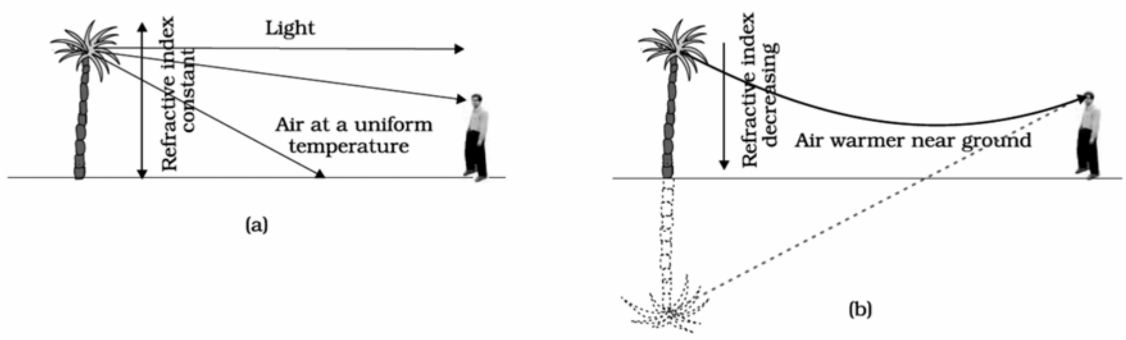
Some noteworthy points while taking into consideration the total internal refraction are:

- When light gets reflected by a surface, normally some fraction of it gets transmitted. The reflected ray, therefore, is always less intense than the incident ray, howsoever smooth the reflecting surface may be. In total internal reflection, on the other hand, no transmission of light takes place.
- The angle of incidence corresponding to an angle of refraction 90º, say AO_3N, is called the critical angle (i_c) for the given pair of media.
- We see from Snell's law that if the relative refractive index is less than one then, since the maximum value of sin r is unity, there is an upper limit to the value of sin i for which the law can be satisfied, that is, $i = i_c$ such that $\sin i_c = n_{21}$
- For values of i larger than i_c, Snell's law of refraction cannot be satisfied, and hence no refraction is

possible. The refractive index of denser medium 2 with respect to rarer medium 1 will be $n_{12} = \frac{1}{\sin(i_c)}$.

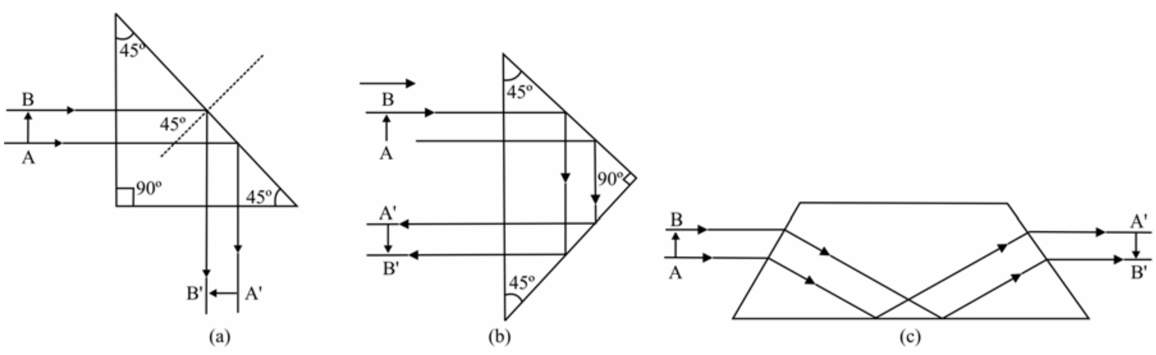
Examples and uses of total internal reflection

- **Mirage**: On still summer days, the air near the ground may become hotter than air further up. The refractive index of air increases with its density. Hotter air is less dense, and so has smaller refractive index than cooler air. So, light from a tall object such as a tree passes through a medium whose refractive index decreases towards the ground. Thus a ray of light from such an object gets bent and is totally internally reflected. Such inverted images of distant high objects cause the optical illusion called a **mirage**, specially common in hot deserts.



(a) A tree is seen by an observer at its place when the air above the ground is at uniform temperature, (b) When the layers of air close to the ground have varying temperature with hottest layers near the ground, light from a distant tree may undergo total internal reflection, and the apparent image of the tree may create an illusion to the observer that the tree is near a pool of water.

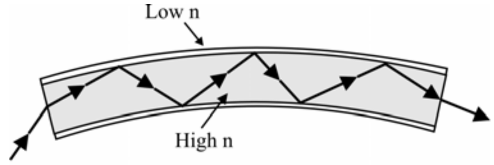
- **Diamond**: Total internal reflection is the main cause of the brilliance of diamonds. Its critical angle (24.4°) is very small, so that once light gets into diamond, it is very likely to be totally reflected internally. By cutting the diamond suitably, multiple internal reflections can be made to occur.



Prisms designed to bend rays by 90° and 180° or to invert image without changing its size make use of total internal reflection.

- **Prism**: Prisms make use of total internal reflection to bend light by 90° or by 180°, or to invert images without changing their size.

- **Optical fibres**: Optical fibres consist of many long high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding. A bundle of optical fibres can be put to several uses. It can be used as a

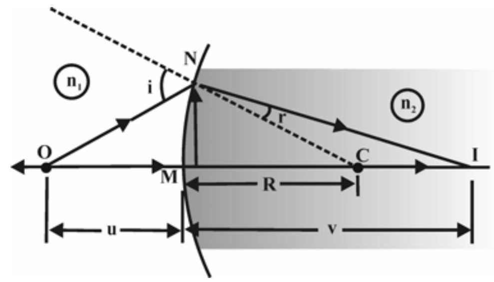


Light undergoes successive total internal reflections as it moves through an optical fibre.

'light pipe' in medical and optical examination. It can also be used for optical signal transmission. Optical fibres have also been used for transmitting and receiving electrical signals which are converted to light by suitable transducers. The main requirement is that there should be very little absorption of light as it has to travel long distances, inside the optical fibre.

Refraction at a Spherical Surface

Any small part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface. The normal at the point of incidence is perpendicular to the tangent plane to the surface at that point and, therefore, passes through the centre of curvature of the surface.



Consider refraction by a single spherical surface as shown in the figure. The figure shows the geometry of formation of image I of an object point O on the principal axis of the spherical surface with centre of curvature C, and radius of curvature R. The rays are incident from a medium of refractive index n_1 to another of refractive index n_2. We take the aperture (or the lateral size) of the surface to be small compared to other distances involved, so that small angle approximation can be made, wherever appropriate. In particular, NM will be taken to be nearly equal to the length of the perpendicular from N to the principal axis.

We have

$$\tan \angle NOM = \angle NOM = \frac{MN}{OM} \quad \text{(for small angles)},$$

$$\tan \angle NCM = \angle NCM = \frac{MN}{MC} \quad \text{(for small angles)},$$

$$\tan \angle NIM = \angle NIM = \frac{MN}{MI} \quad \text{(for small angles)}.$$

Now, for ΔNOC, i is the exterior angle. Therefore,

$$i = \angle NOM + \angle NCM$$

$$= \frac{MN}{OM} + \frac{MN}{MC} \qquad (1)$$

Similarly,

$$r = \angle NCM - \angle NIM$$

i.e., $$r = \frac{MN}{MC} - \frac{MN}{MI} \qquad (2)$$

Now, by Snell's law

$n_1 \sin i = n_2 \sin r$

Or for small angles

$n_1 i = n_2 r$

Substituting i and r from Eqs. (1) and (2), we get

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \qquad (3)$$

Here, OM, MI, MC represent magnitudes of distances. Applying the New Cartesian sign convention,

$$OM = -u,\ MI = +v,\ MC = +R.$$

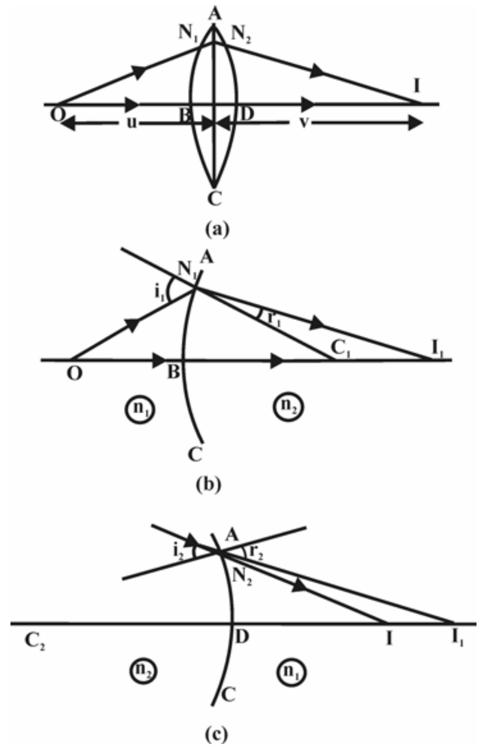
We get

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Refraction by a Lens

Lens - A thin lens is a transparent optical medium bounded by two spherical surfaces. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, we obtain the thin lens formula and the lens maker's formula.

The image formation can be seen in terms of two steps:
The first refracting surface forms the image I_1 of the object O. The image I_1 acts as a virtual object for formation of image I by the second surface.



Applying Eq. $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ to the first interface ABC, we get:

$$\frac{n_1}{OB}+\frac{n_2}{BI_1}=\frac{n_2-n_1}{BC_1} \qquad (1)$$

A similar procedure applied to the second interface ADC gives,

$$-\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC_2} \qquad (2)$$

For a thin lens, $BI_1 = DI_1$. Adding Equations (1) and (2) we get:

$$\frac{n_1}{OB}+\frac{n_1}{DI}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \qquad (3)$$

Special case

If the object is at infinity, $OB = \infty$ and I is at the focus of the lens so that DI = f, the focal length of the lens (f positive for a convex lens). Thus. Eq. (3) gives

$$\frac{n_1}{f}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \qquad (4)$$

By the sign convention,
$BC_1 = +R_1$, $DC_2 = -R_2$
So we get:

$$\frac{1}{f}=(n_{21}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \textbf{(lens maker's formula)} \qquad (5)$$

This equation is useful to design lenses of desired focal length using surfaces of suitable radii of curvature.
From Eqs. (3) and (4) we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=\frac{n_1}{f}$$

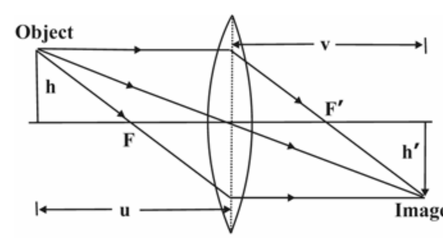
Again, in the thin lens approximation, B and D are both close to the optical centre of the lens. Applying the sign convention,
OB = - u, DI = +v, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \textbf{(thin lens formula)}$$

Note: This formula is true for a concave lens also. In that case R_1 is negative, R_2 positive and therefore f is negative.

Image of object through the lens

To find the image of an object by a lens, we can, in principle, take any two rays coming from an object point and trace their paths using the laws of refraction and find the point where the refracted rays meet (or appear to meet). In practice, however, it is convenient to choose any two of the following rays:

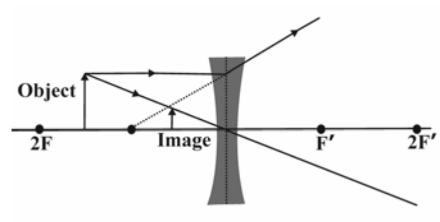


(a)

(i) A ray from the object parallel to the principal axis of the lens after refraction passes through the second principal focus F' (in a convex lens) or appears to diverge (in a concave lens) from the first principal focus F.

(ii) A ray of light, passing through the optical centre of the lens, emerges without any deviation after refraction.

(iii) A ray of light passing through the first principal focus (for a convex lens) or appearing to meet at it (for a concave lens) emerges parallel to the principal axis after refraction.



(b)

The figures illustrate these rules for a convex and a concave lens.

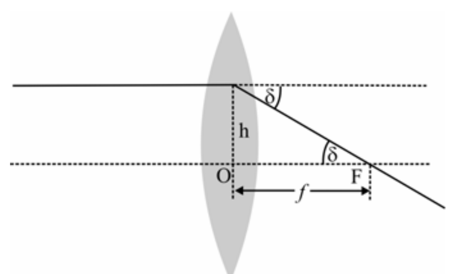
Magnification (m) – Magnification produced by a lens is defined as the ratio of the size of the image to that of the object. The size of the object h is always taken to be positive, but image size h is positive for erect image and negative for an inverted image. Proceeding in the same way as for spherical mirrors, it is easily seen that for a lens

$$m = \frac{h'}{h} = +\frac{v}{u}$$

Thus, for erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

Power of a lens

Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it. Clearly, a lens of shorter focal length bends the incident light more, while converging it in case of a convex lens and diverging it in case of a concave lens. The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical center.



Power of Lens

$$\tan\delta = \frac{h}{f};$$

if h = 1 then $\tan\delta = \frac{1}{f}$

or $\delta = \frac{1}{f}$ for small value of δ.

Thus, $p = \frac{1}{f}$

The SI unit for power of a lens is dioptre (D): 1D = 1m–1. The power of a lens of focal length of 1 metre is one dioptre. Power of a lens is positive for a converging lens and negative for a diverging lens. Thus, when an optician prescribes a corrective lens of power + 2.5 D, the required lens is a convex lens of focal length + 40 cm. A lens of power of – 4.0 D means a concave lens of focal length – 25 cm.

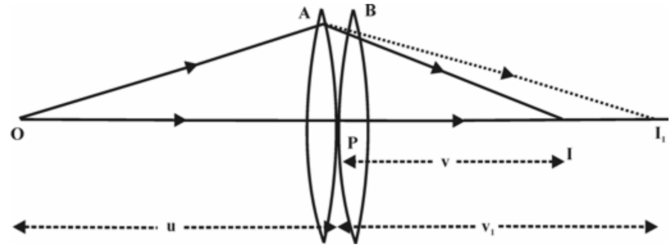
<u>**Combination of Thin Lenses in Contact**</u>

Two lenses in contact

Consider two lenses A and B of focal length f_1 and f_2 placed in contact with each other. Since the lenses are thin, we take the optical centres of the lenses to be coincident.

For the image formed by the first lens A,
We get

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$



(1)

For the image formed by the second lens B, we get

$$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2} \qquad (2)$$

Adding Eqs (1) and (2), we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{ where } \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

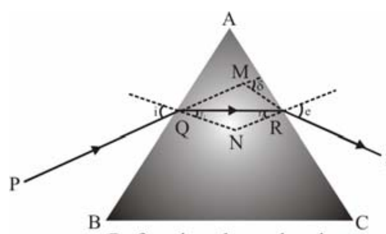
n lenses in contact

The equation is given as $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ where $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+.......$

Power is given as $P = P_1 + P_2 + P_3 + \ldots\ldots\ldots$ where P is the net power of the lens combination.
Note- Lens combination helps to increase the magnification and sharpness of the image.
Magnification
The total magnification m of the combination is a product of magnification (m_1, m_2, m_3,….,) of individual lenses: thus $m = m_1 \times m_2 \times m_3 \times \ldots$..

Refraction in a Prism

Figure shows the passage of light through a prism ABC.



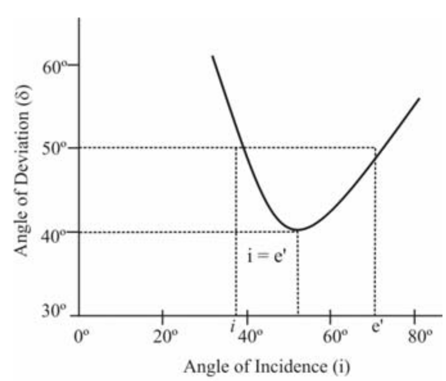
Refraction through prism

(i). The angles of incidence and refraction at the first face AB are i and r_1.
(ii). The angle of incidence (from glass to air) at the second face AC is r_2 and the angle of refraction or emergence is e.
(iii). The angle between the emergent ray RS and incident ray PQ is called the **angle of deviation, δ.**

In the quadrilateral AQNR, two of the angles (at the vertices Q and R) are right angles. Therefore, the sum of the other angles of the quadrilateral is 180°.
$\angle A + \angle QNR = \angle 180°$
from the triangle QNR, $r_1 + r_2 + \angle QNR = 180°$
Comparing these two equations, we get $r_1 + r_2 = A$
The total deviation δ is the sum of deviations at the two faces:
$\delta = (i - r_1) + (e - r_2)$
i.e. $\delta = i + e - A$

<u>Angle of deviation and angle of incidence</u>

Any given value of δ corresponds to two values i and e.
At the minimum deviation $\delta = D_m$, i = e which implies $r_1 = r_2$ and we have $r=\frac{A}{2}$
Also $D_m = 2i - A$, or $i = (A + D_m)/2$



Plot of angle of deviation (δ) versus angle of incidence (i) for a triangular prism.

The refractive index of the prism (or generally, the refractive index of the material of the prism with respect to the medium outside) is:

$$n_{21}=\frac{n_2}{n_1}=\frac{\sin[(A+D_m)/2]}{\sin[A/2]}$$

Dispersion by a Prism

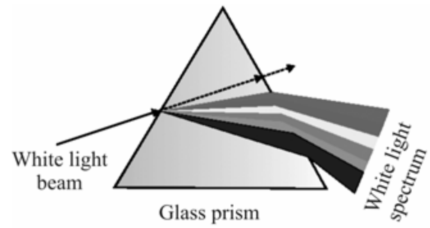
Dispersion is the splitting of light into its component colours and the pattern of colour components of light is called its **spectrum**.

(i). When a narrow beam of sunlight is incident on a glass prism, the emergent light is seen to be consisting of several colours.

(ii). The different component colours in sequence are: violet, indigo, blue, green, yellow, orange and red (given by the acronym VIBGYOR). The red light bends the least, while the violet light bends the most. White light itself consists of colours which are separated by the prism.

(iii). Colour is associated with wavelength of light. In the visible spectrum, red light is at the long wavelength end (~700 nm) while the violet light is at the short wavelength end (~ 400 nm).

(iv). Dispersion takes place because the refractive index of medium for different colours is different. Red light bends less than violet.



Dispersion of sunlight or white light on passing through a glass prism. The relative deviation of different colours shown in highly exaggerated

Note:
Thick lenses could be assumed as made of many prisms, therefore, thick lenses show chromatic aberration due to dispersion of light.

Some natural phenomena due to sunlight

The interplay of light with things around us gives rise to several beautiful phenomena. The spectacle of colour that we see around us all the time is possible only due to sunlight. The blue of the sky, white clouds, the redhue at sunrise and sunset, the rainbow, the brilliant colours of some pearls, shells, and wings of birds, are just a few of the natural wonders we are used to. We describe some of them here from the point of view of physics.

The rainbow

The rainbow is an example of the dispersion of sunlight by the water drops in the atmosphere. This is a phenomenon due to combined effect of dispersion, refraction and reflection of sunlight by spherical water droplets of rain. The conditions for observing a rainbow are that the sun should be

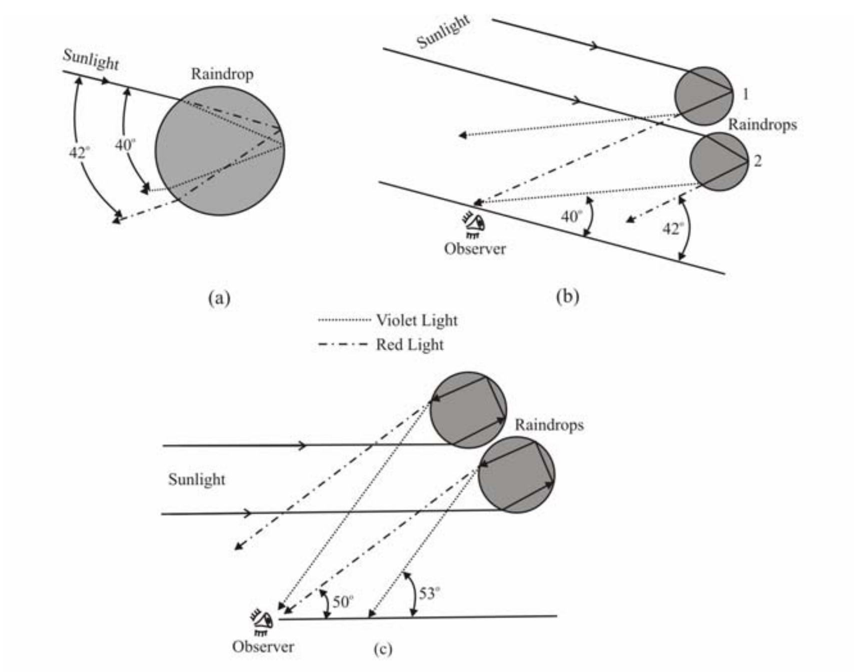


Figure : Rainbow: (a) The sun rays incident on a water drop get refracted twice and reflected internally by a drop; (b) Enlarge view of internal reflection and refraction of a ray of light inside a drop form primary rainbow; and (c) secondary rainbow is formed by rays undergoing internal reflection twice inside the drop.

shining in one part of the sky (say near western horizon) while it is raining in the opposite part of the sky (say eastern horizon). An observer can therefore see a rainbow only when his back is towards the sun.

In order to understand the formation of rainbows, consider the following:

1. The sunlight is first refracted as it enters a raindrop (as in the figure (a)), which causes the different wavelengths (colours) of white light to separate. Longer wavelength of light (red) are bent the least while the shorter wavelength (violet) are bent the most.
2. Next, these component rays strike the inner surface of the water drop and get internally reflected if the angle between the refracted ray and normal to the drop surface is greater than the critical angle (48º, in this case).
3. The reflected light is refracted again as it comes out of the drop as shown in the figure. It is found that the violet light emerges at an angle of 40º related to the incoming sunlight and red light emerges at an angle of 42º. For other colours, angles lie in between these two values.

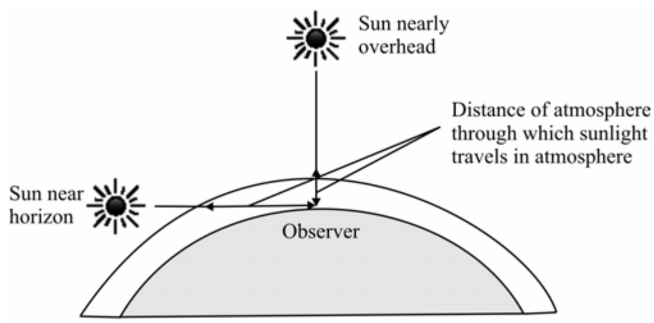
Primary rainbow- The figure (b) explains the formation of primary rainbow. We see that red light from drop 1 and violet light from drop 2 reach the observers eye. The violet from drop 1 and red light from drop 2 are directed at level above or below the observer. Thus the observer sees a rainbow with red colour on the top and violet on the bottom. Thus, the primary rainbow is a result of three-step process, that is, refraction, reflection and refraction.

Secondary Rainbow- When light rays undergoes two internal reflections inside a raindrop, instead of one as in the primary rainbow, a secondary rainbow is formed as shown in the figure (c). It is due to four-step process. The intensity of light is reduced at the second reflection and hence the secondary rainbow is fainter than the primary rainbow. Further, the order of the colours is reversed in it as is clear from the figure (c).

Scattering of light in earth's atmosphere

As the sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering

The various colours observed around us such as the blue colour of sky, white colour of clouds, red colour of setting sun etc. are because of scattering.



Sunlight travels through a longer distance in the atmosphere at sunset and sunrise.

Blueness of sky

Light of shorter wavelengths is scattered much more than the light of longer wavelengths. Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, because of a even shorter wavelength. But our eyes are more sensitive to blue than violet, hence we see the sky as blue.

Whiteness of clouds

Large particles like dust and water droplets present in the atmosphere behave differently. The relevant quantity here is the relative size of the wavelength of light λ, and the scatterer (of typical size, say, a). For a $<< \lambda$, one has Rayleigh scattering which is proportional to $\left(\frac{1}{\lambda^4}\right)$. For a $>> \lambda$, i.e., large scattering objects (for example, raindrops, large dust or ice particles) this is not true; all wavelengths are scattered nearly equally. Thus clouds which have droplets of water, with a$>>\lambda$,are generally white.

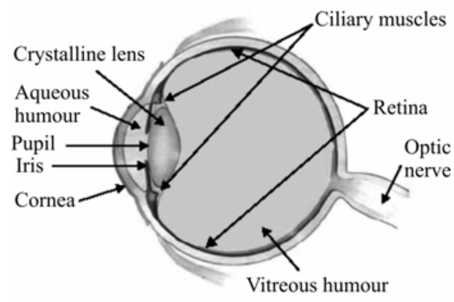
Reddishness observed in sun and moon

At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere (as shown in figure). Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon

The Eye and Optical instruments

The Eye

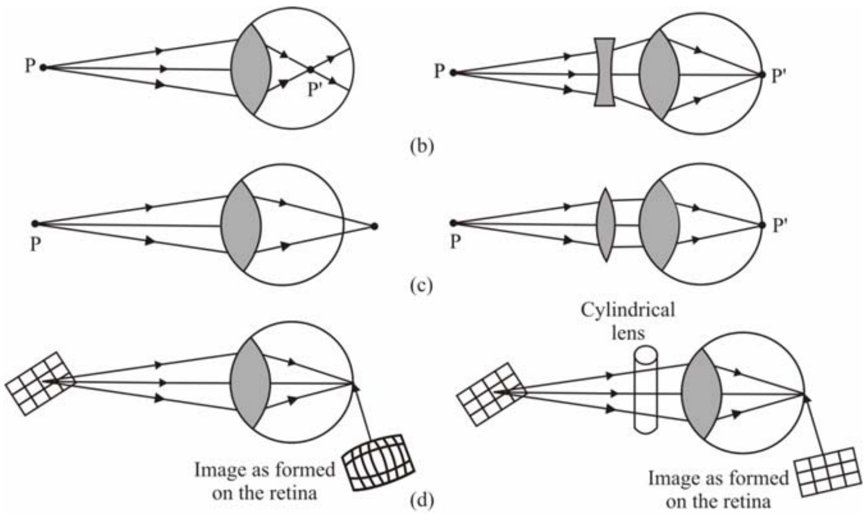
Sclera - The white part of the eye is called **sclera.**
Cornea –Light enters the eye through a curved front surface, the cornea.
Iris - The iris is the coloured part of the eye.
Pupil – It is a circular hole in the middle of the iris**.** Pupil regulates the amount of light passing through the retina to the back of the eye.
Retina - The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information.
The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles.
Accommodation – The ability of the lens to change its focus based on the image distance is called **accommodation**.
Working of the eye in brief - After entering from cornea light passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina then sends the information to the brain



(a)
The structure of the eye

<u>Defects in Eye</u>
A number of optical defects of eye are common.
Myopia - The lens may converge incident light to a point well before the retina. This is called **nearsightedness or myopia**. This means that the eye is producing too much convergence in the incident beam. To compensate this, we interpose a concave lens between the eye and the object, with the right diverging effect. The image will now be focused on the retina figure (b).

Hypermetropia
Similarly, if the lens focusses at a point behind the retina, convergent lens is needed to compensate. This defect is called **farsightedness or hypermetropia** figure (c).
Astigmatism - Another common defect of vision is called **astigmatism**. This occurs when the cornea is not spherical in shape. For example, the cornea could have a larger curvature in the vertical plane than in the horizontal plane. If one looks at a horizontal wire or line, focusing in the vertical plane is needed for a sharp image. Astigmatism results in lines in one direction being well focused while those in a perpendicular direction are not. It is corrected by a lens with one surface which is of cylindrical rather than spherical shape. A cylindrical surface focuses rays in one plane but not in a perpendicular plane. By choosing the radius of curvature and axis direction of the cylindrical surface, astigmatism can be corrected. This can occur along with myopia or hypermetropia.
Presbyopia – A defect (usually associated with old age) in eye where the eye loses its ability to focus objects close to eye i.e. the least distance of distinct vision increases (as much as 200 cm) .

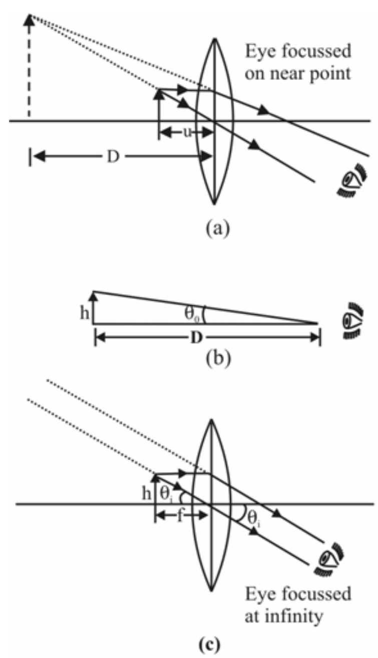


(b) shortsighted or myopic eye and its correction; (c) farsighted or hypermetropic eye and its correction; and (d) astigmatic eye and its correction.

The Microscope

Simple microscope

A simple magnifier or microscope is a converging lens of small focal length as shown in the figure. In order to use such a lens as a microscope, the lens is held near the object, one focal length away or less, and the eye is positioned close to the lens on the other side. The idea is to get an erect, magnified and virtual image of the object at a distance so that it can be viewed comfortably, i.e., at 25 cm or more. If the object is at a distance f, the image is at infinity. However, if the object is at a distance slightly less than the focal length of the lens, the image is virtual and closer than infinity. Although the closest comfortable distance for viewing the image is when it is at the near point (distance D $\cong$ 25 cm), it causes some strain on the eye. Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye. We show both cases, the first in Fig. (a), and the second in Fig. (b) and (c).



A simple microscope; (a) the magnifying lens is located such that the image is at the near point, (b) the angle subtanded by the object, is the same as that at the near point, and (c) the object near the focal point of the lens; the image is far off but closer than infinity.

<u>Magnification by microscope</u>

The linear magnification m, for the image formed at the near point D, by a simple microscope can be obtained by using the relation

$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right) = \left(1 - \frac{v}{f}\right)$$

Now according to our sign convention, v is negative, and is equal in magnitude to D. Thus, the magnification is

$$m = \left(1 + \frac{D}{f}\right)$$

Since D is about 25 cm, to have a magnification of six, one needs a convex lens of focal length, f = 5 cm.

Note that m = h′/h where h is the size of the object and h′ the size of the image. This is also the ratio of the angle subtended by the image to that subtended by the object, if placed at D for comfortable viewing. (Note that this is not the angle actually subtended by the object at the eye, which is h/u.) What a single-lens simple magnifier achieves is that it allows the object to be brought closer to the eye than D.

Image at infinity

We will now find the magnification when the image is at infinity. In this case we will have to obtain the angular magnification. Suppose the object has a height h. The maximum angle it can subtend, and be clearly visible (without a lens), is when it is at the near point, i.e., a distance D. The angle subtended is then given by $\tan\theta_0 = \left(\frac{h}{D}\right) \approx \theta_0$

We now find the angle subtended at the eye by the image when the object is at u. From the relations

$$\frac{h'}{h} = m = \frac{v}{u}$$

We have the angle subtended by the image

$$\tan\theta_i = \frac{h'}{-v} = \frac{h}{-v}.\frac{v}{u} = \frac{h}{-u} \approx \theta$$

The angle subtended by the object, when it is at u = –f

$$\theta_i = \left(\frac{h}{f}\right)$$

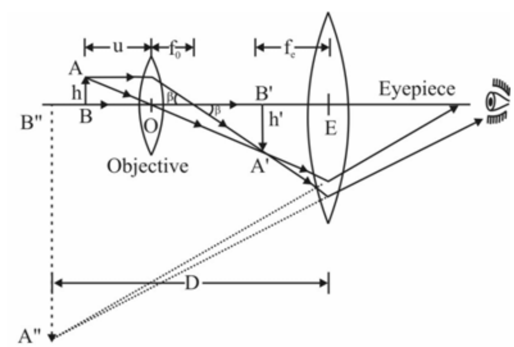
As is clear from Fig. The angular magnification is, therefore

$$m = \left(\frac{\theta_i}{\theta_0}\right) = \frac{D}{f}$$

This is one less than the magnification when the image is at the near point, but the viewing is more comfortable and the difference in magnification is usually small. In subsequent discussions of optical instruments (microscope and telescope) we shall assume the image to be at infinity.

Compound microscope

A simple microscope has a limited maximum magnification (≤ 9) for realistic focal lengths. For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope. A schematic diagram of a compound microscope is shown in the figure. The lens nearest the object, called the objective, forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the eyepiece, which functions essentially like a simple microscope or magnifier, produces the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.



Ray diagram for the formation of image by a compound microscope.

We now obtain the magnification due to a compound microscope. The ray diagram of Figure shows that the (linear) magnification due to the objective, namely h′/h, equals.

$$m_o = \frac{h'}{h} = \frac{L}{f_0}$$

where we have used the result

$$\tan\beta = \left(\frac{h}{f_0}\right) = \left(\frac{h'}{L}\right)$$

Here h′ is the size of the first image, the object size being h and f_o being the focal length of the objective. The first image is formed near the focal point of the eyepiece. The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope.

Magnification

As the first inverted image is near the focal point of the eyepiece, we use the result from the discussion above for the simple microscope to obtain the (angular) magnification me due to it , when the final image is formed at the near point, is

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

When the final image is formed at infinity, the angular magnification due to the eyepiece is

$m_e = (D/f_e)$

Image at infinity

Thus, the total magnification when the image is formed at infinity, is

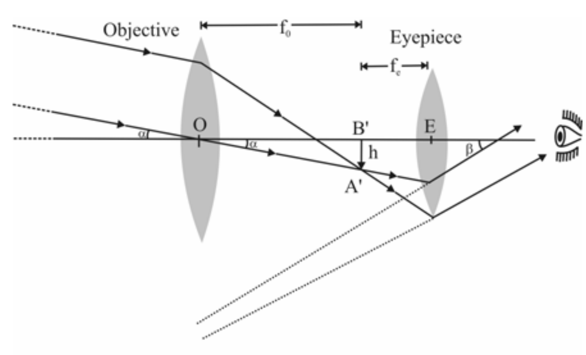
$$m = m_o m_e = \left(\frac{L}{f_0}\right)\left(\frac{D}{f_e}\right)$$

Note:

- To achieve a large magnification of a small object (hence the name microscope), the objective and eyepiece should have small focal lengths. In practice, it is difficult to make the focal length much smaller than
  1 cm. Also large lenses are required to make L large.
  Example: For an objective with f_o = 1.0 cm, and an eyepiece with focal length f_e = 2.0 cm, and a tube length of 20 cm, the magnification is
  $$m = m_o m_e = \left(\frac{L}{f_o}\right)\left(\frac{D}{f_o}\right)$$
  $$= \frac{20}{1} \times \frac{25}{2} = 250$$
- Various other factors such as illumination of the object, contribute to the quality and visibility of the image.
- In modern microscopes, multicomponent lenses are used for both the objective and the eyepiece to improve image quality by minimising various optical aberrations (defects) in lenses.

Telescope

The telescope is used to provide angular magnification of distant objects (as shown in Figure). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.



A refracting telescope

Magnification of telescope

The magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye. Hence

$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_o}{f_e}$$

In this case, the length of the telescope tube is $f_o + f_e$.

Note: Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect.

Refracting telescope

Refracting telescopes can be used both for terrestrial and astronomical observations.

Example: Consider a telescope whose objective has a focal length of 100 cm and the eyepiece a focal length of 1 cm. The magnifying power of this telescope is m = 100/1 = 100.

Let us consider a pair of stars of actual separation 1′ (one minute of arc). The stars appear as though they are separated by an angle of 100 × 1′ = 100′ =1.67º.

Some other noteworthy points regarding refracting telescopes are:

- The main considerations with an astronomical telescope are its light gathering power and its resolution or resolving power. The former clearly depends on the area of the objective. With larger diameters, fainter objects can be observed.
- The resolving power, or the ability to observe two objects distinctly, which are in very nearly the same direction, also depends on the diameter of the objective. So, the desirable aim in optical telescopes is to make them with objective of large diameter.
- The largest lens objective in use has a diameter of 40 inch (~1.02 m). It is at the Yerkes Observatory in Wisconsin, USA.

Reflecting telescopes

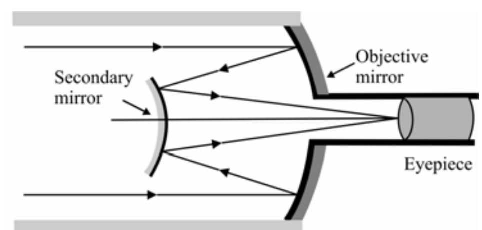
Very big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions. For these reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called **reflecting telescopes**.

Advatages -They have several advantages.

1. There is no chromatic aberration in a mirror.
2. If a parabolic reflecting surface is chosen, spherical aberration is also removed.
3. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim.

Cassegrain telescope

One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. One must have an eyepiece and the observer right there, obstructing some light (depending on the size of the observer cage). This is what is done in the very large 200 inch (~5.08 m) diameters, Mt. Palomar telescope, California. The viewer sits near the focal point of the mirror, in a small cage. Another solution to the problem is to deflect the light being focussed by another mirror. One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, is shown in Fig. This is known as a Cassegrain telescope, after its inventor. It has the advantages of a large focal length in a short telescope.



A refracting telescope. Schematic diagram of a reflecting telescope (Cassegrain)

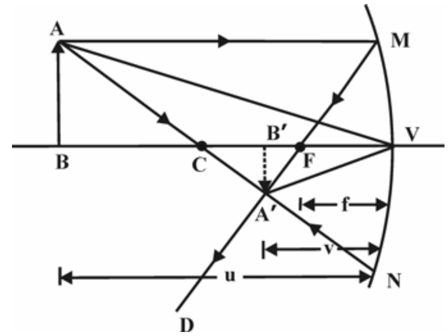
Note:

- The largest telescope in India is in Kavalur, Tamil Nadu. It is a 2.34 m diameter reflecting telescope (Cassegrain). It was ground, polished, set up, and is being used by the Indian Institute of Astrophysics, Bangalore.
- The largest reflecting telescopes in the world are the pair of Keck telescopes in Hawaii, USA, with a reflector of 10 metre in diameter.

SOLVED EXAMPLES

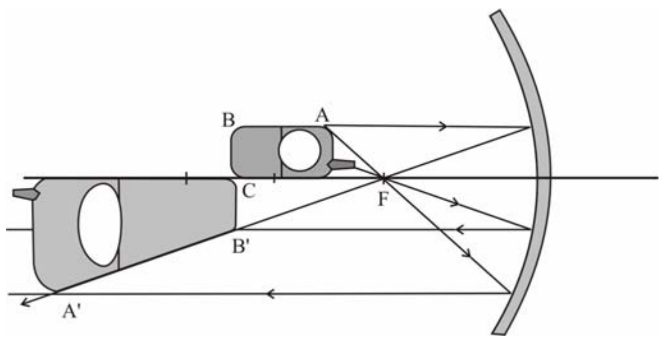
NCERT Solved Examples

NCERT 1: Suppose that the lower half of the concave mirror's reflecting surface in Figure is covered with an opaque (non-reflective) material. What effect will this have on the image of an object placed in front of the mirror?



Solution: You may think that the image will now show only half of the object, but taking the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low (in this case, half).

NCERT 2: A mobile phone lies along the principal axis of a concave mirror, as shown in Figure. Show by suitable diagram, the formation of its image. Explain why the magnification is not uniform. Will the distortion of image depend on the location of the phone with respect to the mirror?



Solution: The ray diagram for the formation of the image of the phone is shown in Figure. The image of the part which is on the plane perpendicular to principal axis will be on the same plane. It will be of the same size, i.e., B′C = BC. You can yourself realise why the image is distorted.

NCERT 3: An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Solution: The focal length f = –15/2 cm = – 7.5 cm

(i) The object distance u = –10 cm.

$$\frac{1}{v}+\frac{1}{-10}=\frac{1}{-7.5}$$

or $$v=\frac{10\times 7.5}{-2.5}=-30\,\text{cm}$$

The image is 30 cm from the mirror on the same side as the object.

Also, magnification $m = -\frac{v}{u}=-\frac{(-30)}{(-10)}=-3$

The image is magnified, real and inverted.

(ii) The object distance u = –5 cm.

$$\frac{1}{v}+\frac{1}{-5}=\frac{1}{-7.5}$$

or $$v=\frac{5\times 7.5}{(7.5-5)}=15\,\text{cm}$$

This image is formed at 15 cm behind the mirror. It is a virtual image.

Magnification $m = -\frac{v}{u} = -\frac{15}{(-5)} = 3$

The image is magnified, virtual and erect.

NCERT 4: Suppose while sitting in a parked car, you notice a jogger approaching towards you in the side view mirror of R = 2 m. If the jogger is running at a speed of 5 m s^{-1}, how fast the image of the jogger appear to move when the jogger is (a) 39 m, (b) 29 m, (c) 19 m, and (d) 9 m away.

Solution: From the mirror equation, we get

$$v = \frac{fu}{u - f}$$

For convex mirror, since R = 2 m, f = 1 m. Then

for u = –39 m, $v = \frac{(-39)\times 1}{-39-1} = \frac{39}{40}$ m

Since the jogger moves at a constant speed of 5 m s^{-1}, after 1 s the position of the image v (for u = –39 + 5 = –34) is (34/35)m.

The shift in the position of image in 1 s is

$$\frac{39}{40} - \frac{34}{35} = \frac{1365 - 1360}{1400} = \frac{5}{1400} = \frac{1}{280}\,m$$

Therefore, the average speed of the image when the jogger is between 39 m and 34 m from the mirror, is (1/280) m s^{-1}

Similarly, it can be seen that for u = –29 m, –19 m and –9 m, the speed with which the image appears to move is

$\frac{1}{150}\,m\,s^{-1}, \frac{1}{60}\,m\,s^{-1}$ and $\frac{1}{10}\,m\,s^{-1}$, respectively.

Although the jogger has been moving with a constant speed, the speed of his/her image appears to increase substantially as he/she moves closer to the mirror. This phenomenon can be noticed by any person sitting in a stationary car or a bus. In case of moving vehicles, a similar phenomenon could be observed if the vehicle in the rear is moving closer with a constant speed.

NCERT 5: The earth takes 24 h to rotate once about its axis. How much time does the sun take to shift by 1º when viewed from the earth?

Solution: Time taken for 360° shift = 24 h

Time taken for 1° shift = 24/360 h = 4 min.

NCERT 6: Light from a point source in air falls on a spherical glass surface (n = 1.5 and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Solution: Here u = – 100 cm, v =?, R = + 20 cm, n_1 = 1, and n_2 = 1.5.

We then have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20}$$

or v = + 100 cm

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

NCERT 7: A magician during a show makes a glass lens with n = 1.47 disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

Solution: The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This gives 1/f = 0 or $f \to \infty$. The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

NCERT 8: (i) If f = 0.5 m for a glass lens, what is the power of the lens? (ii) The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. Its focal length is 12 cm. What is the refractive index of glass? (iii) A convex lens has 20 cm focal length in air. What is focal length in water? (Refractive index of air-water = 1.33, refractive index for air-glass = 1.5.)

Solution: (i) Power = +2 dioptre.

(ii) Here, we have f = + 12 cm, R_1 = +10 cm, R_2 = –15 cm.

Refractive index of air is taken as unity.

We use the lens formula. The sign convention has to be applied for f, R_1 and R_2.

Substituting the values, we have

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

This gives n = 1.5.

(iii) For a glass lens in air, n_2 = 1.5, n_1 = 1, f = + 20 cm. Hence, the lens formula gives

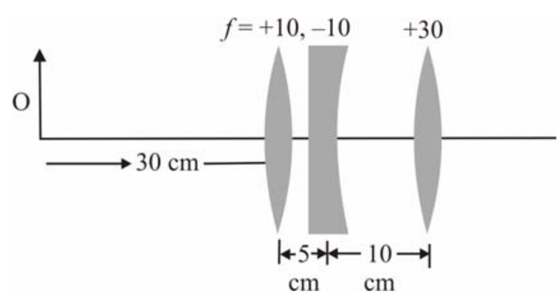
$$\frac{1}{20} = 0.5\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

For the same glass lens in water, n_2 = 1.5, n_1 = 1.33. Therefore,

$$\frac{1.33}{f} = (1.5-1.33)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Combining these two equations, we find f = + 78.2 cm.

NCERT 9: Find the position of the image formed by the lens combination given in the Figure.



Solution: Image formed by the first lens

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

or v_1 = 15 cm

The image formed by the first lens serves as the object for the second. This is at a distance of (15 – 5) cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens.

$$\frac{1}{v_2} - \frac{1}{10} = \frac{1}{-10}$$

or $v_2 = \infty$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens.

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

or $\frac{1}{v_3} - \frac{1}{\infty} = \frac{1}{30}$

or $v_3 = 30$ cm

The final image is formed 30 cm to the right of the third lens.

NCERT 10: What focal length should the reading spectacles have for a person for whom the least distance of distinct vision is 50 cm?

Solution: The distance of normal vision is 25 cm. So if a book is at u = –25 cm, its image should be formed at v = –50 cm. Therefore, the desired focal length is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

or $\frac{1}{f} = \frac{1}{-50} - \frac{1}{-25} = \frac{1}{50}$

or f = + 50 cm (convex lens).

NCERT 11: (a) The far point of a myopic person is 80 cm in front of the eye. What is the power of the lens required to enable him to see very distant objects clearly?

(b) In what way does the corrective lens help the above person? Does the lens magnify very distant objects? Explain carefully.

(c) The above person prefers to remove his spectacles while reading a book. Explain why?

Solution: (a) Solving as in the previous example, we find that the person should use a concave lens of focal length = – 80 cm, i.e., of power = – 1.25 dioptres.

(b) No. The concave lens, in fact, reduces the size of the object, but the angle subtended by the distant object at the eye is the same as the angle subtended by the image (at the far point) at the eye.

The eye is able to see distant objects not because the corrective lens magnifies the object, but because it brings the object (i.e., it produces virtual image of the object) at the far point of the eye which then can be focused by the eye-lens on the retina.

(c) The myopic person may have a normal near point, i.e., about 25 cm (or even less). In order to read a book with the spectacles, such a person must keep the book at a distance greater than 25 cm so that the image of the book by the concave lens is produced not closer than 25 cm. The angular size of the book (or its image) at the greater distance is evidently less than the angular size when the book is placed at 25 cm and no spectacles are needed. Hence, the person prefers to remove the spectacles while reading.

NCERT 12: (a) The near point of a hypermetropic person is 75 cm from the eye. What is the power of the lens required to enable the person to read clearly a book held at 25 cm from the eye?

(b) In what way does the corrective lens help the above person? Does the lens magnify objects held near the eye? (c) The above person prefers to remove the spectacles while looking at the sky. Explain why?

Solution: (a) u = – 25 cm, v = – 75 cm

1/f = 1/25 – 1/75, i.e., f = 37.5 cm.

The corrective lens needs to have a converging power of + 2.67 dioptres.

(b) The corrective lens produces a virtual image (at 75 cm) of an object at 25 cm. The angular size of this image is the same as that of the object. In this sense the lens does not magnify the object but merely brings the object to the near point of the hypermetric eye, which then gets focussed on the retina. However, the angular size is greater than that of the same object at the near point (75 cm) viewed without the spectacles.

(c) A hypermetropic eye may have normal far point i.e., it may have enough converging power to focus parallel rays from infinity on the retina of the shortened eyeball. Wearing spectacles of converging lenses (used for near vision) will amount to more converging power than needed for parallel rays. Hence the person prefers not to use the spectacles for far objects.

Additional Solved Examples

Example 1: An object is placed at a distance of 25cm on the axis of a concave mirror having focal length of 20cm. Find the image distance, type of the image and it's lateral magnification.

Solution:

object distance, u = - 25 cm

focal length, f = - 20 cm

The general equation for spherical mirrors is,

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\therefore \quad -\frac{1}{20}=-\frac{1}{25}+\frac{1}{v}$$

$$\therefore v=\frac{20\times 25}{-5}=-100\,\text{cm}$$

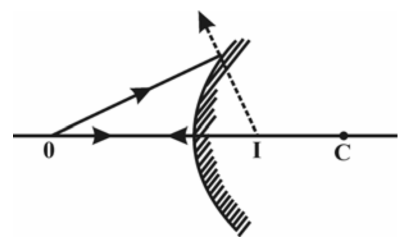
Since the image distance is negative, the image is on the same side as the object. Thus it is a real image.

Lateral magnification, $m=-\frac{v}{u}=-\frac{(-100)}{(-25)}=-4$

Thus the image is magnified 4 times and the negative sign shows that it is inverted.

Example 2: A convex mirror has its radius of curvature 20 cm. Find the position of the image of an object placed at a distance of 12 cm from the mirror.

Solution: The situation is shown in figure. Here u = – 12 cm and R = + 20 cm. We have,



$$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$$

or, $\frac{1}{v}=\frac{2}{R}-\frac{1}{u}$

$$=\frac{2}{20\text{ cm}}-\frac{1}{-12\text{cm}}=\frac{11}{60\text{ cm}}$$

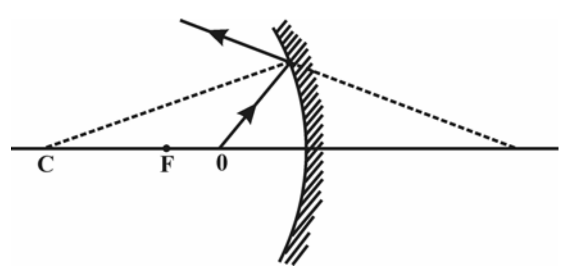
or, $v=\frac{60}{11}$ cm.

The positive sign of v shows that the image is formed on the right side of the mirror. It is a virtual image.

Example 3: An object is placed on the principal axis of a concave mirror of focal length 10 cm at a distance of 8 cm from the pole. Find the position and the nature of the image.

Solution: Here u = – 8 cm and f = – 10 cm.

We have, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$



or, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

$$=\frac{1}{-10\text{cm}}=\frac{1}{-8\text{cm}}$$

$$=\frac{1}{40\text{cm}}$$

or, $v=40\text{cm}$.

The positive sign shows that the image is formed at 40 cm from the pole on the other side of the mirror (figure). As the image is formed beyond the mirror, the reflected rays do not intersect, the image is thus virtual.

Example 4: At what distance from a convex mirror of focal length 2.5 m should a boy stand so that his image has a height equal to half the original height? The principal axis is perpendicular to the height.

Solution: We have,

$$m = -\frac{v}{u} = \frac{1}{2}$$

or, $v = -\frac{u}{2}$.

Also, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

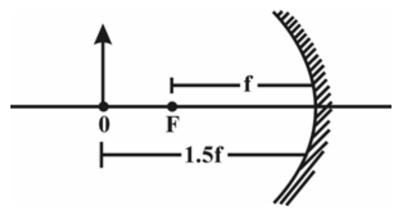
or, $\frac{1}{u} + \frac{1}{-u/2} = \frac{1}{2.5\text{m}}$

or, $-\frac{1}{u} = \frac{1}{2.5\text{ m}}$

or, u = – 2.5 m.

Thus, he should stand at a distance of 2.5 m from the mirror.

Example 5: An object of length 2.5 cm is placed at a distance of 1.5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principal axis. Find the length of the image. Is the image erect or inverted?



Solution: The given situation is shown in figure. The focal length F = –f, and u = –1.5 f. We have,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{F} \text{ or, } \frac{1}{-1.5f} + \frac{1}{v} = \frac{1}{-f}$$

or, $\frac{1}{v} = \frac{1}{1.5f} - \frac{1}{f} = \frac{-1}{3f}$

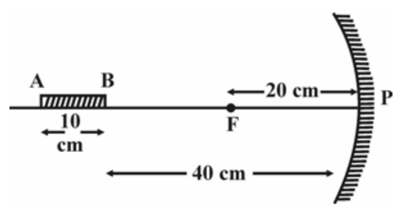
or, v = –3 f.

Now, $m = -\frac{v}{u} = \frac{3f}{-1.5f} = -2$

or, $\frac{h_2}{h_1} = -2$ or, $h_2 = -2h_1 = -5.0\text{cm}$.

The image is 5 cm long. The minus sign shows that it is inverted.

Example 6: As shown in the figure a thin rod AB of length 10 cm is placed on the principal axis of a concave mirror such that it's end B is at a distance of 40 cm from the mirror. If the focal length of the mirror is 20 cm, find the length of the image of the rod.



Solution: f = 20 cm and the end B is at distance 40 cm = 2f = R. Thus the image of B is formed at B only.

Now for end A,

u = - 50 cm, f = - 20 cm

In $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, putting these values

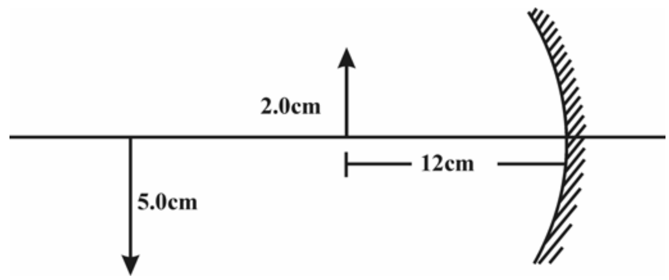
$$-\frac{1}{50} + \frac{1}{v} = -\frac{1}{20}$$

$$\therefore \quad \frac{1}{v} = \frac{1}{50} - \frac{1}{20} = \frac{20-50}{20 \times 50} = -\frac{30}{1000}$$

$\therefore \quad v = -\frac{100}{3} = -33.3\,\text{cm}$

This image is on the same side as the object.
Now, length of the image = 40 – 33.3 = 6.70 cm

Example 7: A 2.0 cm high object is placed on the principal axis of a concave mirror at a distance of 12 cm from the pole. If the image is inverted, real and 5.0 cm high, find the location of the image and the focal length of the mirror.



Solution: The magnification is $m = -\frac{v}{u}$

or, $\frac{-5.0\text{ cm}}{2.0\text{ cm}} = \frac{-v}{-12\text{ cm}}$

or, $v = -30$ cm.

The image is formed at 30 cm from the pole on the side of the object. We have,

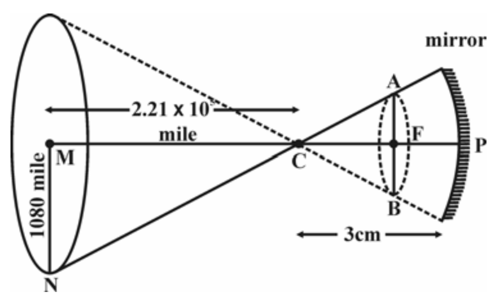
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{-30\text{ cm}} + \frac{1}{-12\,\text{cm}} = -\frac{7}{60\text{ cm}}$$

or, $f = -\frac{60\text{ cm}}{7} = -8.6$ cm.

The focal length of the mirror is 8.6 cm.

Example 8: Diameter of the moon is 2160 miles. It is 2.21×10^5 miles away from a concave mirror having radius of curvature 3 m. Find the diameter of the image formed in the mirror. Neglect the radius of curvature of the mirror in comparison to the distance of the moon. The mirror is placed such that it's principal axis passes through the centre of the moon.



Solution: Since the moon is very far away from the mirror, it's image will be formed at the principal focus of the mirror. This image is shown by AB in the diagram. Form the geometry of the diagram,

$$\frac{AF}{MN} = \frac{CF}{CM}$$

$$\therefore AF = \frac{CF \times MN}{CM}$$

$$= \frac{1.5 \times 1080}{2.21 \times 10^5} \left(\because \text{in comparison to MP, CP is negligible} = 2.21 \times 10^5 \text{ mile}\right)$$

$$\therefore \text{Diameter of the image} = \frac{2 \times 1.5 \times 1080}{2.21 \times 10^5}$$

$$\therefore = 14.7 \times 10^{-3}\text{ m}$$

Example 9: Derive the formula for lateral magnification, $m = \frac{f}{f - u}$ for spherical mirrors; where f = focal length and u = object distance.

Solution: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

$= \frac{u-f}{uf}$

$\therefore v = \frac{fu}{u-f}$

$\therefore \frac{v}{u} = \frac{f}{u-f}$

Now, $m = -\frac{v}{u} = \frac{f}{f-u}$

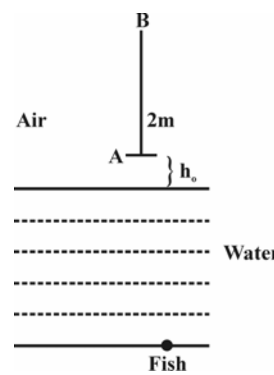
Example 10: A printed page is kept pressed by a glass cube (μ = 1.5) of edge 6.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top?

Solution: The thickness of the cube = t = 6.0 cm. The shift in the position of the printed letters is

$$\Delta t = \left(1 - \frac{1}{\mu}\right)t$$

$$= \left(1 - \frac{1}{1.5}\right) \times 6.0 \text{ cm} = 2.0 \text{ cm.}$$

Example 11: A swimmer is diving in a swimming pool vertically with a velocity of 2 m s^{-1}. What will be the velocity as seen by a stationary fish at the bottom of the pool, right below the diver? Refractive index of water is 1.33.



Solution: In the figure, vertical distance 2 m is shown by AB. The height of A from the surface of water is h_o. Suppose it's apparent height is h_i ($h_i > h_o$).

$$\therefore \frac{h_i}{h_o} = \frac{n(\text{water})}{n(\text{air})}$$

$$\therefore h_i = h_o \times 1.33 \qquad (1)$$

Now the real height of B, $h_o' = (h_o + 2)$m

$\therefore$ if it's apparent height is h_i' then

$$\frac{h_i'}{h_o'} = \frac{n(\text{water})}{n(\text{air})} = 1.33$$

$$\therefore h_i' = h_o' \times 1.33$$

$$= (h_o + 2) \times 1.33 \qquad (2)$$

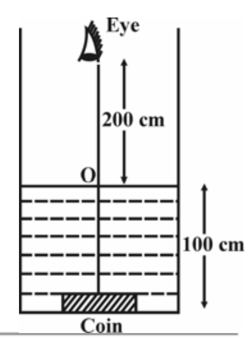
From equations (1) and (2), the apparent distance, seen by the fish

$$= h_i' - h_i = (h_o + 2) \times 1.33 - h_o \times 1.33$$

$$= 2 \times 1.33 = 2.66\text{m}$$

So the fish will see the swimmer falling with a speed of 2.66 m s^{-1}.

Example 12: As shown in the figure a coin is lying at the bottom of a barrel filled with 100 cm of water. If viewed normally from 200 cm above the surface of the water, what will be the apparent depth of the coin? Is this image real or virtual? (Refractive index of water is 1.33).



Solution: For a curved surface,

$$-\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{R} \qquad (1)$$

It can also be used for a plane surface which is a special case of a curved surface with . Thus for a plane surface,

$$-\frac{n_1}{u}+\frac{n_2}{v}=\frac{n_2-n_1}{\infty}=0 \qquad (2)$$

While using equation (1) remember that n_1 is the refractive index of the medium in which the object is lying and n_2 is the refractive index of the material in which the refracted ray travels; so

n_1 = refractive index of water = 1.33 and n_2 = 1 (refractive index of air).

According to our sign convention., u = - 100 cm, v =?

Putting there values in equation (2)

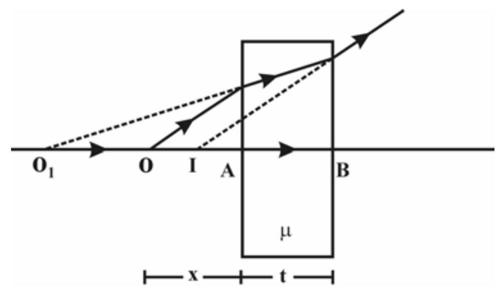
$$-\frac{1.33}{(-100)}+\frac{1}{v}=0$$

$$\therefore \qquad v=-75.2\,\text{cm}$$

Hence the apparent depth of the coin is 75.2 cm. Since v is negative, it shows that the image is virtual.

Example 13: A point object O is placed in front of a transparent slab at a distance x from its closer surface. It is seen from the other side of the slab light incident nearly normally to the slab. The thickness of the slab is t and its refractive index is μ. Show that the apparent shift in the position of the object is independent of x and find its value.

Solution:



The situation is shown in figure. Because of the refraction at the first surface, the image of O is formed at O_1. For this refraction, the real depth is AO = x and the apparent depth is AO_1. Also, the first medium is air and the second is the slab. Thus,

$$\frac{x}{AO_1}=\frac{1}{\mu} \text{ or, } AO_1=\mu x.$$

The point O_1 acts as the object for the refraction at the second surface. Due to this refraction, the image of O_1 is formed at I. Thus,

$$\frac{BO_1}{BI}=\mu$$

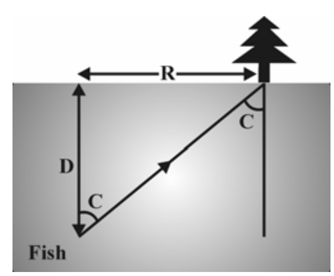
$$\text{or, } \frac{AB+AO_1}{BI}=\mu \text{ or, } \frac{t+\mu x}{BI}=\mu$$

$$\text{or, } BI=x+\frac{t}{\mu}.$$

The net shift is $OI=OB-BI=(x+t)-\left(x+\frac{t}{\mu}\right)$

$$=t\left(1-\frac{1}{\mu}\right), \text{ which is independent of x.}$$

Example 14: A fish is at some depth in a lake. It is at horizontal distance R = 1.5 m from the edge of the lake. If it is just able to see a tree on the edge of the lake what is it's depth? (Refractive index of water 1.33)



Solution: As shown in the figure, the line joining the fish and the base of the tree makes critical angle C with the base of the tree. Thus the fish can see the whole tree.

Now, $\sin C = \frac{1}{n}$

From the figure, $\sin C = \frac{R}{\sqrt{R^2 + D^2}}$

$$\therefore \frac{R}{\sqrt{R^2 + D^2}} = \frac{1}{n}$$

$$\therefore R^2 + D^2 = R^2 n^2$$

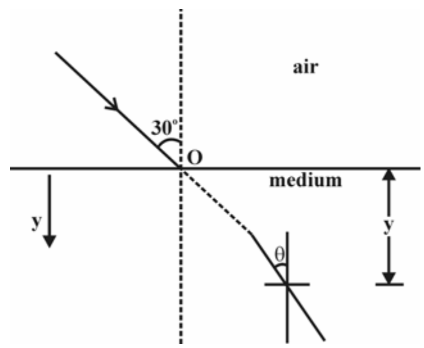
$$\therefore D = R(n^2 - 1)^{\frac{1}{2}}$$

$$= 1.5[(1.33)^2 - 1]^{\frac{1}{2}}$$

$$\therefore D = 1.3\text{m}$$

Example 15: As shown in figure, a ray of light in air is incident at $30°$ on a medium and proceeds ahead in the medium. The refractive index of this medium varies with distance y as given by,

$$n(y) = 1.6 + \frac{0.2\text{cm}^2}{(y + 1\text{cm})^2}$$



where y is in cm.
What is the angle formed by the ray with the normal, at a very large depth?

Solution: Suppose the angle is θ at distance y in the medium.
Applying Snell's law at this point,

$n(y)\sin\theta = C, \text{where } C = \text{constant} (1)$

This formula is true for all the points.
Applying it to point O,

$n(0)\sin 30° = C$

But, $n(0) = 1.6 + \frac{0.2}{(0+1)^2} = 1.8$

$$\therefore 1.8 \times \frac{1}{2} = C$$

$$\therefore C = 0.9$$

Putting this value in (1),

$n(y)\sin\theta = 0.9$

$$\therefore \left\{1.6 + \frac{0.2}{(y+1)^2}\right\}\sin\theta = 0.9$$

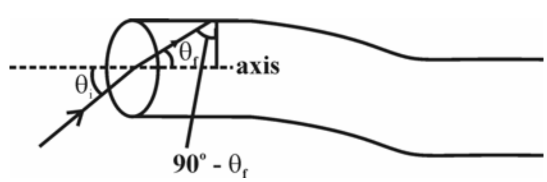
$$\therefore \sin\theta = \frac{0.9}{1.6 + \frac{0.2}{(y+1)^2}}$$

When y is very large, taking $y \to \infty$

$$\sin\theta = \frac{0.9}{1.6}$$

$$\therefore \theta = 34°14'$$

Example 16: A light ray is entering an optical fibre as shown in the figure. What should be the minimum value of the refractive index (n) of the fibre so that the ray does not come out of the fibre?



Solution: If the critical angle for the material of the fibre w.r.t air is C, then for total internal reflection to occur (from the diagram),

$90° - \theta_f > C$

$\therefore \sin(90° - \theta_f) > \sin C$

but, $\sin C = \frac{1}{n}; n =$ refrective index of the material of the fibre

$\therefore \sin(90° - \theta_f) > \frac{1}{n}$

$\therefore n\cos\theta_f > 1$ (1)

Applying Snell's law at the point where the ray enters the fibre,

$\sin\theta_i = n\sin\theta_f$

$\therefore \sin\theta_f = \frac{\sin\theta_i}{n}$

$\therefore \cos\theta_f = \sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{\sin^2\theta_i}{n^2}}$

$\therefore n\cos\theta_f = \sqrt{n^2 - \sin^2\theta_i}$

Using this in equation (1),

$\sqrt{n^2 - \sin^2\theta_i} > 1$

We require that the ray should not come out of the fibre for any angle of incidence.

Now, the maximum value of $\sin\theta_i = 1$. Thus, if the above condition is satisfied for $\theta_i = 90°$, it would be satisfied far any value of θ_i,

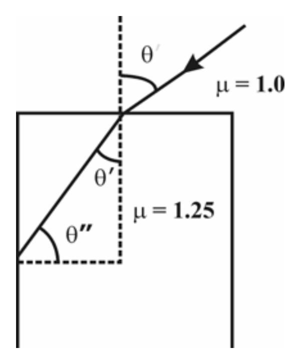
$\therefore \sqrt{n^2 - 1} > 1$

$\therefore n^2 - 1 > 1$

$\therefore n > \sqrt{2}$

Thus the value of n should be greater than $\sqrt{2}$. Thus, if the value of refractive index is greater than $\sqrt{2}$, then rays incident at any angle will undergo total internal reflection.

Example 17: Consider the situation shown in figure. Find the maximum angle θ for which the light suffers total internal reflection at the vertical surface.



Solution: The critical angle for this case is

$\theta'' = \sin^{-1}\frac{1}{1.25} = \sin^{-1}\frac{4}{5}$

or, $\sin\theta'' = \frac{4}{5}$

Since $\theta'' = \frac{\pi}{2} - \theta'$, we have $\sin\theta' = \cos\theta'' = 3/5$. From Snell's law,

$\frac{\sin\theta}{\sin\theta'} = 1.25$

or, $\sin\theta = 1.25 \times \sin\theta'$

$= 1.25 \times \frac{3}{5} = \frac{3}{4}$

or, $\theta = \sin^{-1}\frac{3}{4}$.

If θ'' is greater than the critical angle, θ will be smaller than this value. Thus, the maximum value of θ, for which total reflection takes place at the vertical surface, is $\sin^{-1}(3/4)$.

Example 18: A beaker contains water upto a height h_1 and oil above water upto another height h_2. Find the apparent shift in the position of the bottom of the beaker when viewed from above. Refractive index of water is μ_1 and that of oil is μ_2.

Solution: The apparent shift of the bottom due to the water is

$$\Delta t_1 = \left(1 - \frac{1}{\mu_1}\right) h_1$$

and due to the oil is

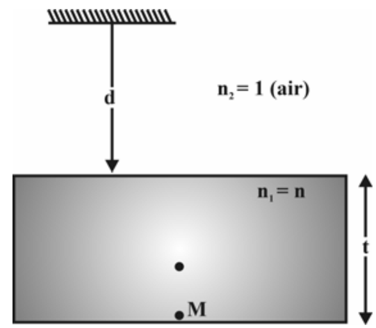
$$\Delta t_2 = \left(1 - \frac{1}{\mu_2}\right) h_2 .$$

The total shift $= \Delta t_1 + \Delta t_2 = \left(1 - \frac{1}{\mu_1}\right) h_1 + \left(1 - \frac{1}{\mu_2}\right) h_2$.

Example 19: The critical angle for water is 48.2°. Find its refractive index.

Solution: $\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin 48.2^\circ} = 1.34$.

Example 20: As shown in the figure, a marking M is made in a medium of thickness t and refractive index n. As shown, a plane mirror is kept at height d from the upper surface of the medium. Find the position of the image of M obtained by the mirror.



Solution: $\frac{h_i}{h_o} = \frac{n_2}{n_1} = \frac{1}{n}$

$\therefore h_i = \frac{h_o}{n} = \frac{t}{n} (\because h_o = t)$

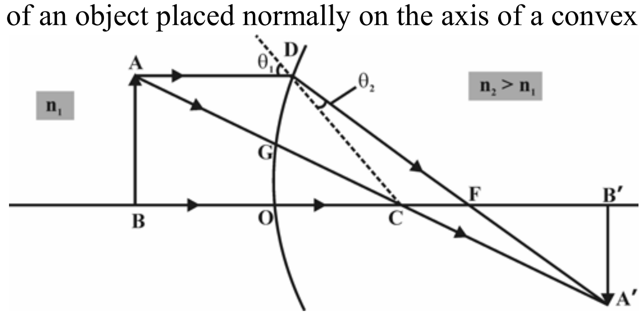
Thus, M is seen at depth $\frac{t}{n}$ from the upper surface of the medium.

$\therefore$ Distance of this image from the mirror $= d + \frac{t}{n}$

$\therefore$ the image formed by the mirror is obtained at the same distance, $d + \frac{t}{n}$, on the upper side of the mirror.

Example 21: Draw the ray diagram to obtain the image of an object placed normally on the axis of a convex spherical refracting surface.

Solution: The ray diagram is as shown below.



AB is the object. According to Snell's law, ray AD parallel to the axis, passes through the principal focus F after refraction at D. Note that F is not the centre of curved surface or the centre of the sphere from where the curved surface is cut. As shown in the figure ray AG is normal to the curved surface. (C is the centre of curvature), so it goes along path GC undeviated and meets ray DA′; i.e., the image of A is formed at A′. Ray BO moving along the axis goes along the path OCB′. From the symmetry it can be said that A′B′ is the real image of object AB.

Example 22: For a thin lens prove that when the heights of the object and the image are equal, object distance and image distance are equal to 2f.

Solution: Here, $|h| = |h'|$

$\therefore |v| = |u|$

Using the equation for lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-u}=\frac{1}{f}$$

$$\therefore \frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

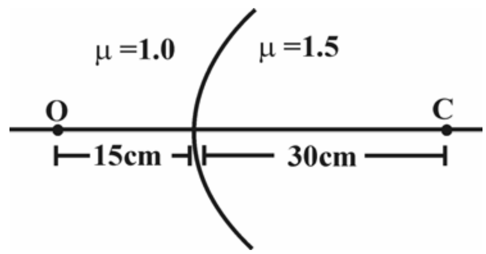
$$\therefore \frac{2}{v}=\frac{1}{f}$$

$\therefore v = 2f$

$\therefore u = v = 2f$

Example 23: Locate the image of the point object O in the situation shown in figure. The point C denotes the centre of curvature of the separating surface.

Solution: Here u = – 15 cm, R = 30 cm, $\mu_1 = 1$ and $\mu_2 = 1.5$. We have,



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

or, $\frac{1.5}{v}-\frac{1.0}{-15\text{cm}}=\frac{1.5-1}{30\,\text{cm}}$

or, $\frac{1.5}{v}=\frac{0.5}{30\,\text{cm}}-\frac{1}{15\,\text{cm}}$

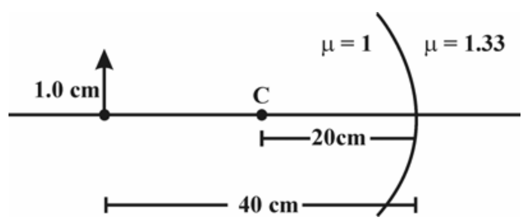
or, $v = -30$ cm.

The image is formed 30 cm left to the spherical surface and is virtual.

Example 24: Find the size of the image formed in the situation shown in figure.

Solution: Here u = – 40 cm, R = – 20 cm, $\mu_1 = 1$, $\mu_2 = 1.33$.

We have,



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

or, $\frac{1.33}{v}-\frac{1}{-40\,\text{cm}}=\frac{1.33-1}{-20\,\text{cm}}$

or, $\frac{1.33}{v}=-\frac{1}{40\,\text{cm}}-\frac{0.33}{20\,\text{cm}}$

or, $v = -32$ cm.

The magnification is

$$m=\frac{h_2}{h_1}=\frac{\mu_1 v}{\mu_2 v}$$

or, $\frac{h_2}{1.0\text{ cm}}=\frac{-32\,\text{cm}}{1.33\times(-40\text{cm})}$

or, $h_2 = +0.6$ cm.

The image is erect.

Example 25: A biconvex lens has radii of curvature 20 cm each. If the refractive index of the material of the lens is 1.5, what is its focal length?

Solution: In a biconvex lens, centre of curvature of the first surface is on the positive side of the lens and that of the second surface is on the negative side. Thus, R_1 = 20 cm and R_2 = – 20 cm.

We have,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20\text{ cm}} - \frac{1}{-20\text{cm}}\right)$

or, f = 20 cm.

Example 26: An object of length 2.0 cm is placed perpendicular to the principal axis of a convex lens of focal length 12 cm. Find the size of the image if the object is at a distance of 8.0 cm from the lens.

Solution: We have u = – 8.0 cm., and f = + 12 cm

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

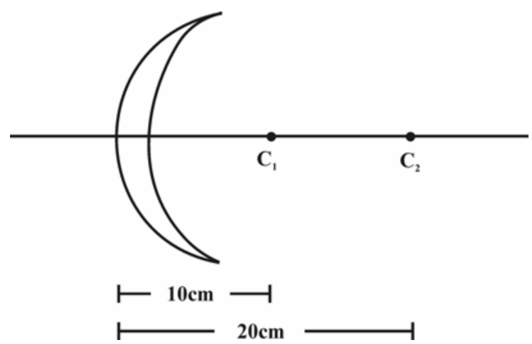
$$\frac{1}{v} = \frac{1}{12\,\text{cm}} + \frac{1}{-8.0\text{ cm}}$$

or, v = – 24 cm.

Thus, m = $\frac{v}{u} = \frac{-24\text{cm}}{-8.0\text{cm}}$ = 3.

Thus, $h_2 = 3\,h_1 = 3 \times 2.0$ cm = 6.0 cm. The positive sign shows that the image is erect.

Example 27: Calculate the focal length of the thin lens shown in figure. The points C_1 and C_2 denote the centres of curvature.



Solution: As is clear from the figure, both the radii of curvature are positive. Thus, R_1 = + 10 cm and R_2 = + 20 cm. The focal length is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{10\text{ cm}} - \frac{1}{20\text{ cm}}\right)$$

$$= 0.5 \times \frac{1}{20\,\text{cm}} = \frac{1}{40\text{ cm}}$$

or, f = 40 cm.

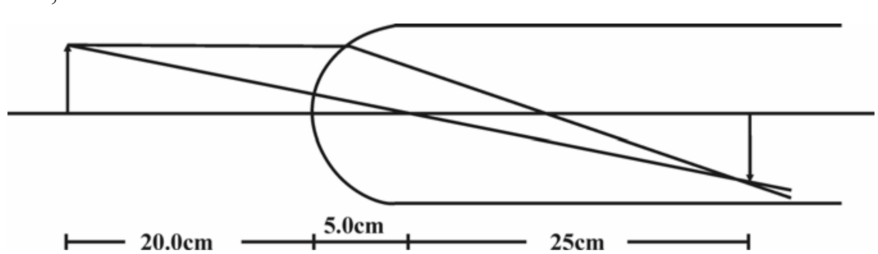
Example 28: One end of a horizontal cylindrical glass rod $(\mu = 1.5)$ of radius 5.0 cm is rounded in the shape of a hemisphere. An object 0.5 mm high is placed perpendicular to the axis of the rod at a distance of 20.0 cm from the rounded edge. Locate the image of the object and find its height.

Solution: Taking the origin at the vertex, u = – 20.0 cm and R = 5.0 cm.

We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

or,

$$\frac{1.5}{v} = \frac{1}{-20.0\text{ cm}} + \frac{0.5}{5.0\text{ cm}} = \frac{1}{20\text{ cm}}$$



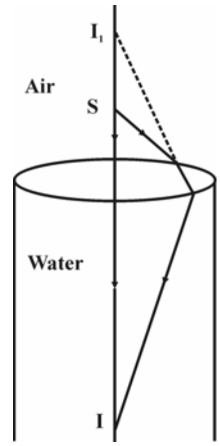
or, v = 30 cm.

The image is formed inside the rod at a distance of 30 cm from the vertex.

The magnification is $m = \frac{\mu_1 v}{\mu_2 u}$

$$= \frac{30\text{ cm}}{-1.5 \times 20\text{ cm}} = -1.$$

Thus, the image will be of same height (0.5 mm) as the object but it will be inverted.



Example 29: A long cylindrical tube containing water is closed by an equiconvex lens of focal length 10 cm in air. A point source is placed along the axis of the tube outside it at a distance of 21 cm from the lens. Locate the final image of the source. Refractive index of the material of the lens = 1.5 and that of water = 1.33.

Solution:

The light from the source S gets refracted at the air-glass interface and then at the glass-water interface. Referring to the figure, let us take vertically downward as the positive direction of the axis.

If the image due to the refraction at the first surface is formed at an image-distance v_1 from the surface, we have,

$$\frac{1.5}{v_1} - \frac{1}{u} = \frac{1.5-1}{R}, \qquad .....(i)$$

where R is the radius of curvature of the surface. As the lens is equiconvex, the radius of curvature of the second surface is − R. Also, the image formed by the first surface acts as the object for the second surface. Thus,

$$\frac{1.33}{v} - \frac{1.5}{v_1} = \frac{1.33-1.5}{-R} \qquad ....(ii)$$

Adding (i) and (ii),

$$\frac{1.33}{v} - \frac{1}{u} = \frac{1}{R}(0.5+0.17) = \frac{0.67}{R}$$

or, $\frac{1.33}{v} - \frac{1}{-21\text{cm}} = \frac{0.67}{R}$

or, $\frac{4}{3v} + \frac{1}{21\text{cm}} = \frac{2}{3R}$

or, $\frac{1}{v} = \frac{1}{2R} - \frac{1}{28\text{cm}}$ .....(iii)

The focal length of the lens in air is 10 cm. Using

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

$$\frac{1}{10\text{cm}} = (1.5-1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

or, R = 10 cm.

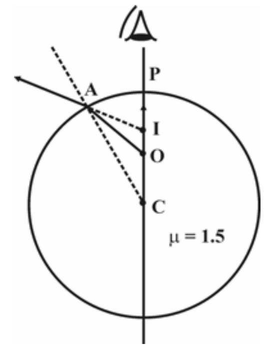
Thus, by (iii),

$$\frac{1}{v} = \frac{1}{20\text{cm}} - \frac{1}{28\text{cm}}$$

or, v = 70 cm.

The image is formed 70 cm inside the tube.

Example 30: There is a small air bubble inside a glass sphere $(\mu = 1.5)$ of radius 10 cm. The bubble is 4.0 cm below the surface and is viewed normally from the outside (figure). Find the apparent depth of the bubble.



Solution: The observer sees the image formed due to refraction at the spherical surface when the light from the bubble goes from the glass to the air.

Here u = – 4.0 cm, R = – 10 cm, μ_1 = 1.5 and μ_2 = 1.

We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

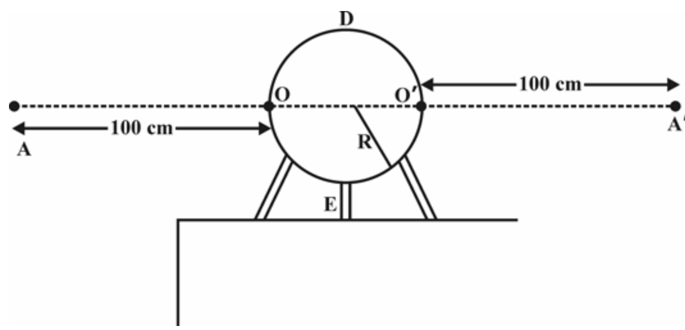
or, $$\frac{1}{v} - \frac{1.5}{-4.0\,\text{cm}} = \frac{1-1.5}{-10\text{ cm}}$$

or, $$\frac{1}{v} = \frac{0.5}{10\text{ cm}} - \frac{1.5}{4.0\text{ cm}}$$

or, v = – 3.0 cm.

Thus, the bubble will appear 3.0 cm below the surface.

Example 31: As shown in the figure a crystal ball of radius 25 cm and refractive index 1.5 is kept on a table. An object A is kept at a distance of 100 cm from the surface DOE on the axis. Find the position of it's image formed by the crystal ball.



Solution: Here, the rays coming from A are refracted twice, at DOE and DO′E, and then the final image is formed. We can use the formula,

$$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R} \qquad (1)$$

for both the surfaces.

For the first surface DOE, AO = u = - 100 cm. The refractive index of the medium containing the object is n_1 = 1 (for air). R = + 25 cm. For the medium to the right of O, n_2 = 1.5.

Putting these values in equation (1),

$$-\frac{1.00}{-100} + \frac{15}{v} = \frac{15 - 1.00}{25}$$

$$\therefore v = 150\,\text{cm}$$

This distance is positive, which shows that the image is formed 150 cm to the right of the first refracting surface.

For the refraction by the second surface, the image formed by the first surface taken as the object (virtual object !!) and it's distance should be taken from the second surface. In our example this distance is (150 – 50) = 100 cm, as shown in the diagram.

Note that the sign convention used for the first surface will continue for the second surface also. That is, all distances to the right of O′ are to be taken positive and to the left are negative.

∴ for the second surface, R = -25 cm and u = 100 cm

Also, the ray refracted by the first surface, travels through the medium of glass and is incident on the second surface. Thus, for the second surface n_1 = 1.50 (Refractive index of glass) and since the ray finally goes in air, n_2 = 1. Putting values in equation (1).

$$-\frac{1.5}{100} + \frac{1}{v} = \frac{1 - 1.5}{-25}$$

$$\therefore v = 28.6\,\text{cm}$$

So the final image is at distance 28.6 cm to the right of surface DO′E.

Example 32: Light from a point source in air falls on a spherical glass surface (n=1.5, radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position is the image formed?

Solution: We use the formula

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

Here,

u = -100 cm, R = + 20 cm, n_1 = 1, and n_2 = 1.5

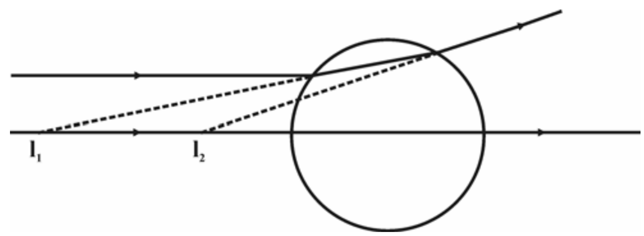
We then have

$$\frac{1.5}{v}+\frac{1}{100}=\frac{0.5}{20}$$

or v = +100 cm.

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

Example 33: A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial, (i) find the position of the image due to refraction at the first surface and the position of the final image, and (ii) draw a ray diagram showing the positions of both the images.



Solution:

The ray diagram is shown in figure. The equation for refraction at a spherical surface is

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}. \qquad .....(i)$$

For the first refraction (water to air); $\mu_1=1.33, \mu_2=1, u=\infty$, R = + 2 mm.

Thus, $\frac{1}{v}=\frac{1-1.33}{2\text{ mm}}$

or, v = – 6 mm.

The negative sign shows that the image I_1 is virtual and forms at 6 mm from the surface of the bubble on the water side. The refracted rays (which seem to come from I_1) are incident on the farther surface of the bubble. For this refraction,

$\mu_1=1,\ \mu_2=1.33$, R = – 2 mm.

The object distance is u = – (6 mm + 4 mm) = –10 mm.

Using equation (i),

$$\frac{1.33}{v}+\frac{1}{10\text{ mm}}=\frac{1.33-1}{-2\text{ mm}}$$

or, $\frac{1.33}{v}=\frac{0.33}{2\text{ mm}}-\frac{1}{10\text{ mm}}$

or, v = – 5 mm.

The minus sign shows that the image is formed on the air side at 5 mm from the refracting surface. Measuring from the centre of the bubble, the first image is formed at 8.0 mm from the centre and the second image is formed at 3.0 mm from the centre. Both images are formed on the side from which the incident rays are coming.

Example 34: A thin lens of focal length + 12 cm is immersed in water $(\mu=1.33)$. What is its new focal length?

Solution: We have, $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

when the lens is placed in air, f = 12 cm. Thus,

$$\frac{1}{12\text{ cm}}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or, $\frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{6\text{ cm}}$.

If the focal length becomes f' when placed in water,

$$\frac{1}{f'}=\left(\frac{1.5}{1.33}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\frac{1}{8}\times\frac{1}{6\,\text{cm}}=\frac{1}{48\text{ cm}}$$ or, $f'=48$ cm.

Example 35: The angle of minimum deviation from a prism is 37°. If the angle of prism is 53°, find the refractive index of the material of the prism.

Solution: $$\mu=\frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}=\frac{\sin\frac{53^\circ+37^\circ}{2}}{\sin\frac{53^\circ}{2}}=\frac{\sin 45^\circ}{\sin 26.5^\circ}=1.58.$$

Example 36: For a prism, angle of prism is $60°$ and it's refractive index is 1.5; find (1) angle of incidence corresponding to the angle of minimum deviation and (2) angle of emergence for angle of maximum deviation.

Solution:

(1) For minimum deviation,

$r_1=r_2$ and $A=r_1+r_2$

$\therefore A=2r_1$

or $r_1=\frac{A}{2}=\frac{60}{2}=30°$

Now n = 1.5 and

$n=\frac{\sin i_1}{\sin r_1}$

$\therefore n\sin r_1=\sin i_1$

$\therefore 1.5\times\sin 30°=\sin i_1$

$\therefore 1.5\times 0.5=\sin i_1$

$\therefore i_1=48°35'$

(2) For maximum deviation, $i_1=90°$

$\therefore \quad 1.5=\frac{\sin 90°}{\sin r_1}\ \therefore r_1=41°48'$

$\therefore \quad r_2=A-r_1=60-41°48'=18°12' \qquad (\because r_1+r_2=A)$

Example 37: An equilateral prism is kept in air and for a particular ray, angle of minimum deviation is $38°$. Calculate the angle of minimum deviation if the prism is immersed in water. Refractive index of water is 1.33.

Solution: $\frac{n_g}{n_a}=\frac{\sin\left(\frac{60+38}{2}\right)^o}{\sin 30°}$

Taking $n_a = 1$,

$n_g=\frac{\sin 49°}{\sin 30°}=1.509$

When prism is immersed in water,

$\frac{n_g}{n_w}=\frac{\sin\left(\frac{60+\delta_m}{2}\right)^o}{\sin 30°}$

But $n_w=1.33$

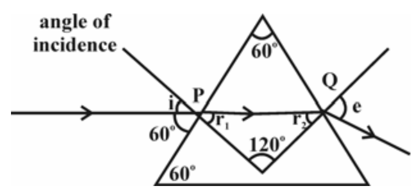
$\therefore \frac{1.509}{1.33}=\frac{\sin\left(\frac{60+\delta_m}{2}\right)^o}{0.5}$

$\therefore \sin\left(\frac{60+\delta_m}{2}\right)^o=\frac{0.5\times 1.509}{1.33}=0.5673$

$\therefore \frac{60+\delta_m}{2}=\sin^{-1}(0.5673)=34^o34'$

$\therefore \delta_m=9^o8'$

Example 38: As shown in the diagram, a ray is incident parallel to the base of an equilateral prism. If the refractive index of the material of the prism is 1.5. Calculate the deviation of the ray.



Solution: It is clear from the figure that angle of incidence $i=30°$.

Using Snell's law at point P, sin i = n sin r_1

$\therefore \sin 30°=1.50\sin r_1$

$\therefore \frac{0.5}{1.5}=\sin r_1$

$\therefore r_1=19.5°$

Now using Snell's law at point Q,

$n\sin r_2=\sin e$ (1)

But, from the geometry of the figure

$r_1+r_2=180°-120°$

$\therefore 19.5°+r_2=60°\Rightarrow r_2=40.5°$

Inserting the value of r_2 in equation (1)

$1.50\times\sin 40.5°=\sin e$

$\therefore \sin e=0.97$

$\therefore e=77°$

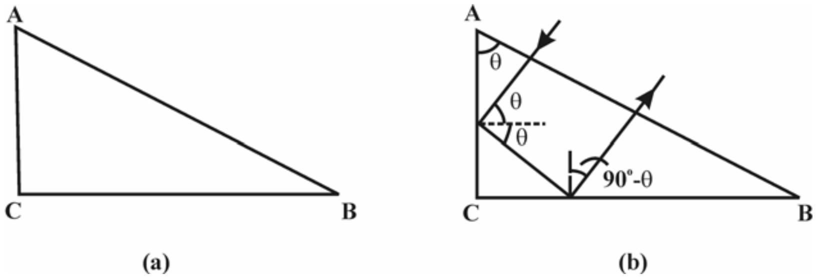
Now

$i+e=A+\delta$

$\therefore \delta=i+e-A=30°+77°-60°=47°$

Example 39: A right prism is to be made by selecting a proper material and the angles A and B ($B\le A$) as shown in figure. It is desired that a ray of light incident normally on AB emerges parallel to the incident direction after two internal reflections. (a) What should be the minimum refractive index μ for this to be possible? (b) For $\mu = 5/3$, is it possible to achieve this with the angle A equal to 60 degrees?

Solution:



(a) Consider the ray incident normally on AB (figure b). The angle of reflection at the surface AC is θ. It is clear from the figure that the angle of incidence at the second surface CB is $90^\circ - \theta$. The emergent ray will be parallel to the incident ray after two total internal reflections. The critical angle θ_c should be less than θ as well as $90^\circ - \theta$. Thus, θ_c should be smaller than or equal to the smaller of θ and $90^\circ - \theta$. i.e., $\theta_c \le \min(\theta, 90^\circ - \theta)$.

As min $(\theta, 90^\circ - \theta) \le 45^\circ, \theta_c \le 45^\circ$

or, $\sin\theta_c \le \frac{1}{\sqrt{2}}$ or, $\frac{1}{\mu} \le \frac{1}{\sqrt{2}}$

or, $\mu \ge \sqrt{2}$.

Thus, the refractive index of the material of the prism should be greater than or equal to $\sqrt{2}$. In this case the given ray can undergo two internal reflections for a suitable θ.

(b) For $\mu = 5/3$, the critical angle θ_c is $\sin^{-1}(3/5) = 37^\circ$.

As the figure suggests, we consider the light incident normally on the face AB. The angle of incidence θ on the surface AC is equal to $\theta = 60^\circ$. As this is larger than the critical angle 37°, total internal reflection takes place here. The angle of incidence at the surface CB is $90^\circ - \theta = 30^\circ$. As this is less than the critical angle, total internal reflection does not take place at this surface.

Example 40: A young boy can adjust the power of his eye-lens between 50 D and 60 D. His far point is infinity. (a) What is the distance of his retina from the eye-lens? (b) What is his near point?

Solution: (a) When the eye is fully relaxed, its focal length is largest and the power of the eye-lens is minimum. This power is 50 D according to the given data. The focal length is $\frac{1}{50}$ m = 2 cm. As the far point is at infinity, the parallel rays coming from infinity are focused on the retina in the fully relaxed condition. Hence, the distance of the retina from the lens equals the focal length which is 2 cm.

(b) When the eye is focused at the near point, the power is maximum which is 60 D. The focal length in this case is $f = \frac{1}{60}$ m = $\frac{5}{3}$ cm. The image is formed on the retina and thus v = 2 cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{2\text{cm}} - \frac{3}{5\text{cm}}$

or, u = – 10 cm.

The near point is at 10 cm.

Example 41: A nearsighted man can clearly see objects up to a distance of 1.5 m. Calculate the power of the lens of the spectacles necessary for the remedy of this defect.

Solution: The lens should form a virtual image of a distance object at 1.5 m from the lens. Thus, it should be

a divergent lens and its focal length should be -1.5 m.
Hence, f = – 1.5 m

or, $P=\frac{1}{f}=-\frac{1}{1.5}m^{-1}=-0.67$ D.

Example 42: Two boys, one 52 inches tall and the other 55 inches tall, are standing at distances 4.0 m and 5.0 m respectively from an eye. Which boy will appear taller?
Solution: The angle subtended by the first boy on the eye is

$$\alpha_1=\frac{52\text{ inch}}{4.0\text{ m}}=13\text{ inch/m}$$

and the angle subtended by the second boy is

$$\alpha_2=\frac{55\text{ inch}}{5.0\text{ m}}=11\text{ inch/m.}$$

As $\alpha_1>\alpha_2$, the first boy will look taller to the eye.

Example 43: The near and far points of a person are at 40 cm and 250 cm respectively. Find the power of the lens he/she should use while reading at 25 cm. With this lens on the eye, what maximum distance is clearly visible?
Solution: If an object is placed at 25 cm from the correcting lens, it should produce the virtual image at 40 cm. Thus, u = – 25 cm, v = – 40 cm.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$=-\frac{1}{40cm}+\frac{1}{25cm}$$

or, $f=\frac{200}{3}cm=+\frac{2}{3}m$

or, $P=\frac{1}{f}=+1.5D.$

The unaided eye can see a maximum distance of 250 cm. Suppose the maximum distance for clear vision is d when the lens is used. Then the object at a distance d is imaged by the lens at 250 cm. We have,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

or, $-\frac{1}{250cm}-\frac{1}{d}=\frac{3}{200cm}$

or, d = – 53 cm.
Thus, the person will be able to see upto a maximum distance of 53 cm.

Example 44: A 10 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification for a normal eye (near point = 25 cm)?
Solution: Maxima angular magnification is achieved when the final image is formed at the near point. Thus, v = – 25 cm. The focal length is $f=\frac{1}{10}$ m = 10 cm.

We have,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

or, $-\frac{1}{25cm}-\frac{1}{u}=\frac{1}{10cm}$

or, $\frac{1}{u} = -\frac{1}{25cm} - \frac{1}{10cm}$

or, $u = -\frac{50}{7} cm = -7.1 cm.$

Example 45: An object is seen through a simple microscope of focal length 12 cm. Find the angular magnification produced if the image is formed at the near point of the eye which is 25 cm away from it.

Solution: The angular magnification produced by a simple microscope when the image is formed at the near point of the eye is given by

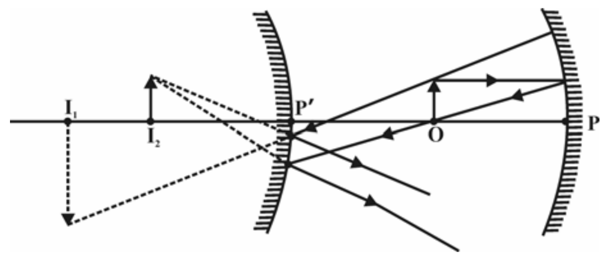
$m = 1 + \frac{D}{f}.$

Here f = 12 cm, D = 25 cm. Hence,

$m = 1 + \frac{25}{12} = 3.08.$

Example 46: An object is placed exactly midway between concave mirrors of radius of curvature 0.2 m. Distance between the mirrors is 0.5 m. Taking the first reflection on the concave mirror, find the type, position and lateral magnification of the final image.

Solution: For the image formed by the concave mirror:



$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

here, u = - 0.25 m; f = - 0.2 m

$\therefore \quad -\frac{1}{0.25} + \frac{1}{v} = -\frac{1}{0.2}$

$\therefore \quad v = -1.0 m$

$\therefore$ the image (I_1) is on the same side as the object and is inverted ($\because$ v is negative) convex mirror is from the right to the left, so the distances measured in this direction are positive.

$\therefore$ object distance is positive. Also f is positive for a convex mirror.

$\therefore$ $u' = 1 - 0.5 = 0.5$ m and $f' = 0.1$ m

$\therefore \frac{1}{0.5} + \frac{1}{v'} = \frac{1}{0.1}$

$\therefore$ $v' = 0.125$ m

$\therefore$ the image (I_2), is formed to the left of the convex mirror and is virtual.

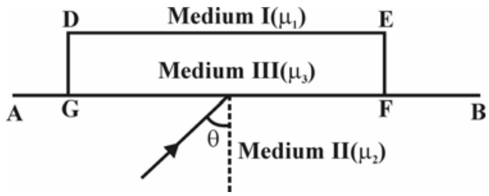
The final lateral magnification $m = \frac{h'_i}{h_o}$; where h'_i = height of the final image

h_o = height of the object

$\therefore m = \frac{h_i'}{h_o'} \times \frac{h_o'}{h_o} = \frac{h_i'}{h_o'} \times \frac{h_i}{h_o}$; where $h_o' = h_i$ = height of the image obtained by the concave mirror

$\therefore m = \frac{v'}{u'} \times \frac{v}{u} = \frac{0.125}{0.5} \times \frac{1}{0.25} = 1$

Example 47: Monochromatic light is incident on the plane interface AB between two media of refractive indices μ_1 and μ_2 $(\mu_2 > \mu_1)$ at an angle of incidence θ as shown in figure. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab DEFG of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.



Solution: We shall use the symbol $\overset{-}{>}$ to mean "infinitesimally greater than".
When the slab is not inserted, $\theta \overset{-}{>} \theta_c = \sin^{-1}(\mu_1/\mu_2)$

or, $\sin\theta \overset{-}{>} \mu_1/\mu_2$

When the slab is inserted, we have two cases $\mu_3 \le \mu_1$ and $\mu_3 > \mu_1$.

Case I: $\mu_3 \le \mu_1$

We have $\sin\theta \overset{-}{>} \mu_1/\mu_2 \ge \mu_3/\mu_2$.

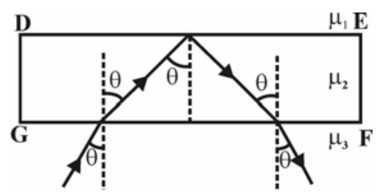
Thus, the light is incident on AB at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II: $\mu_3 > \mu_1$

$\sin\theta \overset{-}{>} \mu_1/\mu_2 \ge \mu_3/\mu_2$.

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (figure).

$$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2} \quad .....(i)$$



or, $\sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta$

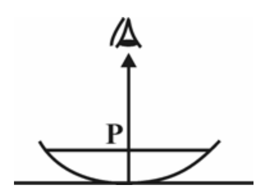
$\overset{-}{>} \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$.

Thus, $\sin\theta' \overset{-}{>} \frac{\mu_1}{\mu_3}$

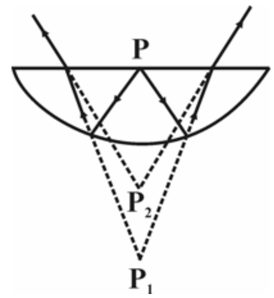
$$\text{or, } \theta' \overset{-}{>} \sin^{-1}\left(\frac{\mu_1}{\mu_3}\right) \quad ......(ii)$$

As the slab has parallel faces, the angle of refraction at the face FG is equal to the angle of incidence at the face DE. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface FG at an angle of incidence θ'. At this face, it will refract into medium II and the angle of refraction will be θ as shown by equation (i). Thus, the total light energy is ultimately reflected back into medium II.

Example 48: A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm (figure). A small dust particle floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.



Solution:



The ray diagram is shown in figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then R = –40 cm. The object distance is u = – 5 cm. Using the mirror equation.

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$$

or, $\frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{-40\text{ cm}}-\frac{1}{-5\text{ cm}}=\frac{6}{40}\text{cm}$

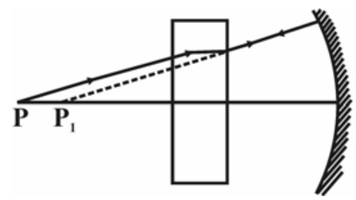
or, v = 6.67 cm.

The positive sign show that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is 6.67 cm + 5.00 cm = 11.67 cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance

$$(11.67\text{ cm})\left(1-\frac{1}{1.33}\right)=2.92\ \text{cm.}$$

Thus, the final image is formed at a point (11.67 – 2.92) cm = 8.75 cm below the water surface.

Example 49: An object is placed 21 cm in front of a concave mirror of radius of curvature 20 cm. A glass slab of thickness 3 cm and refractive index 1.5 is placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. The distance of the nearer surface of the slab from the mirror is 10 cm.

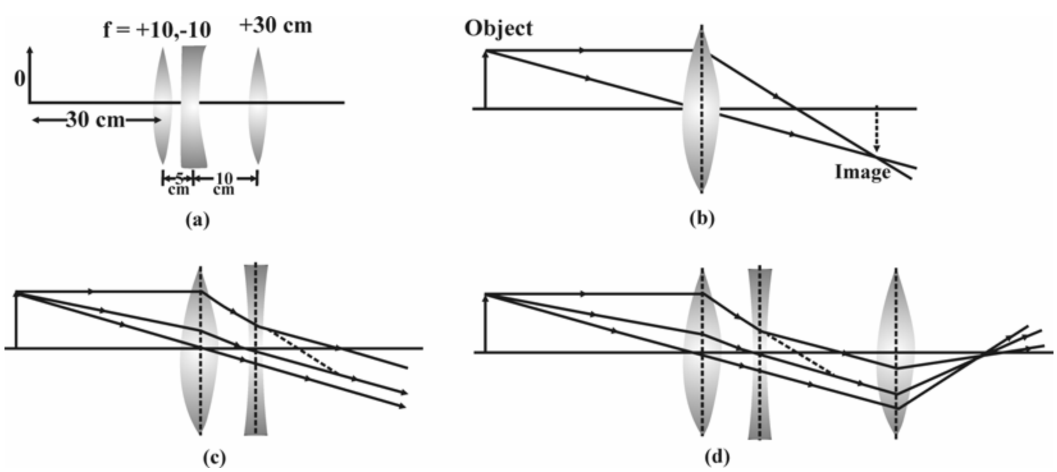


Solution: The situation is shown in figure. Because of the refraction at the two surfaces of the slab, the image of the object P is formed at P_1, shifted towards the mirror by a distance.

$$t\left(1-\frac{1}{\mu}\right)=(3\text{ cm})\left(1-\frac{1}{1.5}\right)=1\text{ cm.}$$

Thus, the rays falling on the concave mirror are diverging from P_1 which is at 21 cm − 1 cm = 20 cm from the mirror. But the radius of curvature is also 20 cm, hence P_1 is at the centre. The rays, therefore, fall normally on the mirror and hence, retrace their path. The final image is formed at P itself.

Example 50: Find the position of the image formed by the lens combination given in the figure (a)



Solution: Image formed by the first lens

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1}$$

$$\frac{1}{v_1}-\frac{1}{-30}=\frac{1}{10}$$

or $v_1 = 15$ cm

The image formed by the first lens serves as the object for the second. This is at a distance of (15 – 5) cm = 10 cm to the right of the second lens. It is a virtual object.

$$\frac{1}{v_2}-\frac{1}{10}=\frac{1}{-10}$$

or $v_2=\infty$

The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens.

$$\frac{1}{v_3}-\frac{1}{u_3}=\frac{1}{f_3}$$

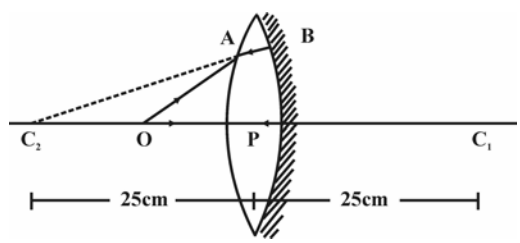
or $\frac{1}{v_3}=\frac{1}{\infty}+\frac{1}{30}$

or $v_3=30\text{cm}.$

The final image is formed 30 cm to the right of the third lens.

Ray diagrams and image formation for the above combination of lenses are given in the Figure (b) – (d). Figure shows the ray diagram and image formation, for the first lens (Figure(b)]; for the first and second lens [Figure(c)] and finally for the combination of all the three lenses [Figure(d)].

Example 51: A biconvex thin lens is prepared from glass $(\mu=1.5)$, the two bounding surfaces having equal radii of 25 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image is formed on the object itself?



Solution: Refer to figure. The object is placed at O.A ray OA starting from O gets refracted into the glass at the first surface and hits the silvered surface along AB. To get the image at the object, the rays should retrace their path after reflection from the silvered surface. This will happen only if AB falls normally on the silvered surface. Thus, AB should appear to come from the centre C_2 of the second surface. Thus, due to the refraction at the first surface, a virtual image of O is formed at C_2.

For this case,

v = – 25 cm, R = + 25 cm, $\mu_1=1, \mu_2=1.5$.

We have,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

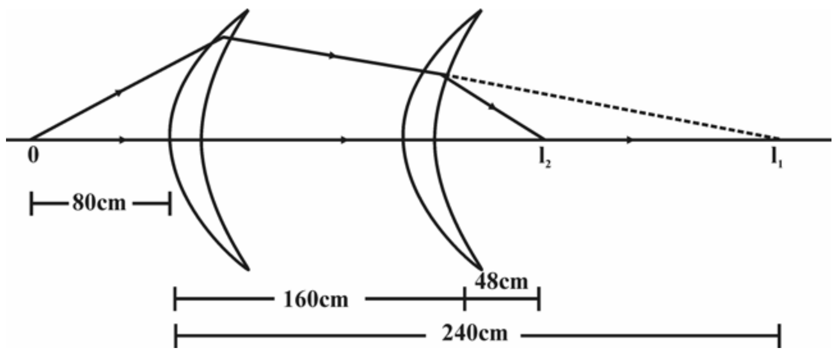
or, $\frac{1.5}{-25\text{ cm}}-\frac{1}{u}=\frac{1.5-1}{25\text{cm}}$

or, $\frac{1}{u}=-\frac{1.5}{25\text{cm}}-\frac{0.5}{25\text{cm}}$

or, u = – 12.5 cm.

Thus, the object should be placed at a distance of 12.5 cm from the lens.

Example 52: A concavo-convex lens made of glass $(\mu = 1.5)$ has surfaces of radii 20 cm and 60 cm. (a) Locate the image of an object placed 80 cm to the left of the lens along the principal axis. (b) A similar lens is placed coaxially at a distance of 160 cm right to it. Locate the position of the image.



Solution: The focal length of the lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{20\,\text{cm}} - \frac{1}{60\,\text{cm}}\right) = \frac{1}{60\,\text{cm}}$$

or, f = 60 cm.

(a) For the image formed by the first lens, u = – 80 cm so that

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{-80\,\text{cm}} + \frac{1}{60\,\text{cm}} = \frac{1}{240\,\text{cm}}$$

or, v = 240 cm.

The first image I_1 would form 240 cm to the right of the first lens.

(b) The second lens intercepts the converging beam as suggested by the figure. The image I_1 acts as a virtual source for the second lens. For the image formed by this lens, u = 240 cm – 160 cm = + 80 cm so that

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{80\,\text{cm}} + \frac{1}{60\,\text{cm}} = \frac{1}{240\,\text{cm}}$$

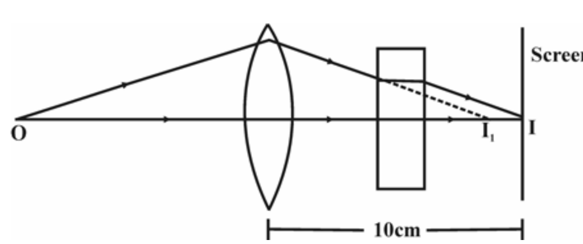
or, v = 34.3 cm.

The final image is formed 34.3 cm to the right of the second lens.

Example 53: A convex lens focuses a distant object on a screen placed 10 cm away from it. A glass plate $(\mu = 1.5)$ of thickness 1.5 cm is inserted between the lens and the screen. Where should the object be placed so that its image is again focused on the screen.

Solution:

The focal length of the lens is 10 cm. The situation with the glass plate inserted is shown in figure. The object is placed at O. The lens would form the image at I_1 but the glass plate intercepts the rays and forms the image at I on the screen.



The shift $I_1I = t\left(1 - \frac{1}{\mu}\right)$

$= (1.5 \text{ cm})\left(1 - \frac{1}{1.5}\right) = 0.5 \text{ cm}.$

Thus, the lens forms the image at a distance of 9.5 cm from itself. Using

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$

$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$

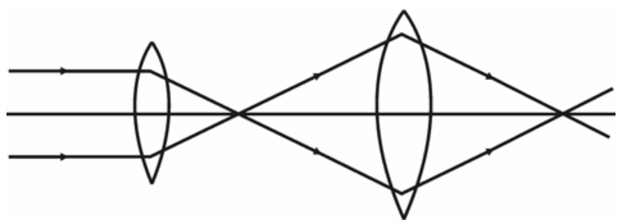
$= \frac{1}{9.5\text{cm}} - \frac{1}{10\text{cm}}$

or, u = 190 cm.

Thus, the object should be placed at a distance of 190 cm from the lens.

Example 54: Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. Find the image of a distant object formed by the combination by (a) using thin lens formula separately for the two lenses and (b) using the equivalent lens. Note that although the combination forms a real image of a distant object on the other side, it is equivalent to a diverging lens as far as the location of the final image is concerned.

Solution:



(a) The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at u = – 40 cm from the second. Using the lens formula for the second lens,

$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{40\text{cm}} + \frac{1}{20\text{cm}}$

or, v = 40 cm.

The final image is formed 40 cm to the right of the second lens.

(b) The equivalent focal length is

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$= \frac{1}{20\text{cm}} + \frac{1}{20\text{cm}} - \frac{60\text{cm}}{(20\text{cm})^2}$

or, F = – 20 cm.

It is a divergent lens. It should be kept at a distance

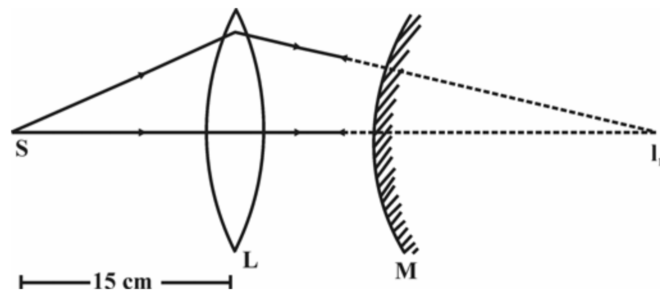
$D = \frac{dF}{f_1}$ behind the second lens.

Here, $D = \frac{(60\text{cm})(-20\text{cm})}{20\text{cm}} = -60\,\text{cm}.$

Thus, the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens i.e., 20 cm to the left of it. It is, therefore, 40 cm to the right of the second lens.

Example 55: A point source S is placed at a distance of 15 cm from a converging lens of focal length 10 cm on its principal axis. Where should a diverging mirror of focal length 12 cm be placed so that a real image is formed on the source itself.

Solution:



The equation for the lens is

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad ....(i)$$

Here u = – 15 cm and f = + 10 cm.

Using equation (i),

$$\frac{1}{v}-\frac{1}{15\text{ cm}}=\frac{1}{10\text{cm}}$$

$$\frac{1}{v}=\frac{1}{10\text{ cm}}-\frac{1}{15\text{ cm}}+\frac{1}{30\text{ cm}}$$

or v = 30 cm.

The positive sign of v shows that the image I_1 is formed to the right of the lens in the figure. The diverging mirror is to be placed to the right in such a way that the light rays fall on the mirror perpendicularly. Then only the rays will retrace their path and form the final image on the object. Thus, the image I_1 formed by the lens should be at the centre of curvature of the mirror.

We have, LI_1 = 30 cm,

MI_1 = R = 2f = 24 cm.

Hence, LM = LI – MI_1 = 6 cm.

Thus, the mirror should be placed 6 cm to the right of the lens.

Example 56: A slide projector produces 500 times enlarged image of a slide on a screen 10 m away. Assume that the projector consists of a single convex lens used for magnification. If the screen is moved 2.0 m closer, by what distance should the slide be moved towards or away from the lens so that the image remains focused on the screen? What is the magnification in this case?

Solution: In the first case, v = 10 m and $\frac{v}{u}=-500$.

Thus, $u=-\frac{v}{500}=-\frac{1}{50}\text{m}=-2.0\,\text{cm}$. The focal length f is given by

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{10\,\text{m}}+\frac{1}{2.0\,\text{cm}}$$

If the screen is moved 2.0 m closer, v = 8.0 m. The object distance u' is given by

$$\frac{1}{v}-\frac{1}{u'}=\frac{1}{f}$$

$$\text{or, } \frac{1}{u'}=\frac{1}{v}-\frac{1}{f}=\frac{1}{8.0\text{m}}-\frac{1}{10\text{m}}-\frac{1}{2.0\text{cm}}$$

$$=\frac{1}{40\text{m}}-\frac{1}{2.0\text{cm}}$$

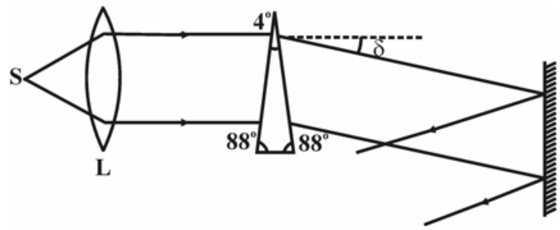
$$=-\frac{1}{2.0\text{cm}}\left(1-\frac{1}{2000}\right)$$

$$\text{or, } u'=-2.0\text{cm}\left(1-\frac{1}{2000}\right)^{-1}$$

$$\approx -2.0\text{cm}\left(1+\frac{1}{2000}\right)=-2.0\text{cm}-\frac{1}{1000}\text{cm}.$$

Thus, the slide should be taken $\frac{1}{1000}$ cm away from the lens.

Example 57: Consider the situation shown in figure. Light from a point source S is made parallel by a convex lens L. The beam travels horizontally and falls on an $88^\circ - 88^\circ - 4^\circ$ prism as shown in the figure. It passes through the prism symmetrically. The transmitted light falls on a vertical mirror. Through what angle should the mirror be rotated so that the image of S is formed on S itself?



Solution: The parallel beam after going through the prism will be deviated by an angle δ. If the mirror is also rotated by this angle δ, the rays will fall normally on it. The rays will be reflected back along the same path and form the image of S on itself.
As the prism is thin, the angle δ is given by

$$\delta = (\mu - 1)A$$

$$= (1.5 - 1) \times 4^\circ = 2^\circ.$$

Thus, the mirror should be rotated by 2°.

Example 58: The refractive indices of silicate flint glass for wavelengths 400 nm and 700 nm are 1.66 and 1.61 respectively. Find the minimum angles of deviation of an equilateral prism made of this glass for light of wavelengths 400 nm and 700 nm.
Solution: The minimum angle of deviation δ_m is given by

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(30^\circ + \frac{\delta_m}{2}\right)}{\sin 30^\circ} = 2\sin\left(30^\circ + \frac{\delta_m}{2}\right).$$

For 400 nm light,

$$1.66 = 2\sin\left(30^\circ + \delta_m/2\right)$$

or, $\sin\left(30^\circ + \delta_m/2\right) = 0.83$

or, $\left(30^\circ + \delta_m/2\right) = 56^\circ$

or, $\delta_m = 52^\circ$.

For 700 nm light,

$$1.61 = 2\sin\left(30^\circ + \delta_m/2\right).$$

This gives $\delta_m = 48^\circ$.

Example 59: A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5 cm. An object has to be placed at a distance of 1.2 cm away from the objective for normal adjustment. (a) Find the angular magnification. (b) Find the length of the microscope tube.
Solution:
(a) If the first image is formed at a distance v from the objective, we have

$$\frac{1}{v} - \frac{1}{(-1.2\,\text{cm})} = \frac{1}{1\,\text{cm}}$$

or, v = 6 cm.

The angular magnification in normal adjustment is

$$m = \frac{v}{u}\frac{D}{f_e} = \frac{6\text{ cm}}{1.2\text{ cm}}.\frac{25\,\text{cm}}{2.5\,\text{cm}} = -50.$$

(b) For normal adjustment, the first image must be in the focal plane of the eyepiece.
The length of the tube is, therefore,
$L = v + f_e = 6\text{ cm} + 2.5\text{ cm} = 8.5\text{ cm}.$

Example 60: A compound microscope consists of an objective of focal length 1.0 cm and an eyepiece of focal length 5.0 cm separated by 12.2 cm. (a) At what distance from the objective should an object be placed to focus it properly so that the final image is formed at the least distance of clear vision (25 cm)? (b) Calculate the angular magnification in this case.

Solution:

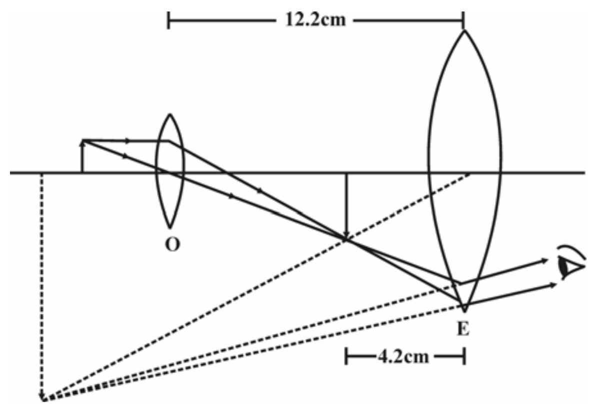
(a) For the eyepiece, fe = + 5 cm.

Using $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$,

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= -\frac{1}{25\text{cm}} - \frac{1}{5\text{cm}}$$

or, $u_e = -\frac{25}{6}\text{ cm} = -4.17\text{ cm} \approx -4.2\text{ cm}.$



As the objective is 12.2 cm away from the eyepiece, the image formed by the objective is 12.2 cm − 4.2 cm = 8.0 cm away from it. For the objective,
$v = +8.0\text{ cm}, f_o = +1.0\text{ cm}.$

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o}$,

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f_o}$$

$$= \frac{1}{8.0\,\text{cm}} - \frac{1}{1.0\,\text{cm}}$$

or, $u = -\frac{8.0}{7.0}\text{cm} = -1.1\text{ cm}.$

(b) The angular magnification is

$$m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$$

$$= \frac{+8.0\,\text{cm}}{-1.1\,\text{cm}}\left(1 + \frac{25\,\text{cm}}{5\,\text{cm}}\right) = -44\,.$$

Example 61: The separation L between the objective (f = 0.5 cm) and the eyepiece (f = 5 cm) of a compound microscope is 7 cm. Where should a small object be placed so that the eye is least strained to see the image? Find the angular magnification produced by the microscope.

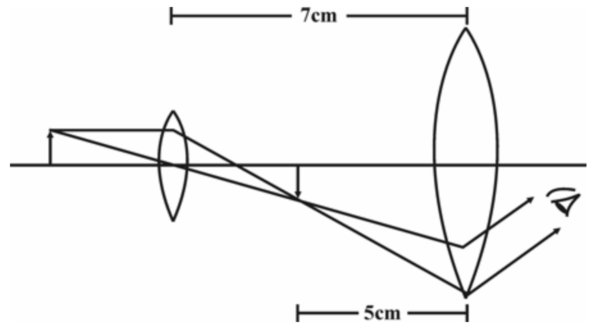
Solution: The eye is least strained if the final image is formed at infinity. In such a case, the image formed by the objective should fall at the focus of he eyepiece. As $f_e = 5$ cm and L = 7 cm, this first image should be formed at 7 cm − 5 cm = 2 cm from the objective. Thus, v = + 2 cm. Also, $f_o = 0.5$ cm. For the objective, using

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_o},$$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f_o}$$

$$=\frac{1}{2\,cm}-\frac{1}{0.5\,cm}$$

or, $u=-\frac{2}{3}cm.$



The angular magnification in this case is

$$m=\frac{v}{u}\frac{D}{f_e}$$

$$=\frac{2\,cm}{-(2/3)\,cm}\frac{25\,cm}{5\,cm}=-15.$$

Example 62: An astronomical telescope has an objective of focal length 200 cm and an eyepiece of focal length 4.0 cm. The telescope is focused to see an object 10 km from the objective. The final image is formed at infinity. Find the length of the tube and the angular magnification produced by the telescope.

Solution: As the object distance 10 km is much larger than the focal length 200 cm, the first image is formed almost at the focus of the objective. It is thus 200 cm from the objective. This image acts as the object for the eyepiece. To get the final image at infinity, this first image should be at the first focus of the eyepiece. The length of the tube is, therefore, 200 cm + 4 cm = 204 cm. The angular magnification in this case

$$m=-\frac{f_0}{f_e}=-\frac{200}{4}=-50.$$

Example 63: A Galilean telescope is constructed by an objective of focal length 50 cm and an eyepiece of focal length 5.0 cm. (a) Find the tube length and magnifying power when it is used to see an object at a large distance in normal adjustment. (b) If the telescope is to focus an object 2.0 m away from the objective, what should be the tube length and angular magnification, the image again forming at infinity?

Solution: $f_o = 50$ cm, $f_e = -5$ cm.

(a) $L = f_o - |f_e| = (50-5)$ cm $= 45$ cm

and $m=-\frac{f_o}{f_e}=\frac{50}{5}=10.$

(b) Using the equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ for the objective,

$$\frac{1}{v}=\frac{1}{f_o}+\frac{1}{u}$$

$$=\frac{1}{50cm}+\frac{1}{-200cm}$$

or, v = 66.67 cm.

The tube length $L=v-|f_e|=(66.67-5)\,cm$

or, L = 61.67 cm.

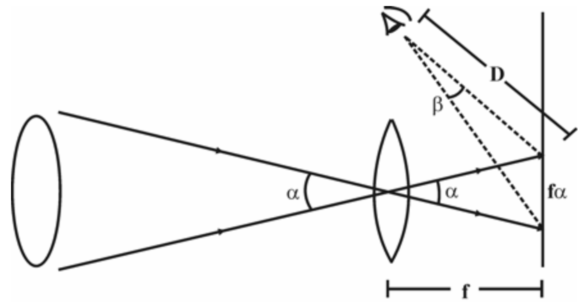
To calculate the angular magnification, we assume that the object remains at large distance from the eye. In this case, the angular magnification

$m = \frac{v}{f_e} = \frac{66.67}{5} = 13.33.$

v is the distance of the first image from the objective which is substituted for f_o.

Example 64: The image of the moon is focused by a converging lens of focal length 50 cm on a plane screen. The image is seen by an unaided eye from a distance of 25 cm. Find the angular magnification achieved due to the converging lens.

Solution: Suppose the moon subtends an angle α on the lens. This also be the angle subtended by the moon on the eye if the moon is directly viewed. The image is formed in the focal plane. The linear size of the image = $f\alpha$ = (50 cm) α.



If this image is seen from a distance of 25 cm, the angle formed by the image on the eye

$|\beta| = \frac{(50\text{cm})|\alpha|}{25\,\text{cm}} = 2|\alpha|.$

The angular magnification is

$\frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right| = -2.$

PROBLEMS

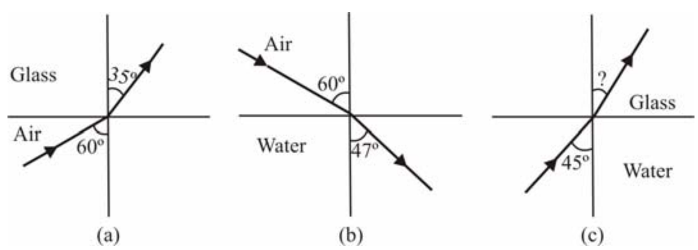
Exercise I

Q 1. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?

Q 2. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Q 3. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 up to the same height, by what distance would the microscope have to be moved to focus on the needle again?

Q 4. Figures (a) and (b) show refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)].



Q 5. A small bulb is placed at the bottom of a tank containing water to a depth of 80cm. What is the area of the surface of water through which light from the bulb can emerge out? Refractive index of water is 1.33. (Consider the bulb to be a point source.)

Q 6. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Q 7. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm?

Q 8. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm and (b) a concave lens of focal length 16cm?

Q 9. An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Q 10. What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses.

Q 11. A compound microscope consists of an objective lens of focal length 2.0cm and an eyepiece of focal length 6.25cm separated by a distance of 15cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25cm) and (b) at infinity? What is the magnifying power of the microscope in each case?

Q 12. A person with a normal near point (25cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Q 13. A small telescope has an objective lens of focal length 144cm and an eyepiece of focal length 6.0cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Q 14. (a) A giant refracting telescope at an observatory has an objective lens of focal length 15m. If an eyepiece of focal length 1.0cm is used, what is the angular magnification of the telescope?

(b) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6m, and the radius of lunar orbit is 3.8×10^8m.

Q 15. Use the mirror equation to deduce that:

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

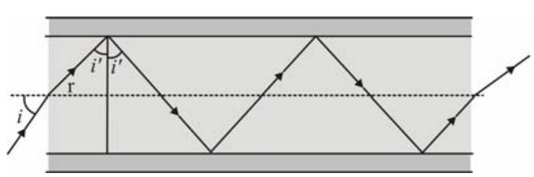
(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Q.16 A small pin fixed on a table top is viewed from above from a distance of 50cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Q.17 (a) Figure shows a cross-section of a 'light pipe' made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure.

(b) What is the answer if there is no outer covering of the pipe?



Q.18 Answer the following questions:

(a) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.

(b) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?

(c) A diver under water, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?

(d) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?
(e) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Q.19 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

Q.20 A screen is placed 90cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20cm. Determine the focal length of the lens.

Q.21 (a) Determine the 'effective focal length' of the combination of the two lenses in Problem 10, if they are placed 8.0cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?
(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40cm. Determine the magnification produced by the two-lens system and the size of the image.

Q.22 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Q.23 You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will
(a) deviate a pencil of white light without much dispersion,
(b) disperse (and displace) a pencil of white light without much deviation.

Q.24 For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Q.25 Does short-sightedness (myopia) or long-sightedness (hypermetropia) imply necessarily that the eye has partially lost its ability of accommodation? If not, what might cause these defects of vision?

Q.26 A myopic person has been using spectacles of power –1.0 dioptre for distant vision. During old age he also needs to use separate reading glass of power + 2.0 dioptres. Explain what may have happened.

Q.27 A person looking at a person wearing a shirt with a pattern comprising vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

Q.28 A man with normal near point (25 cm) reads a book with small print using a magnifying glass: a thin convex lens of focal length 5 cm.
(a) What is the closest and the farthest distance at which he should keep the lens from the page so that he can read the book when viewing through the magnifying glass?
(b) What is the maximum and the minimum angular magnification (magnifying power)

possible using the above simple microscope?

Q.29 A card sheet divided into squares each of size $1mm^2$ is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 9 cm) held close to the eye.
(a) What is the magnification produced by the lens? How much is the area of each square in the virtual image?
(b) What is the angular magnification (magnifying power) of the lens?
(c) Is the magnification in (a) equal to the magnifying power in (b)?
Explain.

Q.30 (a) At what distance should the lens be held from the figure in Problem 29 in order to view the squares distinctly with the maximum possible magnifying power?
(b) What is the magnification in this case?
(c) Is the magnification equal to the magnifying power in this case?
Explain.

Q.31 What should be the distance between the object in Problem 30 and the magnifying glass if the virtual image of each square in the figure is to have an area of 6.25 mm^2. Would you be able to see the squares distinctly with your eyes very close to the magnifier?

Q.32 Answer the following questions:
(a) The angle subtended at the eye by an object is equal to the angle subtended at the eye by the virtual image produced by a magnifying glass. In what sense then does a magnifying glass provide angular magnification?
(b) In viewing through a magnifying glass, one usually positions one's eyes very close to the lens. Does angular magnification change if the eye is moved back?
(c) Magnifying power of a simple microscope is inversely proportional to the focal length of the lens. What then stops us from using a convex lens of smaller and smaller focal length and achieving greater and greater magnifying power?
(d) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?
(e) When viewing through a compound microscope, our eyes should be positioned not on the eyepiece but a short distance away from it for best viewing. Why? How much should be that short distance between the eye and eyepiece?

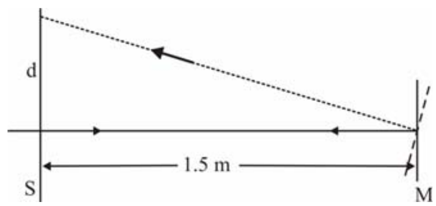
Q.33 An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25cm and an eyepiece of focal length 5cm. How will you set up the compound microscope?

Q.34 A small telescope has an objective lens of focal length 140cm and an eyepiece of focal length 5.0cm. What is the magnifying power of the telescope for viewing distant objects when
(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?
(b) the final image is formed at the least distance of distinct vision (25cm)?

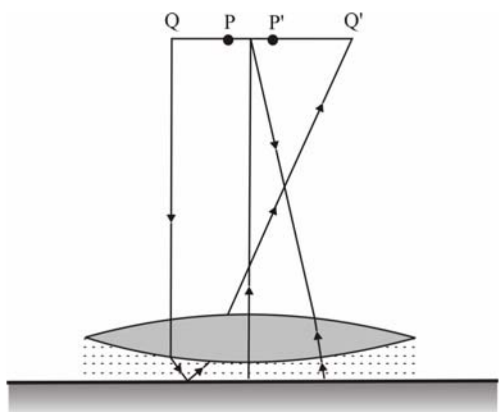
Q.35 (a) For the telescope described in Problem 34 (a), what is the separation between the objective lens and the eyepiece?
(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?
(c) What is the height of the final image of the tower if it is formed at 25cm?

Q.36 A Cassegrain telescope uses two mirrors. Such a telescope is built with the mirrors 20mm apart. If the radius of curvature of the large mirror is 220mm and the small mirror is140mm, where will the final image of an object at infinity be?

Q.37 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Figure. A current in the coil produces a deflection of 3.5^o of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



Q.38 The figure in the problem shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0cm. What is the refractive index of the liquid?



Exercise II

Q 1. State Snell's law of refraction of light.

Q 2. Define critical angle for total internal reflection.

Q 3. What is critical angle? Give one application of total internal reflection.

Q 4. What is critical angle for a material of refractive index $\sqrt{2}$?

Q 5. What is the unit of power of a lens?

Q 6. How do you explain the mirage effect produced in very hot deserts?

Q 7. Derive a relation between the focal length of a convex lens and its radii of curvature.

Q 8. Derive lens formula from curved surface formula.

Q 9. Define linear magnification produced by a lens. Hence derive expression for it. Also define the power of a lens and its unit.

Q 10. Derive Lens Maker's formula for convex surface, when ray travels from a rarer to denser medium.

Q 11. What is dispersion of light?

Q 12. Out of red and blue lights, for which colour is the refractive index of glass greater?

Q 13. What do you mean by dispersive power of the material of a prism?

Q 14. Why does the sky appear blue?

Q 15. Why the rising sun is red in colour?

Q 16. Show that the limiting value of the angle of prism is twice its critical angle.

Q 17. Write a short note on rainbow.

Q 18. Discuss the phenomenon of refraction through a prism and prove that for a prism:
$\mu = \frac{\sin(A + \delta_m)/2}{\sin A/2}$, where A is angle of prism, μ is refractive index of the material of prism and δ_m is the angle of minimum deviation.

Q 19. Define magnifying power of compound microscope.

Q 20. Express the angular magnification of an astronomical telescope in terms of the focal lengths of the objective and the eye-piece.

Q 21. Draw a labelled ray diagram showing the formation of image in a compound microscope. Also define the magnifying power of compound microscope.

Q 22. Draw the course of rays in an astronomical telescope, when the final image is formed at infinity. Also define the magnifying power of the astronomical telescope in this position.

Q 23. Distinguish between magnifying power and resolving power of a telescope.

Q 24. Derive an expression for the angular magnification of a simple microscope.

Q 25. Derive a relation for the magnifying power of a compound microscope.

Q 26. Explain the construction and working of a compound microscope.

Q 27. Explain the construction and working of an astronomical telescope. Calculate its magnifying power, when the image is formed at the least distance of distinct vision.

Q 28. Use the mirror equation to deduce that:
(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.
(b) a convex mirror always produces a virtual image independent of the location of the object.
(c) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.
(d) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Q 29. Describe briefly the construction of prism binoculars. Explain with the help of a ray diagram how the inversion of the image produced by an ordinary telescope is reversed by the use of two right-angled totally reflecting prisms. Explain carefully how the final image is both erect and without any 'lateral inversion'. List some advantages of prism binoculars over an ordinary telescope.

Q 30. A light of wavelength 6000 $\overset{o}{A}$ in air enters a medium with refractive index 1.5. What will be wavelength of light in the medium?

Q 31. A ray of monochromatic light travelling in vacuum with speed c, wavelength λ and frequency ν, enters into a medium of refractive index 1.5. What will be its new wavelength?

Q 32. A ray of light is incident normally on one of the faces of a prism of apex angle 30^0 and refractive index $\sqrt{2}.$ Find the angle of deviation for the ray of light.

Q 33. To print a photograph from a negative, the time of exposure to light from a lamp placed 0.50 m away is 2.5 s. How much exposure time is required if the lamp is placed 1.0 m away?

Q 34. A square wire of side 3.0 cm is placed 25 cm away from a concave mirror of focal length 10 cm. What is the area enclosed by the image of the wire? (The centre of the wire is on the axis of the mirror, with its two sides normal to the axis.)

Q 35. A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. Identity the type of the lens and determine its focal length. What is the size of the image if the size of the needle is 5.0 cm?

Q 36. A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point (25cm). What is the magnifying power of the microscope?

Q 37. (a) An object is placed between two plane mirrors inclined at $60°$ to each other. How many images do you expect to see?

(b) An object is placed between two plane parallel mirrors. Why do the distant images get fainter and fainter?

Q 38. A boy 1.50 m tall with his eye-level at 1.38 m stands before a mirror fixed on a wall. Indicate by means of a ray diagram how the mirror should be positioned so that he can view himself fully. What should be the minimum length of the mirror? Does the answer depend on the eye-level?

Q 39. The bottom of a container is a 4.0 cm thick glass (n = 1.5) slab. The container contains two immiscible liquids A and B of depths 6.0 cm and 8.0 cm, respectively. What is the apparent position of a scratch on the outer surface of the bottom of the glass slab when viewed through the container? Refractive indices of A and B are 1.4 and 1.3, respectively.

Q 40. Using a spectrometer, the following data are obtained for a crown glass prism and a flint glass prism.
Crown glass prism:
Angle of the Prism, A = $72.0°$
Angle of minimum deviation :
$\delta_b = 54.6°$,
$\delta_r = 53.0°$
$\delta_y = 54.0°$,

Flint glass prism:
$A = 60.0°$

$\delta_b = 52.8°$,

$\delta_r = 50.6°$,

$\delta_y = 51.9°$,

where b, r and y refer to particular wavelengths in the blue, red and yellow bands. Compare the dispersive powers of the two varieties of glass prisms.

Q 41. A figure divided into squares each of size 1 mm^2 is being viewed at a distance of 9 cm through a magnifying glass (a converging lens of focal length 10 cm) held close to the eye.
(a) What is the magnification (image size/object size) produced by the lens? How much is the area of each square in the virtual image?
(b) What is the angular magnification (magnifying power) of the lens?
(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Q 42. An amateur astronomer wishes to estimate roughly the size of the Sun using his crude telescope consisting of an objective lens of focal length 200 cm and an eye-piece of focal length 10 cm. By adjusting the distance of the eye-piece from the objective, he obtains an image of the Sun on a screen 40 cm behind the eye-piece. The diameter of the Sun's image is measured to be 6.0 cm. What is his estimate of the Sun's size, given that the average Earth-Sun distance is 1.5×10^{11} m?

Q 43. A terrestrial telescope has an objective of focal length 180 cm and an eye-piece of focal length of 5 cm. The erecting lens has a focal length of 3.5 cm. What is the separation between the objective and the eye-piece? What is the magnifying power of the telescope? Can we use the telescope for viewing astronomical objects?

Q 44. A 35 mm slide with a 24 mm × 36 mm picture is projected on a screen placed 12 m from the slide. The image of the slide picture on the screen measures 1.0 m × 1.5 m. Determine the location of the projection lens and its focal length.

Q 45. (a) Show that for a given brightness of the image on a camera film, the exposure time t is inversely proportional to the square of the aperture size a and directly proportional to the square of the focal length f of the camera lens.
(b) A camera is set at the aperture size f/8 and the exposure time of (1/60)s. How much exposure time is required for receiving the same amount of light if the aperture size is set at f/5.6? How is the depth of field affected by this change?

Q 46. An eye-piece of a telescope consists of two plano-convex lenses L_1 and L_2 each of focal length f separated by a distance of 2f/3. Where should L_1 be placed relative to the focus of the objective lens of the telescope so that the final image through L_2 is seen at infinity?

Flashback

CBSE 2004

Q 1. Draw a ray diagram of an astronomical telescope in the normal adjustment position. Write down the expression for its magnifying power.

(2 out of 70)

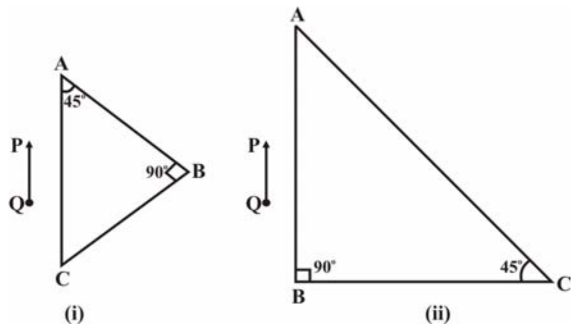
Q2. A spherical surface of radius of curvature R, separates a rarer and a denser medium. Complete the path of the incident ray of light, showing the formation of the real image. Hence derive the relation connecting object distance 'u', image distance 'v', radius of curvature R and the refractive indices n_1 and n_2 of the two media.

Briefly explain, how the focal length of a convex lens changes, with increase in wavelength of incident light.

(5 out of 70)

CBSE 2005

Q 1. A right angled crown glass prism with critical angle 41º is placed before an object PQ, in two positions as shown in the figure (i) and (ii). Trace the paths of rays from P and Q passing through the prisms in the two cases



(2 out of 70)

Q 2. **(a)** Draw a labeled ray diagram to show the formation of the image by a compound microscope. Write the expression for its magnifying power.

(b) How does the resolving power of a compound microscope change, when (i)refractive index of the medium between the object and the objective lens increases and (ii) wavelength of the radiation used is increased.

(3 out of 70)

CBSE 2006

Q 1. Draw a labelled ray diagram to show the image formation in a refracting type astronomical telescope. Why should the diameter of the objective of a telescope be large?

(2 out of 70)

Q 2. A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is

(a) a convex lens of focal length 20 cm.

(b) a concave lens of focal length 16 cm?

Do the required calculations.

(3 out of 70)

CBSE 2007

Q 1. A double convex lens of glass of refractive index 1.6 has its both surfaces of equal radii of curvature of 30 cm each. An object of height 5 cm is placed at a distance of 12.5 cm from the lens. Calculate the size of the image found.

(3 out of 70)

CBSE 2008

Q.1 Why does the bluish colour predominate in a clear sky?

(1 out of 70)

Q.2 How does the angle of minimum deviation of a glass prism of refractive index 1.5 change, if it is immersed in a liquid of refractive index 1.3?

(1 out of 70)

Q.3 Draw a labelled ray diagram, showing the image formation of an astronomical telescope in the normal adjustment position. Write the expression for its magnifying power.

(2 out of 70)

Q.4 Derive the lens formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ for a concave lens, using the necessary ray diagram.

Two lenses of powers 10 D and -5 D are placed in contact.

(i) Calculate the power of the new lens.

(ii) Where should an object be held from the lens, so as to obtain a virtual image of magnification 2?

(5 out of 70)

CBSE 2009

Q.1 Two thin lenses of power + 6D and –2D are in contact. What is the focal length of the combination ? **(1 out of 70)**

Q.2 Define refractive index of a transparent medium.
A ray of light passes through a triangular prism. Plot a graph showing the variation of the angle of deviation with the angle of incidence. **(2 out of 70)**

Q.3 (a) (i) Draw a labeled ray diagram to show the formation of image in an astronomical telescope for a distant object.

(ii) Write three distinct advantages of a reflecting type telescope over a refracting type telescope.

(b) A convex lens of focal length10 cm is placed coaxially 5 cm away from a concave lens of focal length 10 cm. If an object is placed 30 cm in front of the convex lens, find the position of the final image formed by the combined system. **(3 out of 70)**

CBSE 2010

Q.1 A converging lens is kept coaxially in contact with a diverging lens- both the lenses being of equal focal lengths. What is the focal length of the combination ? **(1 out of 70)**

Q.2 (i) Draw a neat labeled ray diagram of an astronomical telescope in normal adjustment. Explain briefly its working.

(ii) An astronomical telescope uses two lenses of powers 10D and 1D. What is its magnifying power in normal adjustment ? **(3 out of 70)**

OR

(i) Draw a neat labeled ray diagram of a compound microscope. Explain briefly its working.

(ii) Why must both the objective and the eye-piece of a compound microscope have short focal lengths ?

CBSE 2011

Q.1 A converging lens has a focal length of 20 cm in air. It is made of a material of refractive index 1.6. It is immersed in a liquid of refractive index 1.3. Calculate its new focal length.

(3 out of 70)

Q.2 Use the mirror equation to show that

(a) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(b) a convex mirror always produces a virtual image independent of the location of the object.

(c) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

(3 out of 70)

Q.3 A compound microscope has an objective of focal length 4 cm and an eyepiece of focal length 10 cm. An object is placed at 6 cm from the objective lens. Calculate the magnifying power of the compound microscope. Also calculate the length of the microscope.

OR

A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece lens of focal length 1.0 cm is used, find the angular magnification of the telescope.
If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.42×10^6 m and the radius of the lunar orbit is 3.8×10^8 m.

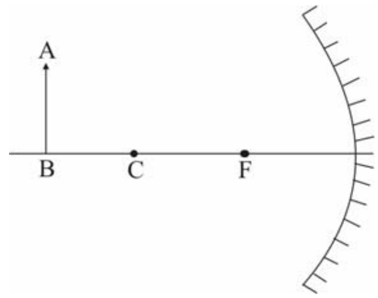
(3 out of 70)

CBSE 2012

Q.1 For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15^o, 25^o and 35^o respectively. In which medium would the velocity of light be minimum.

(1 out of 70)

Q.2 An object AB is kept in front of a concave mirror as shown in the figure.



(i) Complete the ray diagram showing the image formation of the object.
(ii) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?

(2 out of 70)

Q.3 Draw a labeled ray diagram of a reflecting telescope. Mention its two advantages over the refracting telescope.

(2 out of 70)

Q.4 You are given three lenses L_1, L_2 and L_3 each of focal length 15cm. An object is kept at 20 cm in front of L_1, as shown. The final real image is formed at the focus 'I' of c_3. Find the separations between L_1, L_2 and L_3.

(3 out of 70)

CBSE 2013

Q.1 A convex lens of focal length f_l is kept in contace with a concave lens of focal length f_2. Find the focal length of the combination.

(2 out of 70)

Q.2 Draw a labeled ray diagram of a refracting telescope. Define its magnifying power and write the expression for it.

Write two important limitations of a refracting telescope over a reflecting type telescope.

(3 out of 70)

Q.3 One day Chetan's mother developed a severe stomach ache all of a sudden. She was rushed to the doctor who suggested for an immediate endoscopy test and gave an estimate of expenditure for the same. Chetan immediately contacted his class teacher and sheared the information with her. The class teacher arranged for the money and rushed to the hospital. On realizing that Chetan belonged to a below average income group family, even the doctor offered concession for the test fee. The test was conducted successfully.
Answer the following questions based on the above information:
(a) Which principle in optics is made use of in endoscopy?
(b) Briefly explain the values reflected in the action taken by the teacher.
(c) In what way do you appreciate the response of the doctor on the given situation?

(4 out of 70)

ANSWERS

Exercise I

Q 1. v = –54 cm. The image is real, inverted and magnified. The size of the image is 5.0 cm. As u → f, v → ∞; for u < f, image is virtual

Q 2. v = 6.7 cm. Magnification = 5/9 i.e. the size of the image is 2.5 cm. As u → ∞; v → f (but never beyond) while m → 0

Q 3. 1.33; 1.7 cm

Q 4. $n_{ga} = 1.51$; $n_{wa} = 1.32$; $n_{gw} = 1.144$; which gives sin r = 0.6181 i.e. r ≃ 38º

Q 5. r = 0.8 × tan i_c and sin i_c = 1/1.33 ≅ 0.75, where r is the radius (in m) of the largest circle from which light comes out and i_c is the critical angle for water-air interface, Area = 2.6 m^2

Q 6. n ≅ 1.53 and D_m for prism in water ≅ 10º

Q 7. R = 22 cm

Q 8. Here the object is virtual and the image is real. u = +12 cm (object on right; virtual)
(a) f = +20 cm. Image is real and at 7.5 cm from the lens on its right side.(b) f = –16 cm. Image is real and at 48 cm from the lens on its right side.

Q 9. v = 8.4 cm, image is erect and virtual. It is diminished to a size 1.8 cm. As u → ∞, v → f (but never beyond f while m → 0). Note that when the object is placed at the focus of the concave lens (21 cm), the image is located at 10.5 cm (not at infinity as one might wrongly think)

Q 10. A diverging lens of focal length 60 cm

Q 11. (a) v_e = –25 cm and f_e = 6.25 cm give u_e = –5 cm; v_O = (15 – 5) cm = 10 cm, $f_O = u_O$ = – 2.5 cm; Magnifying power = 20 (b) u_O = – 2.59 cm. Magnifying power = 13.5.

Q 12. Angular magnification of the eye-piece for image at 25 cm

$$= \frac{25}{2.5} + 1 = 11; |u_e| = \frac{25}{11}\text{cm} = 2.27\,\text{cm}; \; v_O = 7.2 \text{ cm}$$

Separation = 9.47 cm; Magnifying power = 88

Q 13. 24; 150 cm

Q 14. (a) Angular magnification = 1500 (b) Diameter of the image = 13.7 cm.

Q 15. Apply mirror equation and the condition:
(a) f < 0 (concave mirror); u < 0 (object on left) ; (b) f > 0; u < 0 ; (c) f > 0 (convex mirror) and u < 0
(d) f < 0 (concave mirror); f < u < 0
to deduce the desired result.

Q.16 The pin appears raised by 5.0 cm. It can be seen with an explicit ray diagram that the answer is independent of the location of the slab (for small angles of incidence).

Q.17 (a) sin i'_c = 1.44/1.68 which gives i'_c = 59°. Total internal reflection takes place when i >59^0 or when r < r_{max} = 31° Now, (sin i_{max} /sin r_{max}) =1.68 , which gives i_{max} ≃ 60°. Thus, all incident rays of angles in the range 0 < i < 60° will suffer total internal reflections in the pipe. (If the length of the pipe is finite, which it is in practice, there will be a lower limit on i determined by the ratio of the diameter to the length of the pipe.)
(b) If there is no outer coating, $i'_c = \sin^{-1}(1/1.68)$ = 36.5°. Now, i = 90° will have r = 36.5° and i′= 53.5° which is greater than i'_c Thus, all incident rays (in the range 53.5° < i < 90°) will suffer total internal reflections.

Q.18 (a) Rays converging to a point 'behind' a plane or a convex mirror are reflected to a point in front of the mirror on a screen. In other words, a plane or a convex mirror can produce a real image if the object is virtual.Convince yourself by drawing an appropriate ray diagram.
(b) When the reflected or refracted rays are divergent, the image is virtual.The divergent rays can be converged on to a screen by means of an appropriate converging lens. The convex lens of the eye does just that. The virtual image here serves as an object for the lens to produce a real image. Note, the screen here is not located at the position of the virtual

image. There is no contradiction.

(c) Taller

(d) The apparent depth for oblique viewing decreases from its value for near-normal viewing. Convince yourself of this fact by drawing ray diagrams for different positions of the observer.

(e) Refractive index of a diamond is about 2.42, much larger than that of ordinary glass (about 1.5). The critical angle of diamond is about 24° much less than that of glass A skilled diamond cutter exploits the larger range of angles of incidence (in the diamond), 24° to 90°, to ensure that light entering the diamond is totally reflected from many faces before getting out–thus producing a sparkling effect.

Q.19 For fixed distance s between object and screen, the lens equation does not give a real solution for u or v if f is greater than s/4. Therefore, $f_{max} = 0.75$ m.

Q.20 21.4 cm

Q.21 (a) (i) Let a parallel beam be the incident from the left on the convex lens first.

$f_1 = 30$ cm and $u_1 = -\infty$, give $v_1 = +30$ cm. This image becomes a virtual object for the second lens.

$f_2 = -20$ cm, $u_2 = +(30 - 8)$ cm $= +22$ cm which gives,

$v_2 = -220$ cm. The parallel incident beam appears to diverge from a point 216 cm from the centre of the two-lens system

(ii) Let the parallel beam be incident from the left on the concave lens first: $f_1 = -20$ cm, $u_1 = -\infty$, give $v_1 = -20$ cm. This image becomes a real object for the second lens:

$f_2 = +30$ cm $u_2 = -(20 + 8)$ cm $= -28$ cm which gives, $v_2 = -420$ cm. The parallel incident beam appears to diverge from a point 416 cm on the left of the centre of the two-lens system.

Clearly, the answer depends on which side of the lens system the parallel beam is incident. Further we do not have a simple lens equation true for all u (and v) in terms of a definite constant of the system (the constant being determined by f_1 and f_2 and the separation between the lenses). The notion of effective focal length, therefore, does not seem to be meaningful for this system.

(b) $u_1 = -40$ cm, $f_1 = 30$ cm, gives $v_1 = 120$ cm.

Magnitude of magnification due to the first (convex) lens is 3.

$u_2 = +(120 - 8)$ cm $= +112$ cm (object virtual);

$f_2 = -20$ cm which gives $v_2 = -\frac{112 \times 20}{92}$ cm

Magnitude of magnification due to the second (concave) lens = 20/92.

Net magnitude of magnification = 0.652

Size of the image = 0.98 cm

Q.22 If the refracted ray in the prism is incident on the second face at the critical angle i_c, the angle of refraction r at the first face is $(60° - i_c)$. Now,

$i_c = \sin^{-1}(1/1.524) \simeq 41°$

Therefore, r = 19°

$\sin i = 0.4962; i \simeq 30°$

Q.23 Two identical prisms made of the same glass placed with their bases on opposite sides (of the incident white light) and faces touching (or parallel) will neither deviate nor disperse, but will mearly produce a parallel displacement of the beam.

(a) To deviate without dispersion, choose, say, the first prism to be of crown glass, and take for the second prism a flint glass prism of suitably chosen refracting angle (smaller than that of crown glass prism because the flint glass prism disperses more) so that dispersion due to the first is nullified by the second.

(b) To disperse without deviation, increase the angle of flint glass prism (i.e., try flint glass prisms of greater and greater angle) so that deviations due to the two prisms are equal and opposite. (The flint glass prism angle will still be smaller than that of crown glass because flint glass has higher refractive index than that of crown glass). Because of the adjustments involved for so many colours, these are not meant to be precise arrangements for the purpose required.

Q.24 To see objects at infinity the eye uses its least converging power = (40+20) dioptres = 60 dioptres. This gives a rough idea of the distance between the retina and cornea-eye lens: (5/3) cm. To focus an object at the near point (u = –25 cm) on the retina (v = 5/3 cm), the focal length should be $\left[\frac{1}{25}+\frac{3}{5}\right]^{-1}=\frac{25}{16}\text{cm}$ corresponding to a converging power of 64 dioptres. The power of the eye lens then is (64 – 40) dioptres = 24 dioptres. The range of accommodation of the eye-lens is roughly 20 to 24 dioptres.

Q.25 No, a person may have normal ability of accommodation of the eyelens and yet may be myopic or hypermetropic. Myopia arises when the eye-ball from front to back gets too elongated; hypermetropia arises when it gets too shortened. In practice, in addition, the eye lens may also loose some of its ability of accommodation When the eyeball has the normal length but the eye lens loses partially its ability of accommodation (as happens with increasing age for any normal eye), the 'defect' is called presbyopia and is corrected in the same manner as hypermetropia.

Q.26 The far point of the person is 100 cm while his near point may have been normal (about 25 cm). Objects at infinity produce virtual image at 100 cm (using spectacles). To view closer objects i.e.those which are (or whose images using the spectacles are) between 100 cm and 25 cm, the person uses the ability of accommodation of his eye-lens. This ability usually gets partially lost in old age (presbyopia). The near point of the person recedes to 50 cm. To view objects at 25 cm clearly, the person needs converging lens of power +2 dioptres.

Q.27 The defect (called astigmatism) arises because the curvature of the cornea plus eye-lens refracting system is not the same in different planes. [The eye-lens is usually spherical i.e.has the same curvature on different planes but the cornea is not spherical in case of an astigmatic eye.] In the present casethe curvature in the vertical plane is enough, so sharp images of vertical lines can be formed on the retina. But the curvature is insufficient in the horizontal plane, so horizontal lines appear blurred. The defect can be corrected by using a cylindrical lens with its axis along the vertical. Clearly, parallel rays in the vertical plane will suffer no extra refraction, but those in the horizontal plane can get the required extra convergence due to refraction by the curved surface of the cylindrical lens if the curvature of the cylindrical surface is chosen appropriately.

Q.28 (a) Closest distance = $=4\frac{1}{6}\text{cm}\approx 4.2\text{cm}$

Farthest distance = 5 cm

(b) Maximum angular magnification = [25/(25/6)] = 6 Minimum angular magnification = (25/5) = 5

Q.29 (a) $\frac{1}{v}+\frac{1}{9}=\frac{1}{10}$

i.e., $v=-90\text{cm}$,

Magnitude of magnification = 90/9 = 10.

Each square in the virtual image has an area 10 × 10 × 1 mm^2 = 100 mm^2 = 1 cm^2

(b) Magnifying power = 25/9 = 2.8

(c) No, magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the image even if the image is magnified) to the angular size of the object if placed at the near point (25 cm). Thus, magnification magnitude is |(v/u)| and magnifying power is (25/ |u|). Only when the image is located at the near point |v| = 25 cm, are the two quantities equal.

Q.30 (a) Maximum magnifying power is obtained when the image is at the near point (25 cm)

u = – 7.14 cm.

(b) Magnitude of magnification = (25/ |u|) = 3.5.
(c) Magnifying power = 3.5
Yes, the magnifying power (when the image is produced at 25 cm) is equal to the magnitude of magnification.

Q.31 Magnification

$= \sqrt{(6.25/1)} = 2.5$

$v = +2.5u$

$+\frac{1}{2.5u} - \frac{1}{u} = \frac{1}{10}$

$\text{i.e.,} \; u = -6\text{cm}$

$|v| = 15\text{cm}$

The virtual image is closer than the normal near point (25 cm) and cannot be seen by the eye distinctly.

Q.32 (a) Even though the absolute image size is bigger than the object size the angular size of the image is equal to the angular size of the object. The magnifier helps in the following way: without it object would be placed no closer than 25 cm; with it the object can be placed much closer. The closer object has larger angular size than the same object at 25 cm. It is in this sense that angular magnification is achieved.

(b) Yes, it decreases a little because the angle subtended at the eye is then slightly less than the angle subtended at the lens. The effect is negligible if the image is at a very large distance away. [Note: When the eye is separated from the lens the angles subtended at the eye by the first object and its image are not equal.]

(c) First, grinding lens of very small focal length is not easy. More important, if you decrease focal length, aberrations (both spherical and chromatic) become more pronounced. So, in practice, you cannot get a magnifying power of more than 3 or so with a simple convex lens. However, using an aberration corrected lens system, one can increase this limit by a factor of 10 or so.

(d) Angular magnification of eye-piece is [(25/f_e) + 1] (f_e in cm) which increases if f_e is smaller. Further, magnification of the objective is given by $\frac{v_o}{|u_o|} = \frac{1}{(|u_o|/f_o) - 1}$

which is large when $|u_O|$ is slightly greater than f_o. The microscope is used for viewing very close object. So $|u_o|$ is small and so is f_o.

(e) The image of the objective in the eye-piece is known as 'eye-ring'. All the rays from the object refracted by objective go through the eye-ring. Therefore, it is an ideal position for our eyes for viewing.
If we place our eyes too close to the eye-piece we shall not collect much of the light and also reduce our field of view. If we position our eyes on the eye-ring and the area of the pupil of our eye is greater or equal to the area of the eye-ring, our eyes will collect all the light refracted by the objective. The precise location of the eye-ring naturally depends on the separation between the objective and the eye-piece. When you view through a microscope by placing your eyes on one end, the ideal distance between the eyes and eye-piece is usually built-in the design of the instrument.

Q.33 Assume microscope in normal use i.e. image at 25 cm. Angular magnification of the eye-piece

$= \frac{25}{5} + 1 = 6$

Magnification of the objective

$$= \frac{30}{6} = 5$$

$$\frac{1}{5u_o - u_o} \frac{1}{} - \frac{1}{1.25}$$

which gives u_O= –1.5 cm; v_0= 7.5 cm $|u_e|$= (25/6) cm = 4.17 cm. The separation between the objective and the eye-piece should be (7.5 + 4.17) cm = 11.67 cm. Further the object should be placed 1.5 cm from the objective to obtain the desired magnification.

Q.34 (a) $m = (f_o / f_e) = 28$

(b) $m = \frac{f_o}{f_e}\left[1 + \frac{f_o}{25}\right] = 33.6$

Q.35 (a) $f_O + f_e$ = 145 cm

(b) Angle subtended by the tower = (100/3000) = (1/30) rad.

Angle subtended by the image produced by the objective

$$= \frac{h}{f_o} = \frac{h}{140}$$

Equating the two, h = 4.7 cm.

(c) Magnification (magnitude) of the eye-piece = 6. Height of the final image (magnitude) = 28 cm.

Q.36 The image formed by the larger (concave) mirror acts as virtual object for the smaller (convex) mirror. Parallel rays coming from the object at infinity will focus at a distance of 110 mm from the larger mirror. The distance of virtual object for the smaller mirror = (110 –20) = 90 mm. The focal length of smaller mirror is 70 mm.Using the mirror formula, image is formed at 315 mm from the smaller mirror.

Q.37 The reflected rays get deflected by twice the angle of rotation of the mirror. Therefore, d/1.5 = tan 7°. Hence d = 18.4 cm.

Q.38 n = 1.33

Exercise II

Q 30. 4000 Å

Q 31. $\frac{\lambda}{1.5}$

Q 32. 15°

Q 33. 10s.

Q 34. area $4.0\,cm^2$.

Q 35. Converging lens. f = 30 cm, size of the image = 10 cm.

Q 37. (a) 5 (b) Distant images arise due to multiple reflections. At each reflection, part of the incident intensity of light is lost due to absorption etc.

Q 38. The minimum length of the mirror for a full view is 0.75 m. This minimum length is the same for any level but the positions of the top and bottom edges of the mirror will depend on the eye-level.

Q 39. 4.9cm

Q 40. The ratio of dispersive power of flint-glass to crown-glass is 1.57.

Q 42. His rough estimate of the size of the Sun; $D = 1.5 \times 10^9\,m$ (correct value = $1.39 \times 10^9\,m$).

Q 43. The separation between the objective and eye-piece is 199cm. Magnifying power remains unaltered: 36

Q 44. The projection lens should be 28.1 cm from the slide and have a focal length of 27.5cm.

Flashback

CBSE 2005

Q 2. **(b)** (i)When the refractive index of the medium increases, the resolving power increases.

(ii) When the wavelength of the radiation increases, the resolving power decreases.

CBSE 2006

Q 2. For convex lens – 7.5 cm , for concave lens – 48 cm

CBSE 2007

Q 1. Virtual image of height 10 cm is formed at a distance of 25 cm from the lens on the same side as the object.

CBSE 2008

Q.2 Angle of deviation is decreased **Q.4** (i) + 5D ; (ii) 10 cm

CBSE 2009

Q.1 25 cm

CBSE 2010

Q.1 $f_{eq} = \infty$ **Q.2** (ii) 10

CBSE 2011

Q.1 f = 52cm

Q.3 3.5 or 2.5

Length of microscope = 12 + 7.1
= 19.1 cm

OR

m = 1500
d = 13.5 cm

CBSE 2012

Q.1 Velocity of light will be minimum in A.

Q.4 The distance between L_1 & L_2 is (60 + 15) cm = 75 cm and the distance between L_2 & L_3 can take any real value.

CBSE 2013

Q.1 $f_{eq} = \frac{f_1 f_2}{f_2 - f_1}$ `