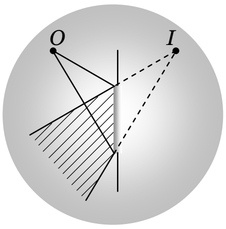
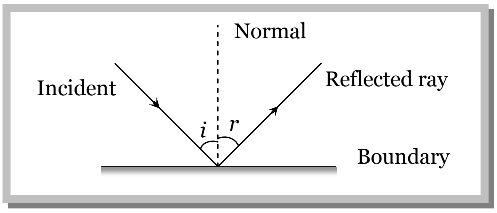


Reflection of Light

When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon, is called reflection of light.



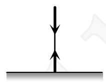
⇒ $\angle i = \angle r$

⇒ After reflection, velocity, wave length and frequency of light remains same but intensity decreases

⇒ There is a phase change of π if reflection takes place from denser medium

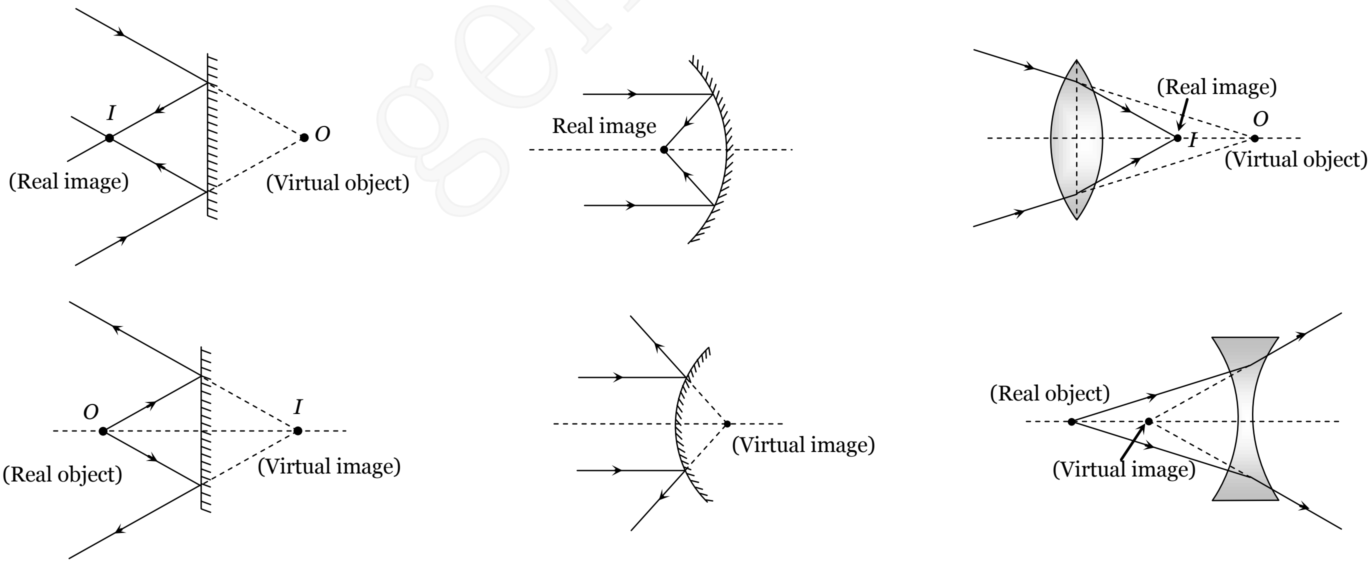
Note : ❑ After reflection velocity, wavelength and frequency of light remains same but intensity decreases.

❑ If light ray incident normally on a surface, after reflection it retraces the path.



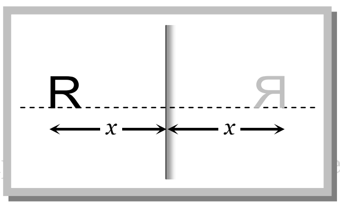
Real and virtual images

If light rays, after reflection or refraction, actually meets at a point then real image is formed and if they appears to meet virtual image is formed.

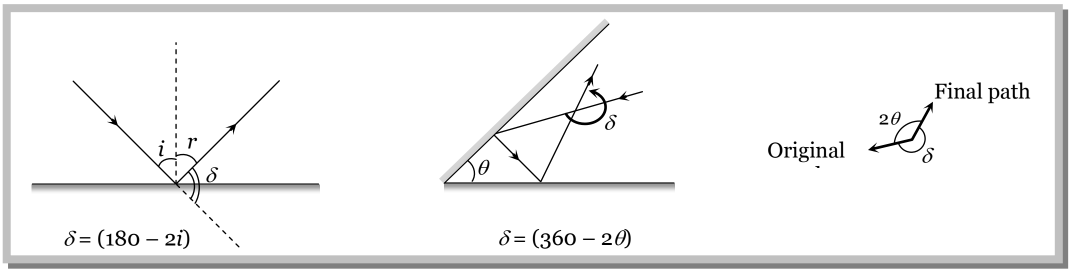


Plane Mirror

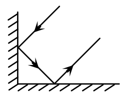
The image formed by a plane mirror is virtual, erect, laterally inverted, equal in size that of the object and at a distance equal to the distance of the object in front of the mirror.



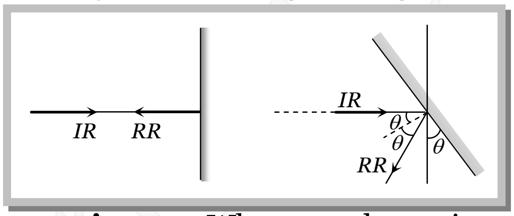
(1) **Deviation :** Deviation produced by a plane mirror and by two inclined plane mirrors.



Note **:** ❑ If two plane mirrors are inclined to each other at 90°, the emergent ray is anti-parallel to incident ray, if it suffers one reflection from each. Whatever be the angle to incidence.



(2) **Rotation :** If a plane mirror is rotated in the plane of incidence through angle θ, by keeping the incident ray fixed, the reflected ray turned through an angle 2θ.



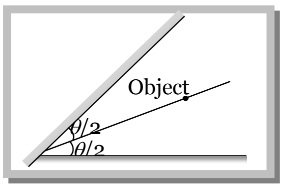
(3) **Images by two inclined plane mirrors :** When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.

(i) $n = \left(\frac{360}{\theta} - 1\right)$; If $\frac{360}{\theta}$ = even integer

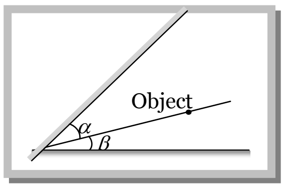
(ii) If $\frac{360}{\theta}$ = odd integer then there are two possibilities

(a) Object is placed symmetrically (b) Object is placed asymmetrically

$n = \left(\frac{360}{\theta} - 1\right)$



$n = \frac{360}{\theta}$



Note **:** ❑ If θ = 0° *i.e.* mirrors are parallel to each other so $n = \infty$ *i.e.* infinite images will be formed.

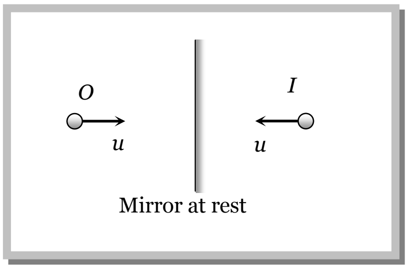
❑ If θ = 90°, $n = \frac{360}{90} - 1 = 3$

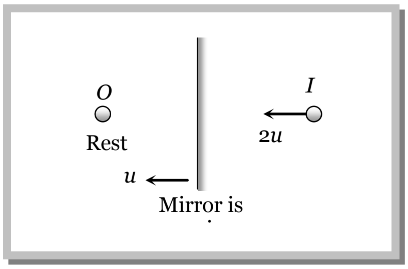
❑ If θ = 72°, $n = \frac{360}{72} - 1 = 4$ (If nothing is said object is supposed to be symmetrically placed).

(4) **Other important informations**

(i) When the object moves with speed u towards (or away) from the plane mirror then image also moves toward (or away) with speed u. But relative speed of image *w.r.t.* object is $2u$.

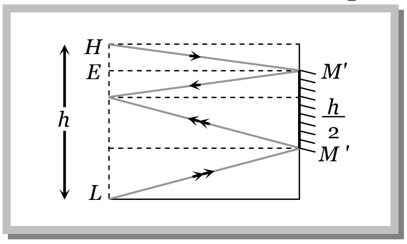
(ii) When mirror moves towards the stationary object with speed u, the image will move with speed $2u$.

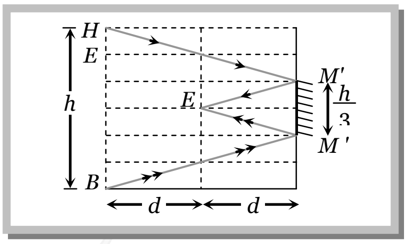




(iii) A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.

(iv) To see complete wall behind himself a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.

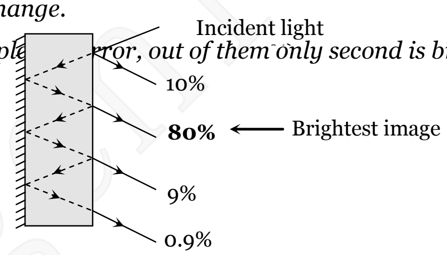




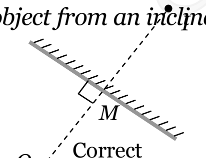
Example

Concepts

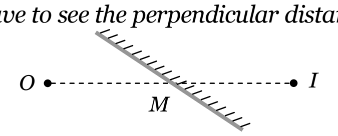
- *The reflection from a denser medium causes an additional phase change of π or path change of $\lambda/2$ while reflection from rarer medium doesn't cause any phase change.*
- *We observe number of images in a thick plane mirror, out of them only second is brightest.*



- *To find the location of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.*



Correct
$OM = MI$

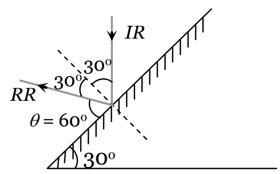


Wrong
$OM = MI$

Example: 1 A plane mirror makes an angle of 30° with horizontal. If a vertical ray strikes the mirror, find the angle between mirror and reflected ray

(a) 30° (b) 45° (c) 60° (d) 90°

Solution : (c) Since angle between mirror and normal is 90° and reflected ray (RR) makes an angle of 30° with the normal so required angle will be $\theta = 60°$.



***Example*: 2** Two vertical plane mirrors are inclined at an angle of 60° with each other. A ray of light travelling horizontally is reflected first from one mirror and then from the other. The resultant deviation is

(a) 60° (b) 120° (c) 180° (d) 240°

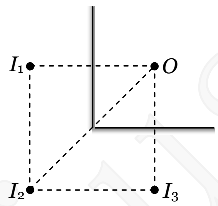
Solution : (d) By using $\delta = (360 - 2\theta)$ $\Rightarrow$ $\delta = 360 - 2 \times 60 = 240^o$

***Example*: 3** A person is in a room whose ceiling and two adjacent walls are mirrors. How many images are formed

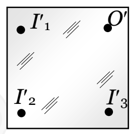
[AFMC 2002]

(a) 5 (b) 6 (c) 7 (d) 8

Solution : (c) The walls will act as two mirrors inclined to each other at 90° and so sill form $\frac{360}{90} - 1 = 3$ images of the person. Now these images with object (Person) will act as objects for the ceiling mirror and so ceiling will form 4 images as shown. Therefore total number of images formed = 3 + 4 = 7



Three images by walls



Four images by

Note : ❑ The person will see only six images of himself (I_1, I_2, I_3, I_1', I_2', I_3')

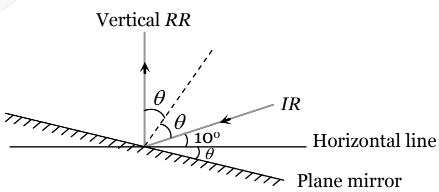
***Example*: 4** A ray of light makes an angle of 10° with the horizontal above it and strikes a plane mirror which is inclined at an angle θ to the horizontal. The angle θ for which the reflected ray becomes vertical is

(a) 40° (b) 50° (c) 80° (d) 100°

Solution : (a) From figure

$\theta + \theta + 10 = 90$

$\Rightarrow \theta = 40^o$



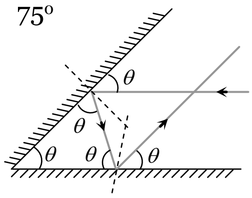
***Example*: 5** A ray of light incident on the first mirror parallel to the second and is reflected from the second mirror parallel to first mirror. The angle between two mirrors is

(a) 30° (b) 60° (c) 75° (d) 90°

Solution : (b) From geometry of figure

$\theta + \theta + \theta = 180^o$

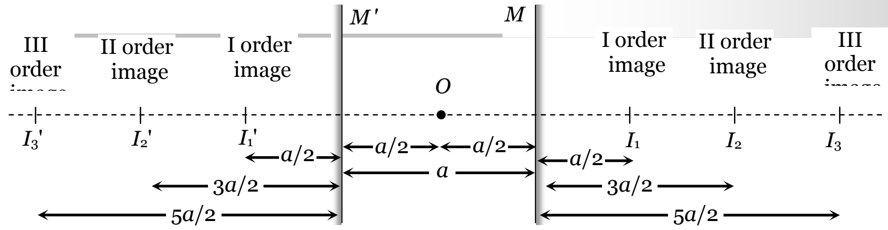
$\Rightarrow \theta = 60^o$



***Example*: 6** A point object is placed mid-way between two plane mirrors distance 'a' apart. The plane mirror forms an infinite number of images due to multiple reflection. The distance between the nth order image formed in the two mirrors is

(a) na (b) $2na$ (c) $na/2$ (d) $n^2 a$

Solution : (b)



From above figure it can be proved that seperation between nth order image formed in the two mirrors = $2na$

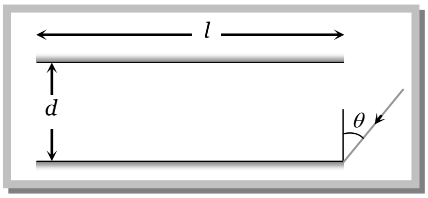
Example: 7 Two plane mirrors P and Q are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of θ at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is

(a) $\frac{l}{d\tan\theta}$

(b) $\frac{d}{l\tan\theta}$

(c) $ld\tan\theta$

(d) None of these

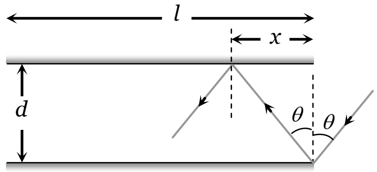


Solution : (a) Suppose n = Total number of reflection light ray undergoes before exist out.

x = Horizontal distance travelled by light ray in one reflection.

So $nx = l$ also $\tan\theta = \frac{x}{d}$

$\Rightarrow \quad n = \frac{l}{d\tan\theta}$



Example: 8 A plane mirror and a person are moving towards each other with same velocity v. Then the velocity of the image is

(a) v (b) $2v$ (c) $3v$ (d) $4v$

Solution : (c) If mirror would be at rest, then velocity of image should be $2v$. but due to the motion of mirror, velocity of image will be $2v + v = 3v$.

Example: 9 A ray reflected successively from two plane mirrors inclined at a certain angle undergoes a deviation of 300°. The number of images observable are

(a) 10 (b) 11 (c) 12 (d) 13

Solution : (b) By using $\delta = (360 - 2\theta) \Rightarrow 300 = 360 - 2\theta$

$\Rightarrow \theta = 30^o$. Hence number of images $= \frac{360}{30} - 1 = 11$

Tricky example: 1

A small plane mirror placed at the centre of a spherical screen of radius R. A beam of light is falling on the mirror. If the mirror makes n revolution. per second, the speed of light on the screen after reflection from the mirror will be

(a) $4\pi nR$ (b) $2\pi nR$ (c) $\frac{nR}{2\pi}$ (d) $\frac{nR}{4\pi}$

Solution : (a) When plane mirror rotates through an angle θ, the reflected ray rotates through an angle 2θ. So spot on the screen will make $2n$ revolution per second

$\therefore$ Speed of light on screen $v = \omega R = 2\pi(2n)R = 4\pi nR$

Tricky example: 2

A watch shows time as 3 : 25 when seen through a mirror, time appeared will be

[RPMT 1997; JIPMER 2001, 2002]

(a) 8 : 35 (b) 9 : 35 (c) 7 : 35 (d) 8 : 25

Solution : (a) For solving this type of problems remember

Actual time = 11 : 60 – given time

So here Actual time = 11 : 60 – 3 : 25 = 8 : 35

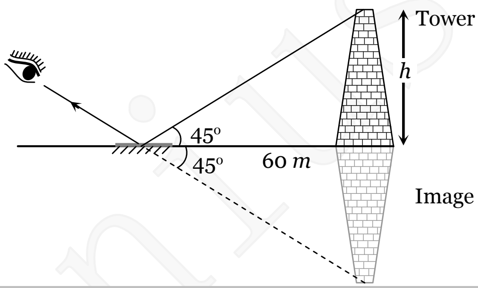
Tricky example: 3

When a plane mirror is placed horizontally on a level ground at a distance of 60 *m* from the foot of a tower, the top of the tower and its image in the mirror subtend an angle of 90° at the eye. The height of the tower will be **[CPMT 1984]**

(a) 30 *m* (b) 60 *m* (c) 90 *m* (d) 120 *m*

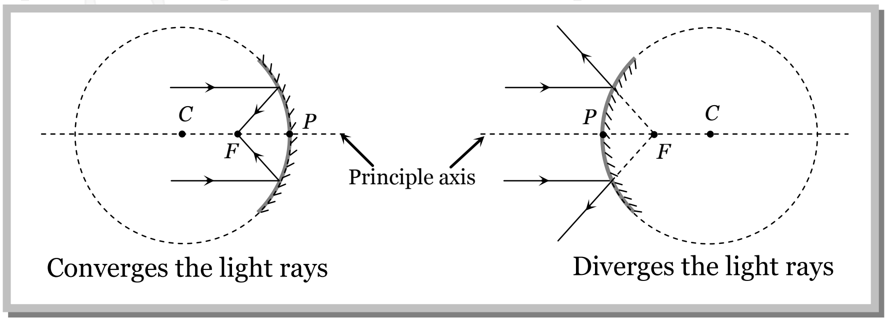
Solution : (b) Form the figure it is clear that $\frac{h}{60} = \tan 45^o$

$\Rightarrow h = 60\ m$



Curved Mirror

It is a part of a transparent hollow sphere whose one surface is polished.



(1) **Some definitions :**

(i) **Pole (P)**	:	Mid point of the mirror
(ii) Centre of curvature (C)	:	Centre of the sphere of which the mirror is a part.
(iii) Radius of curvature (R)	:	Distance between pole and centre of curvature.
		($R_{\text{concave}} = -ve$, $R_{\text{convex}} = +ve$, $R_{\text{plane}} = \infty$)
(iv) Principle axis	:	A line passing through *P* and *C*.
(v) Focus (F)	:	An image point on principle axis for which object is at ∞
(vi) Focal length (f)	:	Distance between *P* and *F*.

(vii) Relation between f and R : $f = \frac{R}{2}$ ($f_{\text{concare}} = -ve$, $f_{\text{convex}} = +ve$, $f_{\text{plane}} = \infty$)

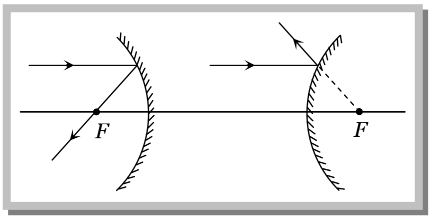
(viii) Power : The converging or diverging ability of mirror

(ix) Aperture : Effective diameter of light reflecting area.
Intensity of image $\propto$ Area $\propto$ (Aperture)^2

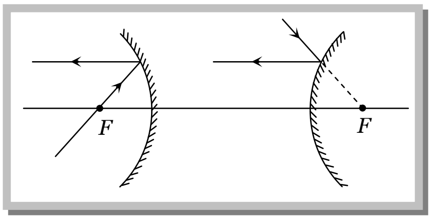
(x) Focal plane : A plane passing from focus and perpendicular to principle axis.

(2) **Rules of image formation and sign convention** :

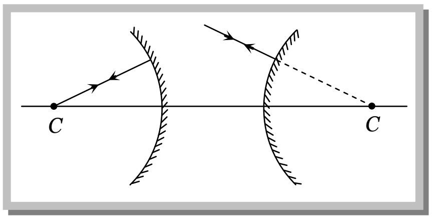
Rule (i)



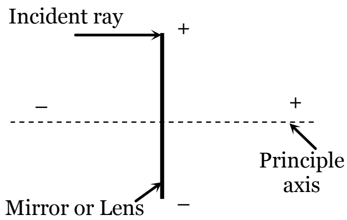
Rule (ii)



Rule (iii)



(3) **Sign conventions :**



(i) All distances are measured from the pole.

(ii) Distances measured in the direction of incident rays are taken as positive while in the direction opposite of incident rays are taken negative.

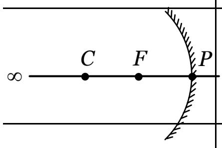
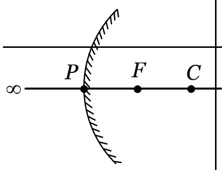
(iii) Distances above the principle axis are taken positive and below the principle axis are taken negative.

Note : ❑ Same sign convention are also valid for lenses.

Use following sign while solving the problem :

Concave mirror				Convex mirror
	Real image (u ≥ f)		**Virtual image (u< f)**	
Distance of object	$u \rightarrow -$		$u \rightarrow -$	$u \rightarrow -$
Distance of image	$v \rightarrow -$		$v \rightarrow +$	$v \rightarrow +$
Focal length	$f \rightarrow -$		$f \rightarrow -$	$f \rightarrow +$
Height of object	$O \rightarrow +$		$O \rightarrow +$	$O \rightarrow +$
Height of image	$I \rightarrow -$		$I \rightarrow +$	$I \rightarrow +$
Radius of curvature	$R \rightarrow -$		$R \rightarrow -$	$R \rightarrow +$
Magnification	$m \rightarrow -$		$m \rightarrow +$	$m \rightarrow +$

(4) **Position, size and nature of image formed by the spherical mirror**

Mirror	Location of the object	Location of the image	Magnification, Size of the image	Nature	
				Real virtual	**Erect inverted**
(a) Concave	At infinity $i.e.$ $u = \infty$	At focus $i.e.$ $v = f$	$m << 1$, diminished	Real	inverted
	Away from centre of curvature ($u > 2f$)	Between f and $2f$ $i.e.$ $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At centre of curvature $u = 2f$	At centre of curvature $i.e.$ $v = 2f$	$m = 1$, same size as that of the object	Real	inverted
	Between centre of curvature and focus : $F < u < 2f$	Away from the centre of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus $i.e.$ $u = f$	At infinity $i.e.$ $v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$ magnified	Virtual	erect
(b) Convex	At infinity $i.e.$ $u = \infty$	At focus $i.e.$, $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

Note : ❑ In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.

❑ Images formed by mirrors do not show chromatic aberration.

❑ For convex mirror maximum image distance is it's focal length.

❑ In concave mirror, minimum distance between a real object and it's real image is zero.
($i.e.$ when $u = v = 2f$)

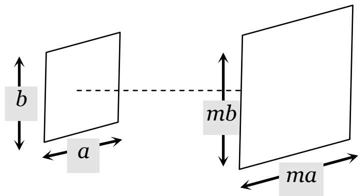
Mirror formula and magnification

For a spherical mirror if u = Distance of object from pole, v = distance of image from pole, f = Focal length, R = Radius of curvature, O = Size of object, I = size of image, m = magnification (or linear magnification), m_s = Areal magnification, A_o = Area of object, A_i = Area of image

(1) **Mirror formula :** $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$; (use sign convention while solving the problems).

Note : ❑ **Newton's formula :** If object distance (x_1) and image distance (x_2) are measured from focus instead of pole then $f^2 = x_1 x_2$

(2) **Magnification :** m = $\frac{\text{Size of object}}{\text{Size of image}}$

Linear magnification		Areal magnification
Transverse	**Longitudinal**	
When a object is placed perpendicular to the principle axis, then linear magnification is called lateral or transverse magnification. It is given by $m=\frac{I}{O}=-\frac{v}{u}=\frac{f}{f-u}=\frac{f-v}{f}$ (* Always use sign convention while solving the problems)	When object lies along the principle axis then its longitudinal magnification $m=\frac{I}{O}=\frac{-(v_2-v_1)}{(u_2-u_1)}$ If object is small; $m=-\frac{dv}{du}=\left(\frac{v}{u}\right)^2$ Also Length of image = $\left(\frac{v}{u}\right)^2 \times \text{Length of object } (L_0)$ $(L_i)=\left(\frac{f}{u-f}\right)^2 .L_o$	 If a 2*D*-object is placed with it's plane perpendicular to principle axis It's Areal magnification $M_s=\frac{\text{Area of image }(A_i)}{\text{Area of object }(A_o)}$ $=\frac{ma\times mb}{ab}=m^2$ $\Rightarrow m_s=m^2=\frac{A_i}{A_o}$

Note : ❑ Don't put the sign of quantity which is to be determined.

❑ If a spherical mirror produces an image '*m*' times the size of the object (*m* = magnification) then *u*, *v* and *f* are given by the followings

$$u=\left(\frac{m-1}{m}\right)f,\quad v=-(m-1)f \quad \text{and} \quad f=\left(\frac{m}{m-1}\right)u \quad \text{(use sign convention)}$$

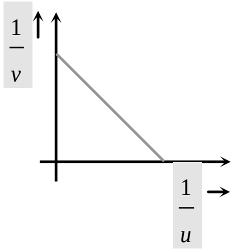
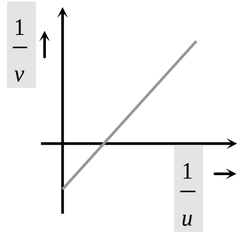
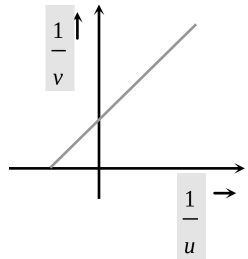
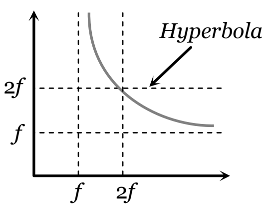
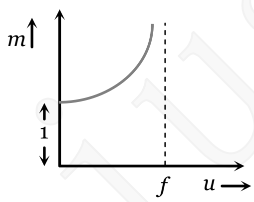
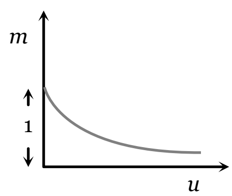
(3) **Uses of mirrors**

(i) **Concave mirror :** Used as a shaving mirror, In search light, in cinema projector, in telescope, by E.N.T. specialists etc.

(ii) **Convex mirror :** In road lamps, side mirror in vehicles *etc.*

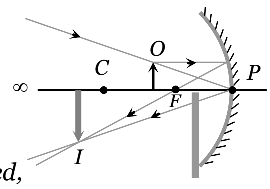
Note : ❑ Field of view of convex mirror is more than that of concave mirror.

Different graphs

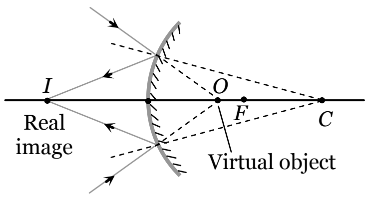
Graph between $\frac{1}{v}$ and $\frac{1}{u}$		
(a) Real image formed by concave mirror 	(b) Virtual image formed by concave mirror 	(c) Virtual image formed by convex mirror 
Graph between u and v for real image of concave mirror	**Graph between u and m for virtual image by concave mirror**	**Graph between u and m for virtual image by convex mirror.**
		

Concepts

- *Focal length of a mirror is independent of material of mirror, medium in which it is placed, wavelength of incident light*
- *Divergence or Convergence power of a mirror does not change with the change in medium.*
- *If an object is moving at a speed v_o towards a spherical mirror along it's axis then speed of image away from mirror is* $v_i = -\left(\frac{f}{u-f}\right)^2 . v_o$ *(use sign convention)*



- *When object is moved from focus to infinity at constant speed, the image will move faster in the beginning and slower later on, towards the mirror.*
- *As every part of mirror forms a complete image, if a part of the mirror is obstructed, full image will be formed but intensity will be reduced.*
- *Can a convex mirror form real images?*
  yes if (distance of virtual object) $u < f$ (focal length)



Example

Example: 10 A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

(a) $(n-1)f$ (b) $\left(\frac{n-1}{n}\right)f$ (c) $\left(\frac{n+1}{n}\right)f$ (d) $(n+1)f$

Solution : (a) By using $m = \frac{f}{f-u}$

Here $m = +\frac{1}{n}$, $f \to +f$ So, $+\frac{1}{n} = \frac{+f}{+f-u} \Rightarrow u = -(n-1)f$

Example: 11 An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

Solution : (c) By using $\frac{I}{O} = \frac{f}{f-u}$

Here $O + 5\,cm$, $f = -\frac{R}{2} = -10\,cm$, $u = -1\,m = -100\,cm$

So, $\frac{I}{+5} = \frac{-10}{-10-(-100)}$ $\Rightarrow$ $I = -\,0.55\,cm.$

Example: 12 An object of length 2.5 *cm* is placed at a distance of 1.5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principle axis. The length of the image is

(a) 5 *cm*, erect (b) 10 *cm*, erect (c) 15 *cm*, erect (d) 5 *cm*, inverted

Solution : (d) By using $\frac{I}{O} = \frac{f}{f-u}$; where $I = ?$, $O = +\,2.5\,cm.$ $f \to -f$, $u = -\,1.5f$

$\therefore \frac{I}{+2.5} = \frac{-f}{-f-(-1.5f)}$ $\Rightarrow$ $I = -5\,cm.$ (Negative sign indicates that image is inverted.)

Example: 13 A convex mirror has a focal length f. A real object is placed at a distance f in front of it from the pole produces an image at

(a) Infinity (b) f (c) $f/2$ (d) $2f$

Solution : (c) By using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{+f} = \frac{1}{v} + \frac{1}{(-f)} \Rightarrow v = \frac{f}{2}$

Example: 14 Two objects A and B when placed one after another infront of a concave mirror of focal length 10 *cm* from images of same size. Size of object A is four times that of B. If object A is placed at a distance of 50 *cm* from the mirror, what should be the distance of B from the mirror

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

Solution : (b) By using $\frac{I}{O} = \frac{f}{f-u} \Rightarrow \frac{I_A}{I_B} \times \frac{O_B}{O_A} = \frac{f-u_B}{f-u_A} \Rightarrow \frac{1}{1} \times \frac{1}{4} = \frac{-10-u_B}{-10-(-50)} \Rightarrow u_B = -20cm$.

Example: 15 A square of side 3 *cm* is placed at a distance of 25 *cm* from a concave mirror of focal length 10 *cm*. The centre of the square is at the axis of the mirror and the plane is normal to the axis. The area enclosed by the image of the wire is

(a) 4 cm^2 (b) 6 cm^2 (c) 16 cm^2 (d) 36 cm^2

Solution : (a) By using $m^2 = \dfrac{A_i}{A_o}$; where $m = \dfrac{f}{f-u}$

Hence from given values $m = \dfrac{-10}{-10-(-25)} = \dfrac{-2}{3}$ and $A_o = 9\,cm^2$ ∴

$A_i = \left(\dfrac{-2}{3}\right)^2 \times 9 = 4\,cm^2$

***Example*: 16** A convex mirror of focal length 10 *cm* is placed in water. The refractive index of water is 4/3. What will be the focal length of the mirror in water

(a) 10 *cm* (b) 40/3 *cm* (c) 30/4 *cm* (d) None of these

Solution : (a) No change in focal length, because *f* depends only upon radius of curvature *R*.

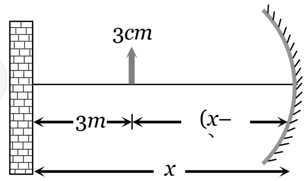
***Example*: 17** A candle flame 3 *cm* is placed at distance of 3 *m* from a wall. How far from wall must a concave mirror be placed in order that it may form an image of flame 9 *cm* high on the wall

(a) 225 *cm* (b) 300 *cm* (c) 450 *cm* (d) 650 *cm*

Solution : (c) Let the mirror be placed at a distance *x* from wall

By using

$\dfrac{I}{O} = \dfrac{-v}{u} \Rightarrow \dfrac{-9}{+3} = \dfrac{-(-x)}{-(x-3)} \Rightarrow x = -4.5m = -450cm.$



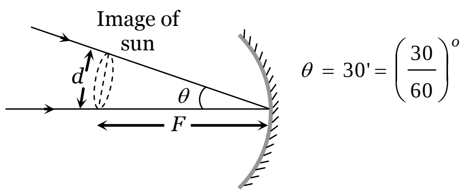
***Example*: 18** A concave mirror of focal length 100 *cm* is used to obtain the image of the sun which subtends an angle of 30'. The diameter of the image of the sun will be

(a) 1.74 *cm* (b) 0.87 *cm* (c) 0.435 *cm* (d) 100 *cm*

Solution : (b) Diameter of image of sun $d = f\theta$

$\Rightarrow d = 100 \times \left(\dfrac{30}{60}\right) \times \dfrac{\pi}{180}$

$\Rightarrow \quad d = 0.87\,cm$.



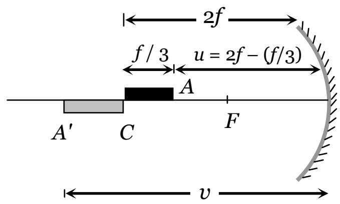
***Example*: 19** A thin rod of length *f* / 3 lies along the axis of a concave mirror of focal length *f*. One end of its magnified image touches an end of the rod. The length of the image is **[MP PET 1995]**

(a) f (b) $\dfrac{1}{2}f$ (c) $2f$ (d) $\dfrac{1}{4}f$

Solution : (b) If end *A* of rod acts an object for mirror then it's image will be *A*' and if $u = 2f - \dfrac{f}{3} = \dfrac{5f}{3}$

So by using $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u} \Rightarrow \dfrac{1}{-f} = \dfrac{1}{v} + \dfrac{1}{\dfrac{-5f}{3}} \Rightarrow v = -\dfrac{5}{2}f$

∴ Length of image $= \dfrac{5}{2}f - 2f = \dfrac{f}{2}$

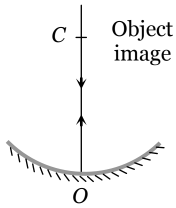


***Example*: 20** A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let *O* be the pole of the mirror and *C* its centre of curvature. A point object is placed at *C*. It has a real image, also located at *C*. If the mirror is now filled with water, the image will be

(a) Real, and will remain at C (b) Real, and located at a point between C and ∞

(c) Virtual and located at a point between C and O (d) Real, and located at a point between C and O

Solution : (d)



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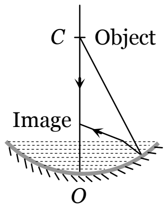


Finally

Tricky example: 4

An object is placed infront of a convex mirror at a distance of 50 *cm*. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 *cm,* it is found that there is no parallel between the images formed by two mirrors. Radius of curvature of mirror will be

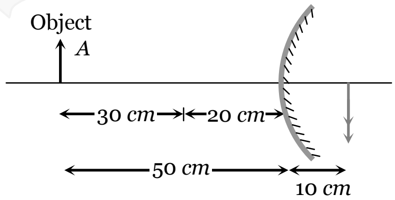
(a) 12.5 *cm* (b) 25 *cm* (c) $\frac{50}{3}\,cm$ (d) 18 *cm*

Solution : (b) Since there is no parallel, it means that both images (By plane mirror and convex mirror) coinciding each other.

According to property of plane mirror it will form image at a distance of 30 *cm* behind it. Hence for convex mirror $u = -50\ cm$, $v = +10\ cm$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

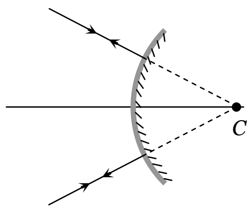
$\Rightarrow f = \frac{25}{2}\,cm$ $\Rightarrow R = 2f = 25cm.$



Tricky example: 5

A convergent beam of light is incident on a convex mirror so as to converge to a distance 12 *cm* from the pole of the mirror. An inverted image of the same size is formed coincident with the virtual object. What is the focal length of the mirror

(a) 24 *cm* (b) 12 *cm* (c) 6 *cm* (d) 3 *cm*



Solution : (c) Here object and image are at the same position so this position must be centre of curvature

$\therefore\ R = 12\ cm \ \Rightarrow f = \frac{R}{2}$

Practice Questions Basic Level

1. A light bulb is placed between two mirrors (plane) inclined at an angle of 60°. Number of images formed are

[NCERT 1980; CPMT 1996, 97; SCRA 1994; AIIMS 1997; RPMT 1999; AIEEE 2002; Orissa JEE 2003; MP PET 2004]

(a) 2 (b) 4 (c) 5 (d) 6

2. Two plane mirrors are inclined at an angle of 72^o. The number of images of a point object placed between them will be **[KCET (Engg. & Med.)1999; BCECE 2003]**

(a) 2 (b) 3 (c) 4 (d) 5

3. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 30^o (b) 60^o (c) 90^o (d) 120^o

4. A man of length h requires a mirror of length at least equal to, to see his own complete image **[MP PET 2003]**

(a) $\frac{h}{4}$ (b) $\frac{h}{3}$ (c) $\frac{h}{2}$ (d) h

5. Two plane mirrors are at 45° to each other. If an object is placed between them then the number of images will be **[MP PMT 200**

(a) 5 (b) 9 (c) 7 (d) 8

6. An object is at a distance of 0.5 m in front of a plane mirror. Distance between the object and image is **[CPMT 2002]**

(a) 0.5 m (b) 1 m (c) 0.25 m (d) 1.5 m

7. A man runs towards a mirror at a speed 15 m/s. The speed of the image relative to the man is **[RPMT 1999; Kerala PET 2002]**

(a) 15 ms^{-1} (b) 30 ms^{-1} (c) 35 ms^{-1} (d) 20 ms^{-1}

8. The light reflected by a plane mirror may form a real image **[KCET (Engg. & Med.) 2002]**

(a) If the rays incident on the mirror are diverging (b) If the rays incident on the mirror are converging

(c) If the object is placed very close to the mirror (d) Under no circumstances

9. A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see his entire height right from toe to head, he uses a plane mirror kept at a distance of 1 m from him. The minimum length of the plane mirror required is **[MP PMT 1993; DPMT 2001]**

(a) 180 cm (b) 90 cm (c) 85 cm (d) 170 cm

10. A small object is placed 10 cm infront of a plane mirror. If you stand behind the object 30 cm from the object and look at its image, the distance focused for your eye will be

(a) 60 cm (b) 20 cm (c) 40 cm (d) 80 cm

11. Two plane mirrors are at right angles to each other. A man stands between them and combs his hair with his right hand. In how many of the images will he be seen using his right hand

(a) None (b) 1 (c) 2 (d) 3

12. A man runs towards mirror at a speed of 15 m/s. What is the speed of his image **[CBSE PMT 2000]**

(a) 7.5 m/s (b) 15 m/s (c) 30 m/s (d) 45 m/s

13. A ray of light is incidenting normally on a plane mirror. The angle of reflection will be **[MP PET 2000]**

(a) 0° (b) 90° (c) Will not be reflected (d) None of these

14. A plane mirror produces a magnification of **[MP PMT/PET 1997]**

(a) – 1 (b) + 1 (c) Zero (d) Between 0 and + ∞

15. When a plane mirror is rotated through an angle θ, then the reflected ray turns through the angle 2θ, then the size of the image **[MP PAT 1996]**

(a) Is doubled (b) Is halved (c) Remains the same (d) Becomes infinite

16. What should be the angle between two plane mirrors so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other

(a) 60° (b) 90° (c) 120° (d) 175°

17. Ray optics is valid, when characteristic dimensions are **[CBSE PMT 1994]**

(a) Of the same order as the wavelength of light (b) Much smaller than the wavelength of light

(c) Of the order of one millimeter (d) Much larger than the wavelength of light

18. It is desired to photograph the image of an object placed at a distance of 3 m from the plane mirror. The camera which is at a distance of 4.5 m from the mirror should be focussed for a distance of

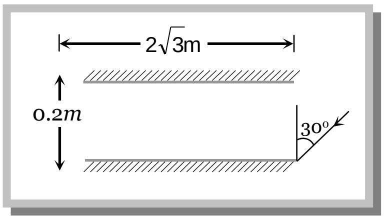
(a) 3 m (b) 4.5 m (c) 6 m (d) 7.5 m

19. Two plane mirrors are parallel to each other an spaced 20 *cm* apart. An object is kept in between them at 15 *cm* from *A*. Out of the following at which point an image is not formed in mirror *A* (distance measured from mirror *A*)

(a) 15 *cm* (b) 25 *cm* (c) 45 *cm* (d) 55 *cm*

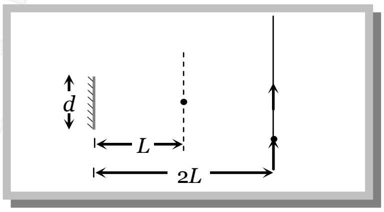
Advance Level

20. Two plane mirrors *A* and *B* are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of *A*. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is



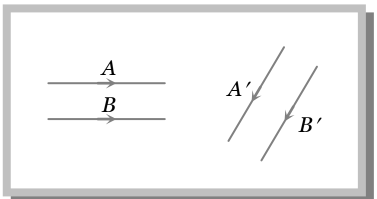
(a) 28
(b) 30
(c) 32
(d) 34

21. A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown. The greatest distance over which he can see the image of the light source in the mirror is



(a) *d*/2
(b) *d*
(c) 2*d*
(d) 3*d*

22. The figure shows two rays *A* and *B* being reflected by a mirror and going as *A*′ and *B*′. The mirror is



(a) Plane (b) Concave
(c) Convex (d) May be any spherical mirror

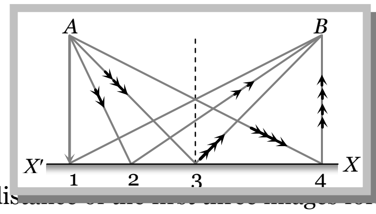
23. An object is initially at a distance of 100 *cm* from a plane mirror. If the mirror approaches the object at a speed of 5 *cm/s,* then after 6 *s* the distance between the object and its image will be

(a) 60 *cm* (b) 140 *cm* (c) 170 *cm* (d) 150 *cm*

24. An object placed in front of a plane mirror is displaced by 0.4 *m* along a straight line at an angle of 30° to mirror plane. The change in the distance between the object and its image is

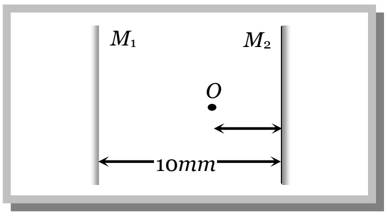
(a) 0.20 *m* (b) 0.40 *m* (c) 0.25 *m* (d) 0.80 *m*

25. A ray of light travels from *A* to *B* with uniform speed. On its way it is reflected by the surface *XX*′. The path followed by the ray to take least time is



(a) 1 (b) 2
(c) 3 (d) 4

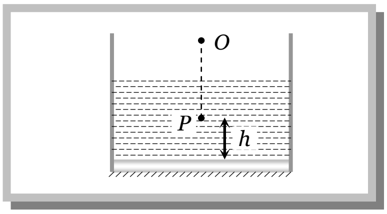
26. A point object *O* is placed between two plan mirrors as shown is fig. The dis[illegible]med by mirror M_2 from it are



(a) 2 *mm,* 8 *mm,* 18 *mm*
(b) 2 *mm,* 18 *mm,* 28 *mm*
(c) 2 *mm,* 18 *mm,* 22 *mm*

(d) 2 *mm*, 18 *mm*, 58 *mm*

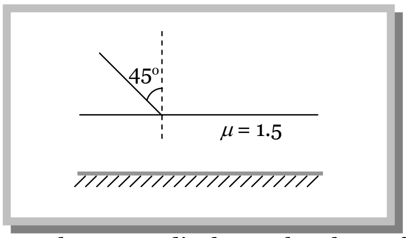
27. A plane mirror is placed at the bottom of the tank containing *a* liquid of refractive index μ . *P* is a small object at a height *h* above the mirror. An observer *O*-vertically above *P* outside the liquid see *P* and its image in the mirror. The apparent distance between these two will be



(a) $2\mu h$ (b) $\frac{2h}{\mu}$

(c) $\frac{2h}{\mu - 1}$ (d) $h\left(1+\frac{1}{\mu}\right)$

28. One side of a glass slab is silvered as shown. A ray of light is incident on the other side at angle of incidence $i = 45^o$. Refractive index of glass is given as 1.5. The deviation of the ray of light from its initial path when it comes out of the slab is



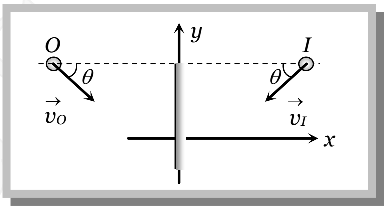
(a) 90°

(b) 180°

(c) 120°

(d) 45°

29. If an object moves towards a plane mirror with a speed *v* at an angle θ to the perpendicular to the plane of the mirror, find the relative velocity between the object and the image



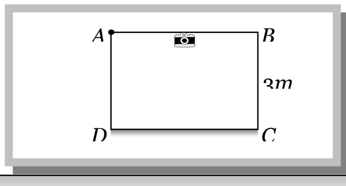
(a) *v*

(b) 2*v*

(c) $2v\cos\theta$

(d) $2v\sin\theta$

30. Figure shows a cubical room *ABCD* will the wall *CD* as a plane mirror. Each side of the room is 3*m*. We place a camera at the midpoint of the wall *AB*. At what distance should the camera be focussed to photograph an object placed at *A*



(a) 1.5 *m* (b) 3 *m*

(c) 6 *m* (d) More than 6 *m*

Reflection of light at spherical surface

Basic Level

31. A man having height 6 *m*, want to see full height in mirror. They observe image of 2*m* height erect, then used mirror is **[J & K CET 2004]**

(a) Concave (b) Convex (c) Plane (d) None of these

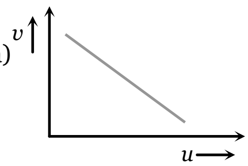
32. An object of length 6*cm* is placed on the principal axis of a concave mirror of focal length *f* at a distance of 4 *f*. The length of the image will be **[MP PET 2003]**

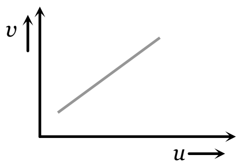
(a) 2 *cm* (b) 12 *cm* (c) 4 *cm* (d) 1.2 *cm*

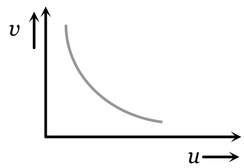
33. Convergence of concave mirror can be decreased by dipping in **[AFMC 2003]**

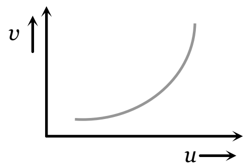
(a) Water (b) Oil (c) Both (d) None of these

34. In an experiment of find the focal length of a concave mirror a graph is drawn between the magnitudes of *u* and *v*. The graph looks like

(a) 

(b) 

(c) 

(d) 

35. An object 2.5 *cm* high is placed at a distance of 10 *cm* from a concave mirror of radius of curvature 30 *cm* The size of the image is **[BVP 2003]**

(a) 9.2 *cm* (b) 10.5 *cm* (c) 5.6 *cm* (d) 7.5 *cm*

36. A diminished virtual image can be formed only in **[MP PMT 2002]**

(a) Plane mirror (b) A concave mirror (c) A convex mirror (d) Concave-parabolic mirror

37. A point object is placed at a distance of 30 *cm* from a convex mirror of focal length 30*cm*. The image will form at **[JIPMER 2002]**

(a) Infinity (b) Focus (c) Pole (d) 15 *cm* behind the mirror

38. The focal length of a convex mirror is 20 *cm* its radius of curvature will be **[MP PMT 2001]**

(a) 10 *cm* (b) 20 *cm* (c) 30 *cm* (d) 40 *cm*

39. A concave mirror of focal length 15 *cm* forms an image having twice the linear dimensions of the object. The position of the object when the image is virtual will be

(a) 22.5 *cm* (b) 7.5 *cm* (c) 30 *cm* (d) 45 *cm*

40. Under which of the following conditions will a convex mirror of focal length *f* produce an image that is erect, diminished and virtual **[AMU (Engg.) 2001]**

(a) Only when $2f > u > f$ (b) Only when $u = f$ (c) Only when $u < f$ (d) Always

41. A concave mirror gives an image three times as large as the object placed at a distance of 20 *cm* from it. For the image to be real, the focal length should be **[SCRA 1998; JIPMER 2**

(a) 10 *cm* (b) 15 *cm* (c) 20 *cm* (d) 30 *cm*

42. A point object is placed at a distance of 10 *cm* and its real image is formed at a distance of 20*cm* from a concave mirror. If the object is moved by 0.1*cm* towards the mirror, the image will shift by about

(a) 0.4 *cm* away from the mirror (b) 0.4 *cm* towards the mirror

(c) 0.8 *cm* away from the mirror (d) 0.8 *cm* towards the mirror

43. The minimum distance between the object and its real image for concave mirror is **[RPMT 1999]**

(a) *f* (b) 2*f* (c) 4*f* (d) Zero

44. An object is placed at 20 *cm* from a convex mirror of focal length 10 *cm*. The image formed by the mirror is **[JIPMER 1999]**

(a) Real and at 20 *cm* from the mirror (b) Virtual and at 20 *cm* from the mirror

(c) Virtual and at 20/3 *cm* from the mirror (d) Real and at 20/3 *cm* from the mirror

45. An object is placed 40 *cm* from a concave mirror of focal length 20 *cm*. The image formed is **[MP PET 1986; MP PMT/PET 19**

(a) Real, inverted and same in size (b) Real, inverted and smaller

(c) Virtual, erect and larger (d) Virtual, erect and smaller

46. Match List I with List II and select the correct answer using the codes given below the lists **[SCRA 1998]**

List I (Position of the object)	List II (Magnification)
(I) An object is placed at focus before a convex mirror	(A) Magnification is $-\infty$
(II) An object is placed at centre of curvature before a concave mirror	(B) Magnification is 0.5
(III) An object is placed at focus before a concave mirror	(C) Magnification is + 1
(IV) An object is placed at centre of curvature before a convex mirror	(D) Magnification is – 1
	(E) Magnification is 0.33

Codes :

(a) I-B, II-D, III-A, IV-E (b) I-A, II-D, III-C, IV-B (c) I-C, II-B, III-A, IV-E (d) I-B, II-E, III-D, IV-C

47. In a concave mirror experiment, an object is placed at a distance x_1 from the focus and the image is formed at a distance x_2 from the focus. The focal length of the mirror would be

(a) $x_1 x_2$ (b) $\sqrt{x_1 x_2}$ (c) $\frac{x_1 + x_2}{2}$ (d) $\sqrt{\frac{x_1}{x_2}}$

48. Which of the following forms a virtual and erect image for all positions of the object **[IIT-JEE 1996]**

(a) Convex lens (b) Concave lens (c) Convex mirror (d) Concave mirror

49. A convex mirror has a focal length *f*. A real object is placed at a distance *f* in front of it from the pole produces an image at **[MP PAT 1996]**

(a) Infinity (b) f (c) $f/2$ (d) $2f$

50. Radius of curvature of concave mirror is 40 *cm* and the size of image is twice as that of object, then the object distance is **[AFMC 1995]**

(a) 60 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

51. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The magnification produced by a convex mirror is always less than one

(b) A virtual, erect, same-sized image can be obtained using a plane mirror

(c) A virtual, erect, magnified image can be formed using a concave mirror

(d) A real, inverted, same-sized image can be formed using a convex mirror

52. Radius of curvature of convex mirror is 40 *cm* and the size of object is twice as that of image, then the image distance is **[AFMC 1995]**

(a) 10 *cm* (b) 20 *cm* (c) 40 *cm* (d) 30 *cm*

53. If an object is placed 10 *cm* in front of a concave mirror of focal length 20 *cm*, the image will be **[MP PMT 1995]**

(a) Diminished, upright, virtual (b) Enlarged, upright, virtual (c) Diminished, inverted, real (d) Enlarged, upright, real

54. An object 1 *cm* tall is placed 4 *cm* in front of a mirror. In order to produce an upright image of 3 *cm* height one needs a **[SCRA 1994]**

(a) Convex mirror of radius of curvature 12 *cm* (b) Concave mirror of radius of curvature 12 *cm*

(c) Concave mirror of radius of curvature 4 *cm* (d) Plane mirror of height 12 *cm*

55. The image formed by a convex mirror of a real object is larger than the object **[CPMT 1994]**

(a) When $u < 2f$ (b) When $u > 2f$ (c) For all values of u (d) For no value of u

56. An object 5 *cm* tall is placed 1 *m* from a concave spherical mirror which has a radius of curvature of 20 *cm*. The size of the image is **[MP PET 1993]**

(a) 0.11 *cm* (b) 0.50 *cm* (c) 0.55 *cm* (d) 0.60 *cm*

57. A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 *cm*. The distance of the object from the mirror is

(a) 5 *cm* (b) 12 *cm* (c) 10 *cm* (d) 20 *cm*

58. Given a point source of light, which of the following can produce a parallel beam of light **[CPMT 1974]**

(a) Convex mirror (b) Concave mirror

(c) Concave lens (d) Two plane mirrors inclined at an angle of 90^o

59. A convex mirror is used to form the image of an object. Then which of the following statements is wrong

(a) The images lies between the pole and the focus (b) The image is diminished in size

(c) The images is erect (d) The image is real

60. A boy stands straight infront of a mirror at a distance of 30 *cm* away from it. He sees his erect image whose height is $\frac{1}{5}^{th}$ of his real height. The mirror he is using is

(a) Plane mirror (b) Convex mirror (c) Concave mirror (d) Plano-convex mirror

61. For the largest distance of the image from a concave mirror of focal length 10*cm*, the object should be kept at

(a) 10 *cm* (b) Infinite (c) 40 *cm* (d) 60 *cm*

62. A dentist uses a small mirror that gives a magnification of 4 when it is held 0.60 *cm* from a tooth. The radius of curvature of the mirror is

(a) 1.60 *cm* (convex) (b) 0.8 *cm* (concave) (c) 1.60 *cm* (concave) (d) 0.8 *cm* (convex)

63. A dice is placed with its one edge parallel to the principal axis between the principal focus and the centre of the curvature of a concave mirror. Then the image has the shape of

(a) Cube (b) Cuboid (c) Barrel shaped (d) Spherical

Advance Level

64. A short linear object of length *l* lies along the axis of a concave mirror of focal length *f* at a distance *u* form the pole of the mirror. The size of the image is approximately equal to **[IIT 1988; BHU 2003]**

(a) $l\left(\frac{u-f}{f}\right)^{1/2}$ (b) $l\left(\frac{u-f}{f}\right)^{2}$ (c) $l\left(\frac{f}{u-f}\right)^{1/2}$ (d) $l\left(\frac{f}{u-f}\right)^{2}$

65. A point object is moving on the principal axis of a concave mirror of focal length 24 *cm* towards the mirror. When it is at a distance of 60 *cm* from the mirror, its velocity is 9 *cm/sec*. What is the velocity of the image at that instant

(a) 5 *cm/sec* towards the mirror (b) 4 *cm/sec* towards the mirror

(c) 4 *cm/sec* away from the mirror (d) 9 *cm/sec* away from the mirror

66. A convex mirror of focal length 10 *cm* forms an image which is half of the size of the object. The distance of the object from the mirror is

(a) 10 *cm* (b) 20 *cm* (c) 5 *cm* (d) 15 *cm*

67. A concave mirror is used to focus the image of a flower on a nearby well 120 *cm* from the flower. If a lateral magnification of 16 is desired, the distance of the flower from the mirror should be

(a) 8 *cm* (b) 12 *cm* (c) 80 *cm* (d) 120 *cm*

68. A thin rod of 5 *cm* length is kept along the axis of a concave mirror of 10 *cm* focal length such that its image is real and magnified and one end touches the rod. Its magnification will be

(a) 1 (b) 2 (c) 3 (d) 4

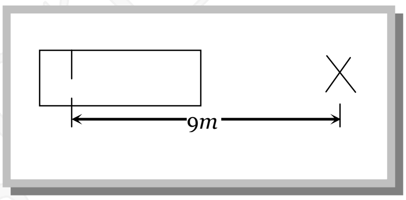
69. A luminous object is placed 20 *cm* from surface of a convex mirror and a plane mirror is set so that virtual images formed in two mirrors coincide. If plane mirror is at a distance of 12 *cm* from object, then focal length of convex mirror, is

(a) 5 *cm* (b) 10 *cm* (c) 20 *cm* (d) 40 *cm*

70. A rear mirror of a vehicle is cylindrical having radius of curvature 10 *cm*. The length of arc of curved surface is also 10 *cm*. If the eye of driver is assumed to be at large distance, from the mirror, then the field of view in radian is

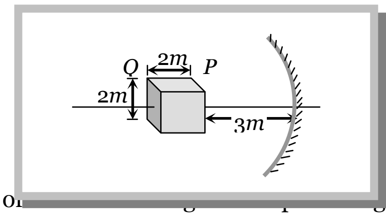
(a) 0.5 (b) 1 (c) 2 (d) 4

71. A vehicle has a driving mirror of focal length 30 *cm*. Another vehicle of dimension $2 \times 4 \times 1.75\ m^3$ is 9 *m* away from the mirror of first vehicle. Position of the second vehicle as seen in the mirror of first vehicle is



(a) 30 *cm*
(b) 60 *cm*
(c) 90 *cm*
(d) 9 *cm*

72. A cube of side 2 *m* is placed in front of a concave mirror focal length 1*m* with its face *P* at a distance of 3 *m* and face *Q* at a distance of 5 *m* from the mirror. The distance between the images of face *P* and *Q* and height of images of *P* and *Q* are



(a) 1 *m*, 0.5 *m*, 0.25 *m*
(b) 0.5 *m*, 1 *m*, 0.25 *m*
(c) 0.5 *m*, 0.25 *m*, 1*m*
(d) 0.25 *m*, 1*m*, 0.5 *m*

73. A concave mirror of radius of curvature 60 *cm* is placed at the bottom o... ...ht of 20 *cm*. The mirror faces upwards with its axis vertical. Solar light falls normally on the surface of water and the image of the sun is formed. If ${}_a\mu_w = \frac{4}{3}$ then with the observer in air, the distance of the image from the surface of water is

(a) 30 *cm* (b) 10 *cm* (c) 7.5 *cm* above (d) 7.5 *cm* below

74. A concave mirror forms an image of the sun at a distance of 12 *cm* from it

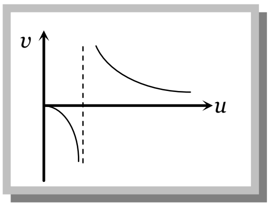
(a) The radius of curvature of this mirror is 6 *cm*
(b) To use it as a shaving mirror, it must be held at a distance of 8-10 *cm* from the face
(c) If an object is kept at a distance of 12 *cm* from it, the image formed will be of the same size as the object
(d) All the above a alternatives are correct

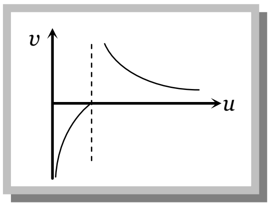
75. A small piece of wire bent into an *L* shape with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 *cm*. If the bend is 20 *cm* from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

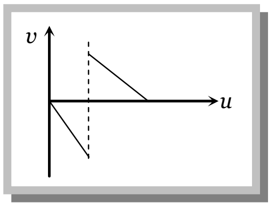
(a) 1 : 2 (b) 3 : 1 (c) 1 : 3 (d) 2 : 1

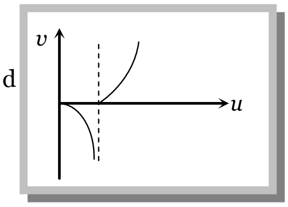
76. As the position of an object (u) reflected from a concave mirror is varied, the position of the image (v) also varies. By letting the u changes from 0 to $+\infty$ the graph between v versus u will be

77.

(a)


b


c


d


78. A concave mirror has a focal length 20 *cm*. The distance between the two positions of the object for which the image size is double of the object size is

(a) 20 *cm* (b) 40 *cm* (c) 30 *cm* (d) 60 *cm*

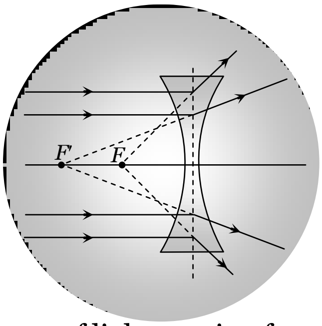
79. A concave mirror of focal length 10 *cm* and a convex mirror of focal length 15 *cm* are placed facing each other 40 *cm* apart. A point object is placed between the mirrors, on their common axis and 15 *cm* from the concave mirror. Find the position and nature of the image produced by the successive reflections, first at concave mirror and then at convex mirror

(a) 2 *cm* (b) 4 *cm* (c) 6 *cm* (d) 8 *cm*

Answer Sheet

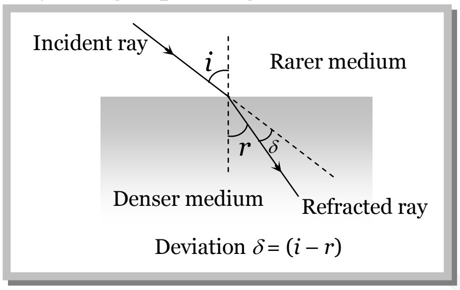
Assignments

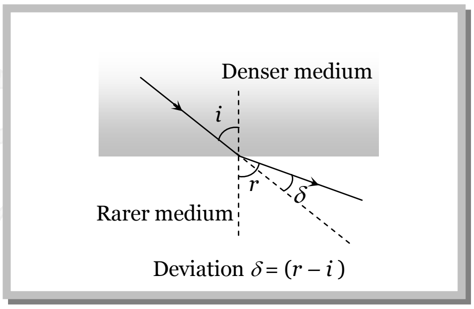
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
c	**c**	**c**	**c**	**c**	**b**	**b**	**b**	**b**	**c**	**b**	**b**	**a**	**b**	**c**	**b**	**d**	**d**	**c**	**b**
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	**a**	**b**	**b**	**c**	**c**	**b**	**a**	**c**	**d**	**b**	**a**	**d**	**c**	**d**	**c**	**d**	**d**	**b**	**d**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	**a**	**d**	**c**	**a**	**a**	**b**	**b, c**	**c**	**d**	**d**	**a**	**b**	**b**	**d**	**c**	**b**	**b**	**d**	**b**
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**		
a	**c**	**b**	**d**	**c**	**a**	**a**	**a**	**a**	**b**	**a**	**d**	**c**	**b**	**b**	**a**	**a**	**c**		



Refraction of Light

The bending of the ray of light passing from one medium to the other medium is called refraction.





Snell's law

The ratio of sine of the angle of incidence to the angle of refraction (r) is a constant called refractive index

i.e. $\frac{\sin i}{\sin r} = \mu$ (a constant). For two media, Snell's law can be written as ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$

$\Rightarrow \mu_1 \times \sin i = \mu_2 \times \sin r$ *i.e.* $\mu \sin\theta =$ constant

Also in vector form : $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{n}} = \boldsymbol{\mu}(\hat{\boldsymbol{r}} \times \hat{\boldsymbol{n}})$

Refractive Index

Refractive index of a medium is that characteristic which decides speed of light in it. It is a scalar, unit less and dimensionless quantity

.(1) **Types :** It is of following two types

Absolute refractive index	Relative refractive index
(i) When light travels from air to any transparent medium then R.I. of medium *w.r.t.* air is called it's absolute R.I. *i.e.* ${}_{\text{air}}\mu_{\text{medium}} = \frac{c}{v}$	(i) When light travels from medium (1) to medium (2) then R.I. of medium (2) *w.r.t.* medium (1) is called it's relative R.I. *i.e.* ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$ (where v_1 and v_2 are the speed of light in medium 1 and 2 respectively).
(ii) Some absolute R.I.	(ii) Some relative R.I. (a) When light enters from water to glass :

$_a\mu_{glass} = \frac{3}{2} = 1.5$, $_a\mu_{water} = \frac{4}{3} = 1.33$

$_a\mu_{diamond} = 2.4$, $_a\mu_{Cs_2} = 1.62$

$_a\mu_{crown} = 1.52$, $\mu_{vacuum} = 1$, $\mu_{air} = 1.0003 \approx 1$

$_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

(b) When light enters from glass to diamond :

$_g\mu_D = \frac{\mu_D}{\mu_g} = \frac{2.4}{1.5} = \frac{8}{5}$

Note : ❑ Cauchy's equation : $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + ...$ $(\lambda_{Red} > \lambda_{violet}$ so $\mu_{Red} < \mu_{violet})$

❑ If a light ray travels from medium (1) to medium (2), then $_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$ → $\mu \propto \frac{1}{\lambda}$, $\mu \propto \frac{1}{v}$, $v \propto \lambda$

(2) **Dependence of Refractive index**

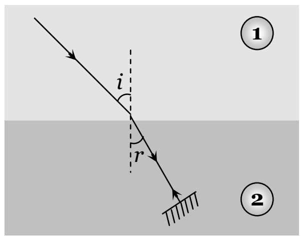
(i) Nature of the media of incidence and refraction.

(ii) Colour of light or wavelength of light.

(iii) Temperature of the media : Refractive index decreases with the increase in temperature.

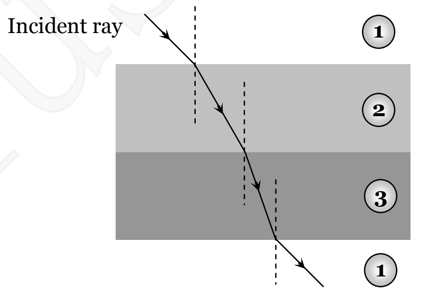
(3) **Principle of reversibility of light and refraction through several media :**

Principle of reversibility



$_1\mu_2 = \frac{1}{_2\mu_1}$

Refraction through several media

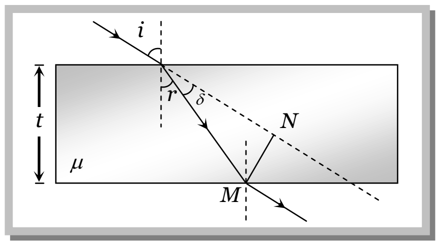


$_1\mu_2 \times {_2\mu_3} \times {_3\mu_1} = 1$

Refraction Through a Glass Slab and Optical Path

(1) **Lateral shift**

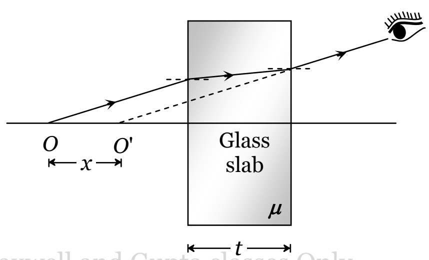
The refracting surfaces of a glass slab are parallel to each other. When a light ray passes through a glass slab it is refracted twice at the two parallel faces and finally emerges out parallel to it's incident direction *i.e.* the ray undergoes no deviation δ = 0. The angle of emergence (e) is equal to the angle of incidence (i)



The Lateral shift of the ray is the perpendicular distance between the incident and the emergent ray, and it is given by ***MN* = *t* sec *r* sin (*i* – *r*)**

Normal shift

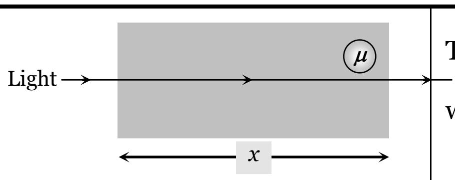
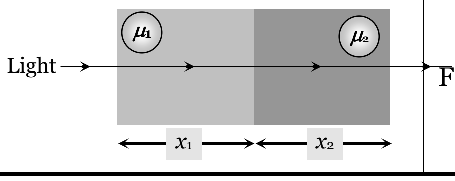
Normal shift $OO' = x = \left(1 - \frac{1}{\mu}\right)t$



Or the object appears to be shifted towards the slab by the distance x

(2) **Optical path :**

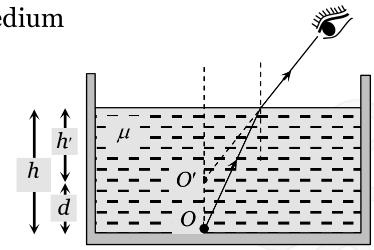
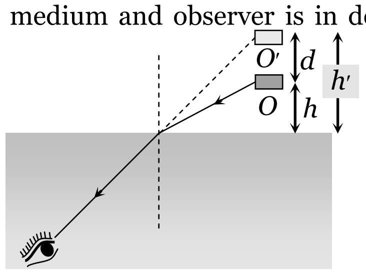
It is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium.

	Time taken by light ray to pass through the medium $= \frac{\mu x}{c}$; where x = geometrical path and μx = optical path
	For two medium in contact optical path $= \mu_1 x_1 + \mu_2 x_2$

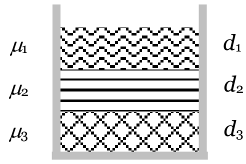
Note : ❑ Since for all media $\mu > 1$, so optical path length (μx) is always greater than the geometrical path length (x).

Real and Apparent Depth

If object and observer are situated in different medium then due to refraction, object appears to be displaced from it's real position. There are two possible conditions.

(1) When object is in denser medium and observer is in rarer medium 	(1) Object is in rarer medium and observer is in denser medium. 
(2) $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{h}{h'}$ Real depth >Apparent depth that's why a coin at the bottom of bucket (full of water) appears to be raised)	(2) $\mu = \frac{h'}{h}$ Real depth < Apparent depth that's why high flying aeroplane appears to be higher than it's actual height.
(3) Shift $d = h - h' = \left(1 - \frac{1}{\mu}\right)h$	(3) $d = (\mu - 1)h$
(4) For water $\mu = \frac{4}{3} \Rightarrow d = \frac{h}{4}$ For glass $\mu = \frac{3}{2} \Rightarrow d = \frac{h}{3}$	(4) Shift for water $d_w = \frac{h}{3}$ Shift for glass $d_g = \frac{h}{2}$

Note : ❑ If a beaker contains various immisible liquids as shown then



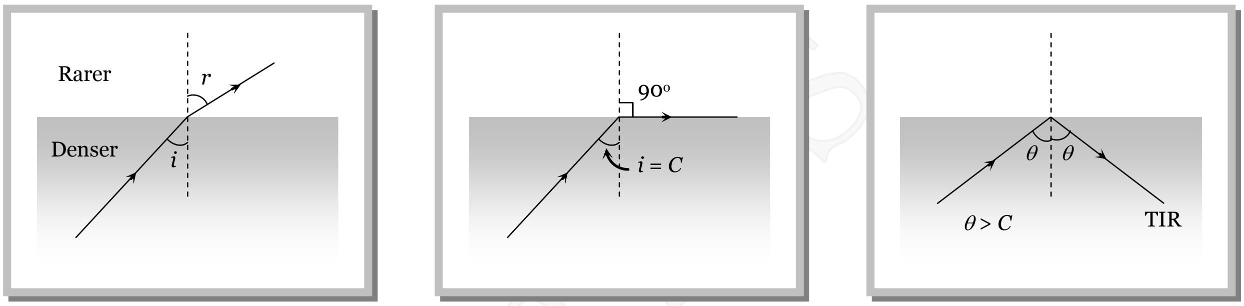
Apparent depth of bottom $= \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \dfrac{d_3}{\mu_3} + ...$

$\mu_{\text{combination}} = \dfrac{d_{AC}}{d_{App.}} = \dfrac{d_1 + d_2 + ...}{\dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + ...}$ (In case of two liquids if $d_1 = d_2$ than $\mu = \dfrac{2\mu_1\mu_2}{\mu_1 + \mu_2}$)

Total Internal Reflection

When a ray of light goes from denser to rarer medium it bends away from the normal and as the angle of incidence in denser medium increases, the angle of refraction in rarer medium also increases and at a certain angle, angle of refraction becomes 90°, this angle of incidence is called critical angle (*C*).

When Angle of incidence exceeds the critical angle than light ray comes back in to the same medium after reflection from interface. This phenomenon is called Total internal reflection (TIR).

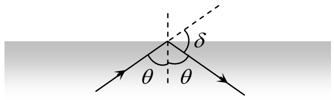


Important formula $\mu = \dfrac{1}{\sin C} = \text{cosec } C$; where $\mu \rightarrow {}_{\text{Rerer}}\mu_{\text{Denser}}$

Note : ❑ When a light ray travels from denser to rarer medium, then deviation of the ray is

$\delta = \pi - 2\theta \Rightarrow \delta \rightarrow \text{max. when } \theta \rightarrow \text{min.} = C$

i.e. $\delta_{max} = (\pi - 2C)$; $C \rightarrow$ critical angle



(1) **Dependence of critical angle**

(i) Colour of light (or wavelength of light) : Critical angle depends upon wavelength as $\lambda \propto \dfrac{1}{\mu} \propto \sin C$

(a) $\lambda_R > \lambda_V \Rightarrow C_R > C_V$

(b) Sin $C = \dfrac{1}{{}_R\mu_D} = \dfrac{\mu_R}{\mu_D} = \dfrac{\lambda_D}{\lambda_R} = \dfrac{v_D}{v_R}$ (for two media) (c) For TIR from boundary of two media $i > \sin^{-1}\dfrac{\mu_R}{\mu_D}$

(ii) Nature of the pair of media : Greater the refractive index lesser will be the critical angle.

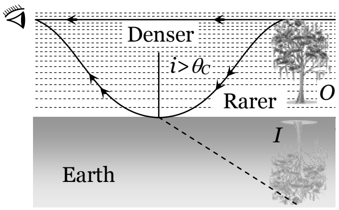
(a) For (glass- air) pair $\rightarrow C_{\text{glass}} = 42^o$ (b) For (water-air) pair $\rightarrow C_{\text{water}} = 49^o$

(c) For (diamond-air) pair $\rightarrow C_{\text{diamond}} = 24^o$

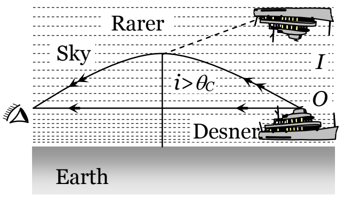
(iii) Temperature : With temperature rise refractive index of the material decreases therefore critical angle increases.

(2) **Examples of total internal reflection (TIR)**

(i)



Mirage : An optical illusion in deserts



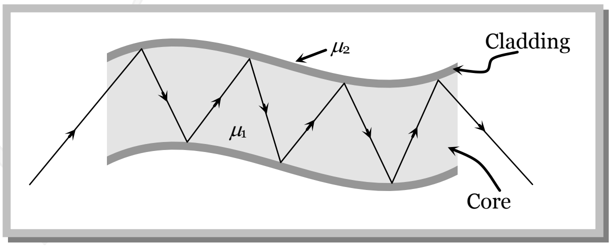
Looming : An optical illusion in cold countries

(ii) **Brilliance of diamond :** Due to repeated internal reflections diamond sparkles.

(iii) **Optical fibre :** Optical fibres consist of many long high quality composite glass/quartz fibres. Each fibre consists of a core and cladding. The refractive index of the material of the core (μ_1) is higher than that of the cladding (μ_2).

When the light is incident on one end of the fibre at a small angle, the light passes inside, undergoes repeated total internal reflections along the fibre and finally comes out. The angle of incidence is always larger than the critical angle of the core material with respect to its cladding. Even if the fibre is bent, the light can easily travel through along the fibre

A bundle of optical fibres can be used as a 'light pipe' in medical and optical examination. It can also be used for optical signal transmission. Optical fibres have also been used for transmitting and receiving electrical signals which are converted to light by suitable transducers.

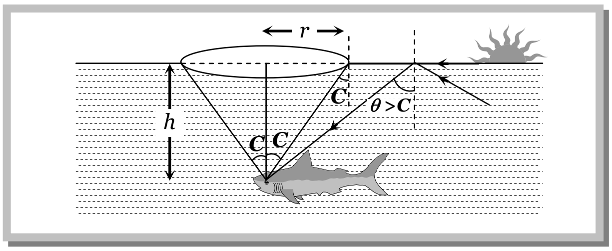


(iv) **Field of vision of fish (or swimmer) :** A fish (diver) inside the water can see the whole world through a cone with.

(a) Apex angle $= 2C = 98^o$

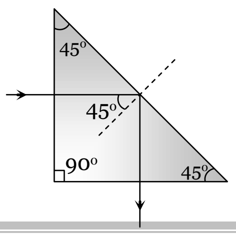
(b) Radius of base $r = h\tan C = \dfrac{h}{\sqrt{\mu^2 - 1}}$

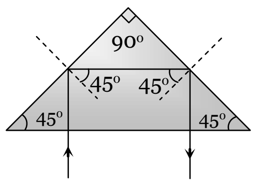
(c) Area of base $A = \dfrac{\pi h^2}{(\mu^2 - 1)}$

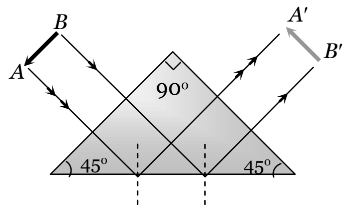


Note : ❑ For water $\mu = \dfrac{4}{3}$ so $r = \dfrac{3h}{\sqrt{7}}$ and $A = \dfrac{9\pi h^2}{7}$.

(v) **Porro prism :** A right angled isosceles prism, which is used in periscopes or binoculars. It is used to deviate light rays through 90^o and 180^o and also to erect the image.







Example

Example: 1 A beam of monochromatic blue light of wavelength 4200 $Å$ in air travels in water $(\mu = 4/3)$. Its wavelength in water will be

(a) 2800 $Å$ (b) 5600 $Å$ (c) 3150 $Å$ (d) 4000 $Å$

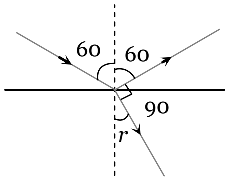
Solution: (c) $\mu \propto \frac{1}{\lambda} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{1}{4/3} = \frac{\lambda_2}{4200} \Rightarrow \lambda_2 = 3150\ Å$

Example: 2 On a glass plate a light wave is incident at an angle of 60º. If the reflected and the refracted waves are mutually perpendicular, the refractive index of material is **[MP PMT 1994; Haryana CEE 1996]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{3}}$

Solution: (b) From figure $r = 30^o$

$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^o}{\sin 30^o} = \sqrt{3}$



Example: 3 Velocity of light in glass whose refractive index with respect to air is 1.5 is 2 × $10^8\ m/s$ and in certain liquid the velocity of light found to be $2.50 \times 10^8\ m/s$. The refractive index of the liquid with respect to air is **[CPMT 1978; MP PET/PMT 1988]**

(a) 0.64 (b) 0.80 (c) 1.20 (d) 1.44

Solution: (c) $\mu \propto \frac{1}{v} \Rightarrow \frac{\mu_{li}}{\mu_g} = \frac{v_g}{v_l} \Rightarrow \frac{\mu_l}{1.5} = \frac{2 \times 10^8}{2.5 \times 10^8} \Rightarrow \mu_l = 1.2$

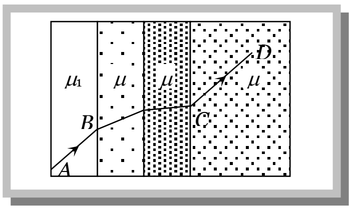
Example: 4 A ray of light passes through four transparent media with refractive indices $\mu_1 . \mu_2, \mu_3$, and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray *CD* is parallel to the incident ray *AB*, we must have

(a) $\mu_1 = \mu_2$

(b) $\mu_2 = \mu_3$

(c) $\mu_3 = \mu_4$

(d) $\mu_4 = \mu_1$



Solution: (d) For successive refraction through difference media $\mu \sin \theta =$ constant.

Here as θ is same in the two extreme media. Hence $\mu_1 = \mu_4$

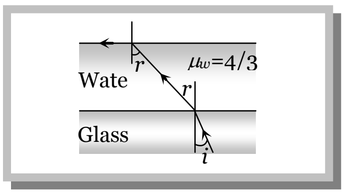
Example: 5 A ray of light is incident at the glass–water interface at an angle *i*, it emerges finally parallel to the surface of water, then the value of μ_g would be

(a) (4/3) sin *i*

(b) 1/ sin *i*

(c) 4/ 3

(d) 1



Solution: (b) For glass water interface ${}_g\mu_\omega = \frac{\sin i}{\sin r}$ ......(i) and For water-air interface ${}_\omega\mu_a = \frac{\sin r}{\sin 90}$ .....(ii)

$\therefore\ {}_g\mu_\omega \times {}_\omega\mu_a = \sin i \quad \Rightarrow \quad \mu_g = \dfrac{1}{\sin i}$

***Example*: 6** The ratio of thickness of plates of two transparent mediums A and B is 6 : 4. If light takes equal time in passing through them, then refractive index of B with respect to A will be

(a) 1.4 (b) 1.5 (c) 1.75 (d) 1.33

Solution: (b) By using $t = \dfrac{\mu x}{c}$

$\Rightarrow \dfrac{\mu_B}{\mu_A} = \dfrac{x_A}{x_B} = \dfrac{6}{4} \Rightarrow {}_A\mu_B = \dfrac{3}{2} = 1.5$

***Example*: 7** A ray of light passes from vacuum into a medium of refractive index μ, the angle of incidence is found to be twice the angle of refraction. Then the angle of incidence is

(a) $\cos^{-1}(\mu/2)$ (b) $2\cos^{-1}(\mu/2)$ (c) $2\sin^{-1}(\mu)$ (d) $2\sin^{-1}(\mu/2)$

Solution: (b) By using $\mu = \dfrac{\sin i}{\sin r} \Rightarrow \mu = \dfrac{\sin 2r}{\sin r} = \dfrac{2\sin r\cos r}{\sin r}$ $(\sin 2\theta = 2\sin\theta\cos\theta)$

$\Rightarrow r = \cos^{-1}\left(\dfrac{\mu}{2}\right)$. So, $i = 2r = 2\cos^{-1}\left(\dfrac{\mu}{2}\right)$.

***Example*: 8** A ray of light falls on the surface of a spherical glass paper weight making an angle α with the normal and is refracted in the medium at an angle β. The angle of deviation of the emergent ray from the direction of the incident ray is

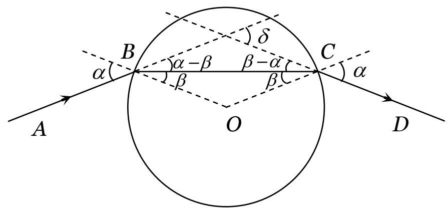
(a) $(\alpha-\beta)$ (b) $2(\alpha-\beta)$ (c) $(\alpha-\beta)/2$ (d) $(\alpha+\beta)$

Solution: (b) From figure it is clear that ΔOBC is an isosceles triangle,

Hence $\angle OCB = \beta$ and emergent angle is α

Also sum of two in terior angles = exterior angle

$\therefore\ \delta = (\alpha-\beta)+(\alpha-\beta) = 2(\alpha-\beta)$

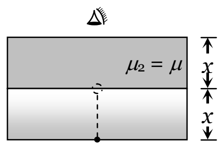


***Example*: 9** A rectangular slab of refractive index μ is placed over another slab of refractive index 3, both slabs being identical in dimensions. If a coin is placed below the lower slab, for what value of μ will the coin appear to be placed at the interface between the slabs when viewed from the top

(a) 1.8 (b) 2 (c) 1.5 (d) 2.5

Solution: (c) Apparent depth of coin as seen from top $= \dfrac{x}{\mu_1} + \dfrac{x}{\mu_2} = x$

$\Rightarrow \dfrac{1}{\mu_1} + \dfrac{1}{\mu_2} = 1 \quad \Rightarrow \dfrac{1}{3} + \dfrac{1}{\mu} = 1 \quad \Rightarrow \mu = 1.5$



***Example*: 10** A coin is kept at bottom of an empty beaker. A travelling microscope is focussed on the coin from top, now water is poured in beaker up to a height of 10 *cm*. By what distance and in which direction should the microscope be moved to bring the coin again in focus

(a) 10 *cm* up ward (b) 10 *cm* down ward (c) 2.5 *cm* up wards (d) 2.5 *cm* down wards

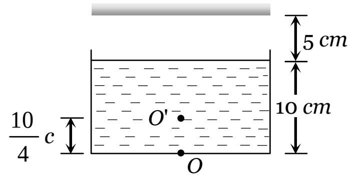
Solution: (c) When water is poured in the beaker. Coin appears to shift by a distance $d = \dfrac{h}{4} = \dfrac{10}{4} = 2.5\,cm$

Hence to bring the coil again in focus, the microscope should be moved by 2.5 *cm* in upward direction.

***Example*: 11** Consider the situation shown in figure. Water $\left(\mu_w = \frac{4}{3}\right)$ is filled in a breaker upto a height of 10 *cm*. A plane mirror fixed at a height of 5 *cm* from the surface of water. Distance of image from the mirror after reflection from it of an object *O* at the bottom of the beaker is

(a) 15 *cm* (b) 12.5 *cm* (c) 7.5 *cm* (d) 10 *cm*

Solution: (b) From figure it is clear that object appears to be raised by $\frac{10}{4} cm\,(2.5\,cm)$



Hence distance between mirror and $O' = 5 + 7.5 = 12.5\,cm$

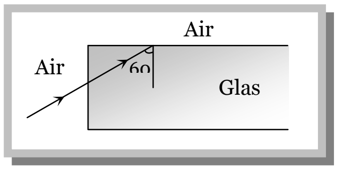
So final image will be formed at 12.5 *cm* behind the plane mirror.

***Example*: 12** The wavelength of light in two liquids '*x*' and '*y*' is 3500 Å and 7000 Å, then the critical angle of *x* relative to *y* will be

(a) 60° (b) 45° (c) 30° (d) 15°

Solution: (c) $\sin C = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{3500}{7000} = \frac{1}{2} \Rightarrow C = 30^o$

***Example*: 13** A light ray from air is incident (as shown in figure) at one end of a glass fiber (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface, so that it undergoes a total internal reflection. How much time would it take to traverse the straight fiber of length 1 *km* **[Orissa JEE 2002]**



(a) 3.33 μ *sec*

(b) 6.67 μ *sec*

(c) 5.77 μ *sec*

(d) 3.85 μ *sec*

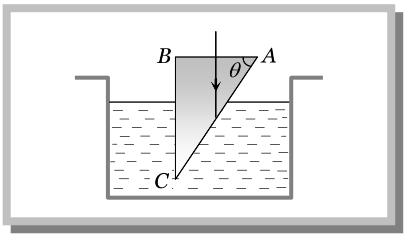
Solution: (d) When total internal reflection just takes place from lateral surface then $i = C$ *i.e.* $C = 60°$

From $\mu = \frac{1}{\sin C} \Rightarrow \mu = \frac{1}{\sin 60} = \frac{2}{\sqrt{3}}$

Hence time taken by light traverse some distance in medium $t = \frac{\mu x}{C}$

$\Rightarrow t = \frac{\frac{2}{\sqrt{3}} \times (1 \times 10^3)}{3 \times 10^8} = 3.85\,\mu\,sec.$

***Example*: 14** A glass prism of refractive index 1.5 is immersed in water $(\mu = 4/3)$. A light beam incident normally on the face *AB* is totally reflected to reach the face *BC* if



(a) $\sin\theta > 8/9$

(b) $2/3 < \sin\theta < 8/9$

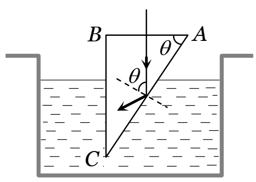
(c) $\sin\theta \le 2/3$

(d) $\cos\theta \ge 8/9$

Solution: (a) From figure it is clear that

Total internal reflection takes place at *AC*, only if $\theta > C$

$\Rightarrow \sin\theta > \sin C \qquad \Rightarrow \sin\theta > \frac{1}{{}_\omega\mu_g}$



$\Rightarrow \sin\theta > \dfrac{1}{9/8}$ $\Rightarrow \sin\theta > \dfrac{8}{9}$

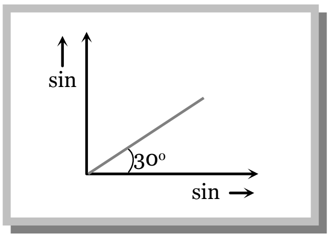
***Example*: 15** When light is incident on a medium at angle *i* and refracted into a second medium at an angle *r*, the graph of sin *i vs* sin *r* is as shown in the graph. From this, one can conclude that

(a) Velocity of light in the second medium is 1.73 times the velocity of light in the I medium

(b) Velocity of light in the I medium is 1.73 times the velocity in the II medium

(c) The critical angle for the two media is given by $\sin i_c = \dfrac{1}{\sqrt{3}}$

(d) $\sin i_c = \dfrac{1}{2}$



Solution: (b, c) From graph $\tan 30^o = \dfrac{\sin r}{\sin i} = \dfrac{1}{{}_1\mu_2} \Rightarrow {}_1\mu_2 = \sqrt{3} \Rightarrow \dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2} = 1.73 \Rightarrow v_1 = 1.75\, v_2$

Also from $\mu = \dfrac{1}{\sin C} \Rightarrow \sin C = \dfrac{1}{{}_{Rarer}\mu_{Denser}} \Rightarrow \sin C = \dfrac{1}{{}_1\mu_2} = \dfrac{1}{\sqrt{3}}$.

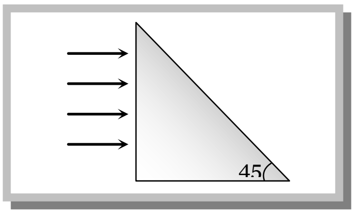
***Example*: 16** A beam of light consisting of red, green and blue colours is incident on a right angled prism. The refractive indices of the material of the prism for the above red, green and blue wavelength are 1.39, 1.44 and 1.47 respectively. The prism will

(a) Separate part of red colour from the green and the blue colours

(b) Separate part of the blue colour from the red and green colours

(c) Separate all the colours from one another

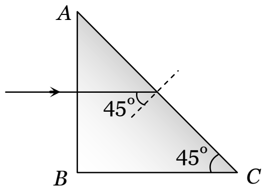
(d) Not separate even partially any colour from the other two colours



Solution: (a) At face *AB*, *i* = 0 so *r* = 0, *i.e.*, no refraction will take place. So light will be incident on face *AC* at an angle of incidence of 45°. The face *AC* will not transmit the light for which $i > \theta_C$, *i.e.*, $\sin i > \sin\theta_C$

or $\sin 45^o > (1/\mu)$ *i.e.*, $\mu > \sqrt{2}$ $(= 1.41)$

Now as $\mu_R < \mu$ while μ_G and $\mu_B > \mu$, so red will be transmitted through the face *AC* while green and blue will be reflected. So the prism will separate red colour from green and blue.



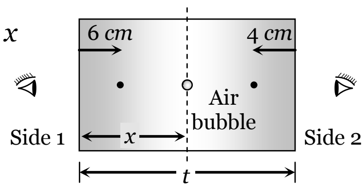
***Example*: 17** An air bubble in a glass slab $(\mu = 1.5)$ is 6 *cm* deep when viewed from one face and 4 *cm* deep when viewed from the opposite face. The thickness of the glass plate is

(a) 10 *cm* (b) 6.67 *cm* (c) 15 *cm* (d) None of these

Solution: (c) Let thickness of slab be *t* and distance of air bubble from one side is *x*

When viewed from side (1) : $1.5 = \dfrac{x}{6} \Rightarrow x = 9cm$

When viewed from side (2) : $1.5 = \dfrac{(t-x)}{4} \Rightarrow 1.5 = \dfrac{(t-9)}{4} \Rightarrow t = 15cm$

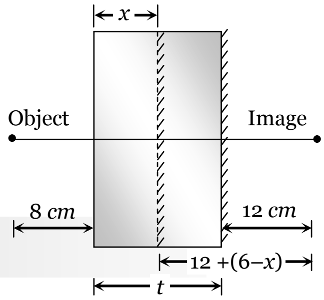


Tricky example: 1

One face of a rectangular glass plate 6 *cm* thick is silvered. An object held 8 *cm* in front of the first face, forms an image 12 *cm* behind the silvered face. The refractive index of the glass is

(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.6

Solution : (c) From figure thickness of glass plate $t = 6\ cm$.



Let x be the apparent position of the silvered surface.

According to property of plane mirror

$x + 8 = 12 + 6 - x \quad \Rightarrow \quad x = 5\ cm$

Al... t ... 6 ... 1.2

Tricky example: 2

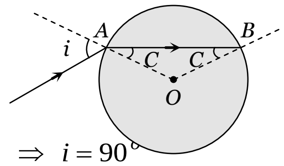
A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere doesn't come out of the sphere

(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$ (c) 90° (d) $\cos^{-1}\left(\frac{1}{3}\right)$

Solution : (c) Ray doesn't come out from the sphere means TIR takes place.

Hence from figure $\angle ABO = \angle OAB = C$

$\therefore \quad \mu = \frac{1}{\sin C} \Rightarrow \sin C = \frac{1}{\mu} = \frac{2}{3}$



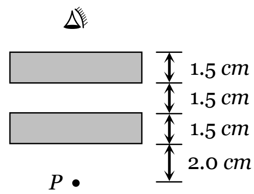
Applying Snell's Law at A $\quad \frac{\sin i}{\sin C} = \frac{3}{2} \Rightarrow \sin i = \frac{3}{2}\sin C = \frac{3}{2} \times \frac{2}{3} = 1 \quad \Rightarrow \quad i = 90^\circ$

Tricky example: 3

The image of point P when viewed from top of the slabs will be

(a) 2.0 *cm* above P (b) 1.5 *cm* above P (c) 2.0 *cm* below P (d) 1 *cm* above P

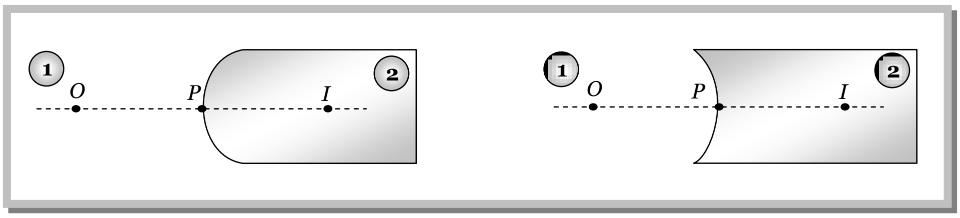
Solution: (d) The two slabs will shift the image a distance



$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1\ cm$$

Therefore, final image will be 1 *cm* above point P.

Refraction From Curved Surface



μ_1 = Refractive index of the medium from which light rays are coming (from object).

μ_2 = Refractive index of the medium in which light rays are entering.

u = Distance of object, v = Distance of image, R = Radius of curvature

Refraction formula : $\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$ (use sign convention while solving the problem)

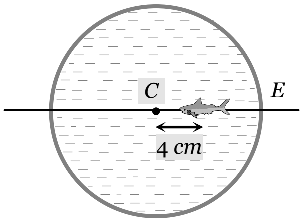
Note : ❑ Real image forms on the side of a refracting surface that is opposite to the object, and virtual image forms on the same side as the object.

❑ Lateral (Transverse) magnification $m = \frac{I}{O} = \frac{\mu_1 v}{\mu_2 u}$.

Specific Example

In a thin spherical fish bowl of radius 10 *cm* filled with water of refractive index 4/3 there is a small fish at a distance of 4 *cm* from the centre *C* as shown in figure. Where will the image of fish appears, if seen from *E*

(a) 5.2 *cm* (b) 7.2 *cm* (c) 4.2 *cm* (d) 3.2 *cm*



Solution : (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

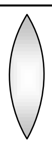
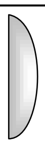
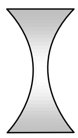
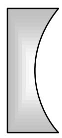
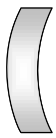
where $\mu_1 = \frac{4}{3}$, $\mu_2 = 1$, $u = -6\,cm$, $v = ?$

On putting values $v = -5.2\,cm$

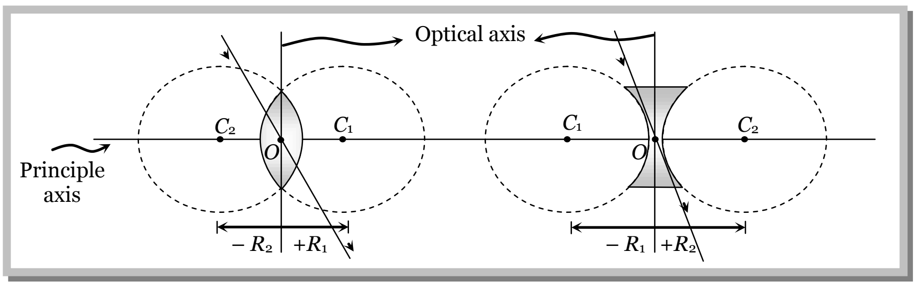
Lens

Lens is a transparent medium bounded by two refracting surfaces, such that at least one surface is spherical.

(1) **Type of lenses**

Convex lens (Converges the light rays)			**Concave lens (Diverges the light rays)**		
					
Double convex	Plano convex	Concavo convex	Double concave	Plane concave	Convexo concave
Thick at middle			Thin at middle		
It forms real and virtual images both			It forms only virtual images		

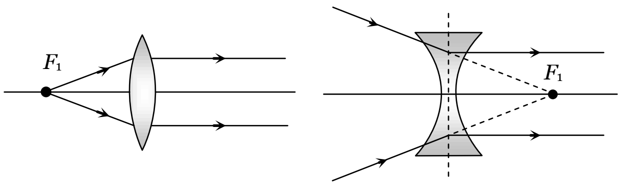
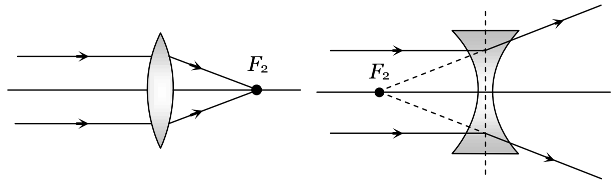
(2) **Some definitions**



C_1, C_2 – Centre of curvature,
R_1, R_2 – Radii of curvature

(i) **Optical centre (*O*) :** A point for a given lens through which light ray passes undeviated (Light ray passes undeviated through optical centre).

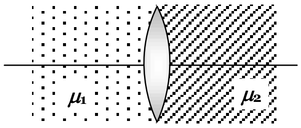
(ii) **Principle focus**

First principle focus	**Second principle focus**
	

Note : ❑ Second principle focus is the principle focus of the lens.

❑ When medium on two sides of lens is same then $|F_1| = |F_2|$.

❑ If medium on two sides of lens are not same then the ratio of two focal lengths $\frac{f_1}{f_2} = \frac{\mu_1}{\mu_2}$



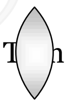
(iii) **Focal length (*f*) :** Distance of second principle focus from optical centre is called focal length

$$f_{\text{convex}} \rightarrow \text{positive}, \quad f_{\text{concave}} \rightarrow \text{negative}, \quad f_{\text{plane}} \rightarrow \infty$$

(iv) **Aperture :** Effective diameter of light transmitting area is called aperture. Intensity of image $\propto (\text{Aperture})^2$

(v) **Power of lens (*P*) :** Means the ability of a lens to converge the light rays. Unit of power is Diopter (*D*).

$$P = \frac{1}{f(m)} = \frac{100}{f(cm)}; \; P_{\text{convex}} \rightarrow \text{positive}, \; P_{\text{concave}} \rightarrow \text{negative}, \; P_{\text{plane}} \rightarrow \text{zero}.$$

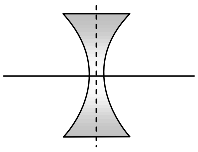
Note : ❑ Thick lens  Thin lens 

$P\uparrow f\downarrow R\downarrow$ $\qquad$ $P\downarrow f\uparrow R\uparrow$

(2) **Image formation by lens**

Lens	**Location of the object**	**Location of the image**	**Nature of image**		
			Magnification	**Real / virtual**	**Erect / inverted**
Convex	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m < 1$ diminished	Real	Inverted
	Away from 2*f* *i.e.* $(u > 2f)$	Between *f* and 2*f* *i.e.* $f < v < 2f$	$m < 1$ diminished	Real	Inverted

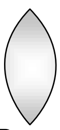
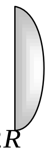
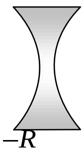
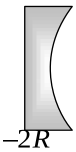
2*f* *f* *f* 2*f*

	At $2f$ or $(u = 2f)$	At $2f$ *i.e.* $(v = 2f)$	$m = 1$ same size	Real	Inverted
	Between f and $2f$ *i.e.* $f < u < 2f$	Away from $2f$ *i.e.* $(v > 2f)$	$m > 1$ magnified	Real	Inverted
	At focus *i.e.* $u = f$	At infinity *i.e.* $v = \infty$	$m = \infty$ magnified	Real	Inverted
	Between optical centre and focus, $u < f$	At a distance greater than that of object $v > u$	$m > 1$ magnified	Virtual	Erect
Concave 	At infinity *i.e.* $u = \infty$	At focus *i.e.* $v = f$	$m < 1$ diminished	Virtual	Erect
	Anywhere between infinity and optical centre	Between optical centre and focus	$m < 1$ diminished	Virtual	Erect

Note : ❑ Minimum distance between an object and it's real image formed by a convex lens is $4f$.
❑ Maximum image distance for concave lens is it's focal length.

(4) **Lens maker's formula**

The relation between f, μ, R_1 and R_2 is known as lens maker's formula and it is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Equiconvex lens	**Plano convex lens**	**Equi concave lens**	**Plano concave lens**
$R_1 = R$ and $R_2 = -R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = R$	$R_1 = \infty$, $R_2 = -R$ $f = \frac{R}{(\mu - 1)}$  for $\mu = 1.5$, $f = 2R$	$R_1 = -R$, $R_2 = +R$ $f = -\frac{R}{2(\mu - 1)}$  for $\mu = 1.5$ $f = -R$	$R_1 = \infty$, $R_2 = R$ $f = \frac{R}{2(\mu - 1)}$  for $\mu = 1.5$, $f = -2R$

(5) **Lens in a liquid**

Focal length of a lens in a liquid (f_l) can be determined by the following formula

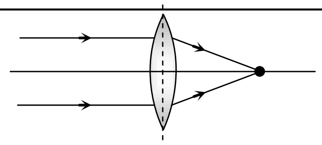
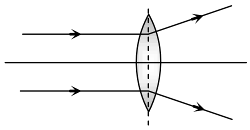
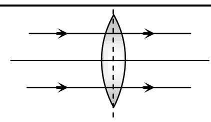
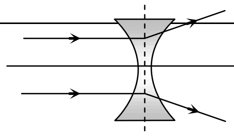
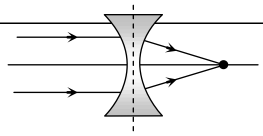
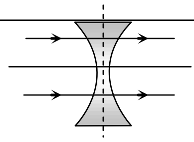
$$\frac{f_l}{f_a} = \frac{({}_a\mu_g - 1)}{({}_l\mu_g - 1)}$$ (Lens is supposed to be made of glass).

Note : ❑ Focal length of a glass lens $(\mu = 1.5)$ is f in air then inside the water it's focal length is $4f$.

❑ In liquids focal length of lens increases (↑) and it's power decreases (↓).

(6) **Opposite behaviour of a lens**

In general refractive index of lens $(\mu_L) >$ refractive index of medium surrounding it (μ_M).

$\mu_L > \mu_M$	$\mu_L < \mu_M$	$\mu_L = \mu_M$
		
		

(7) **Lens formula and magnification of lens**

(i) Lens formula : $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$; (use sign convention)

(ii) Magnification : The ratio of the size of the image to the size of object is called magnification.

(a) Transverse magnification : $m=\frac{I}{O}=\frac{v}{u}=\frac{f}{f+u}=\frac{f-v}{f}$ (use sign convention while solving the problem)

(b) Longitudinal magnification : $m=\frac{I}{O}=\frac{v_2-v_1}{u_2-u_1}$. For very small object $m=\frac{dv}{du}=\left(\frac{v}{u}\right)^2=\left(\frac{f}{f+u}\right)^2=\left(\frac{f-v}{f}\right)^2$

(c) Areal magnification : $m_s=\frac{A_i}{A_o}=m^2=\left(\frac{f}{f+u}\right)^2$, ($A_i$ = Area of image, A_o = Area of object)

(8) **Relation between object and image speed**

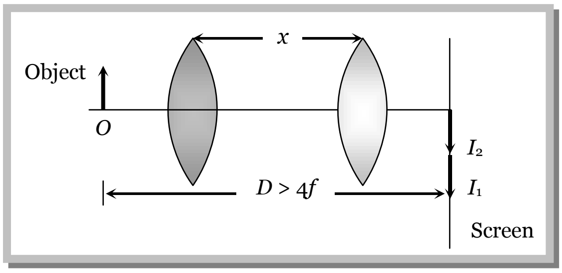
If an object move with constant speed (V_o) towards a convex lens from infinity to focus, the image will move slower in the beginning and then faster. Also $V_i=\left(\frac{f}{f+u}\right)^2.V_o$

(9) **Focal length of convex lens by displacement method**

(i) For two different positions of lens two images (I_1 and I_2) of an object is formed at the same location.

(ii) Focal length of the lens $f=\frac{D^2-x^2}{4D}=\frac{x}{m_1-m_2}$

where $m_1=\frac{I_1}{O}$ and $m_2=\frac{I_2}{O}$

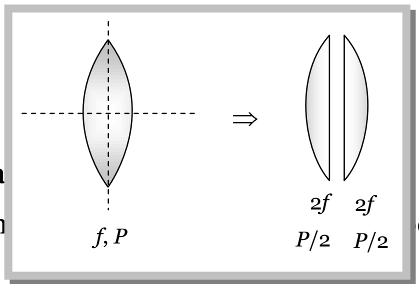


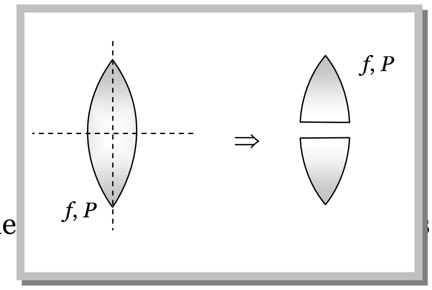
(iii) Size of object $O=\sqrt{I_1.I_2}$

(10) **Cutting of lens**

(i) A symmetric lens is cut along optical axis in two equal parts. Intensity of image formed by each part will be same as that of complete lens.

(ii) A symmetric lens is cut along principle axis in two equal parts. Intensity of image formed by each part will be less compared as that of complete lens.(aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens)





(11) **Combina**

(i) For a systen cal le follows :

$P = P_1 + P_2 + P_3..........$, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$,

$m = m_1 \times m_2 \times m_3 \times..........$ ..

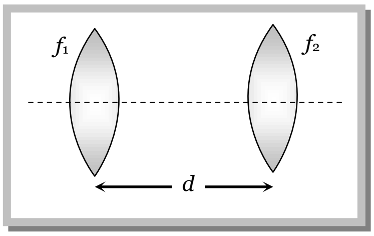
(ii) In case when two thin lens are in contact : Combination will behave as a lens, which have more power or lesser focal length.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow F = \frac{f_1 f_2}{f_1 + f_2} \quad \text{and} \quad P = P_1 + P_2$$

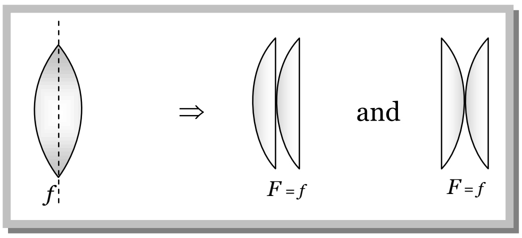
(iii) If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{combination} = \infty$

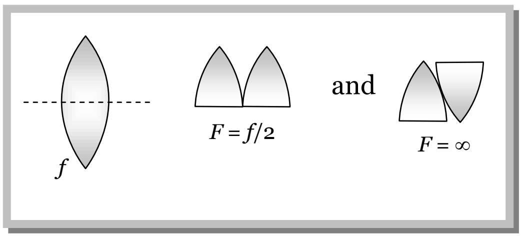
(iv) When two lenses are placed co-axially at a distance d from each other then equivalent focal length (F).

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{and} \quad P = P_1 + P_2 - dP_1 P_2$$



(v) Combination of parts of a lens :



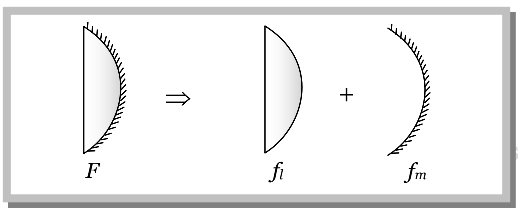


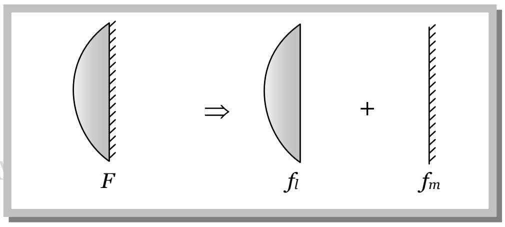
(12) **Silvering of lens**

On silvering the surface of the lens it behaves as a mirror. The focal length of the silvered lens is $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$ where f_l = focal length of lens from which refraction takes place (twice)

f_m = focal length of mirror from which reflection takes place.

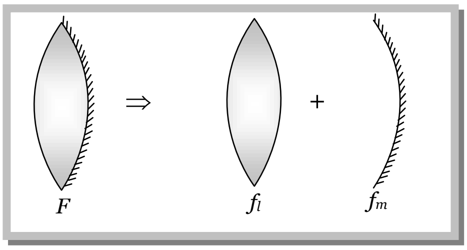
(i) Plano convex is silvered





$f_m = \frac{R}{2},\ f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2\mu}$ $\qquad$ $f_m = \infty,\ f_l = \frac{R}{(\mu-1)}$ so $F = \frac{R}{2(\mu-1)}$

(ii) Double convex lens is silvered



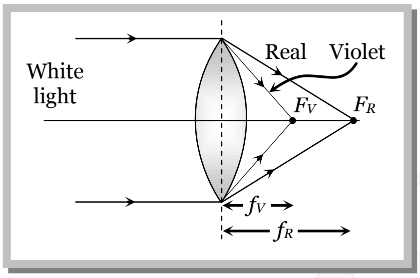
Since $f_l = \frac{R}{2(\mu-1)},\ f_m = \frac{R}{2}$

So $F = \frac{R}{2(2\mu-1)}$

Note : ❑ Similar results can be obtained for concave lenses.

(13) **Defects in lens**

(i) **Chromatic aberration :** Image of a white object is coloured and blurred because μ (hence f) of lens is different for different colours. This defect is called chromatic aberration.



$\mu_V > \mu_R$ so $f_R > f_V$

Mathematically chromatic aberration $= f_R - f_V = \omega f_y$

ω = Dispersion power of lens.

f_y = Focal length for mean colour $= \sqrt{f_R f_V}$

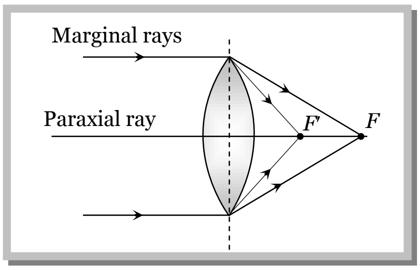
Removal : To remove this defect *i.e.* for Achromatism we use two or more lenses in contact in place of single lens.

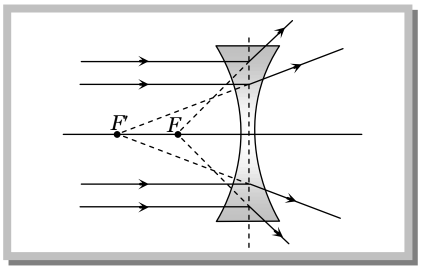
Mathematically condition of Achromatism is : $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ or $\omega_1 f_2 = -\omega_2 f_1$

Note : ❑ Component lenses of an achromatic doublet cemented by canada blasam because it is transparent and has a refractive index almost equal to the refractive of the glass.

(ii) **Spherical aberration :** Inability of a lens to form the point image of a point object on the axis is called Spherical aberration.

In this defect all the rays passing through a lens are not focussed at a single point and the image of a point object on the axis is blurred.





Removal : A simple method to reduce spherical aberration is to use a stop before and infront of the lens. (but this method reduces the intensity of the image as most of the light is cut off). Also by using plano-convex lens, using two lenses separated by distance $d = F - F'$, using crossed lens.

Note : ❑ Marginal rays : The rays farthest from the principal axis.

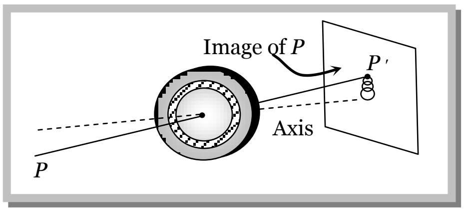
Paraxial rays : The rays close to the principal axis.

❑ Spherical aberration can be reduced by either stopping paraxial rays or marginal rays, which can be done by using a circular annular mask over the lens.

❑ Parabolic mirrors are free from spherical aberration.

(iii) **Coma :** When the point object is placed away from the principle axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called Coma.

It refers to spreading of a point object in a plane ⊥ to principle axis.

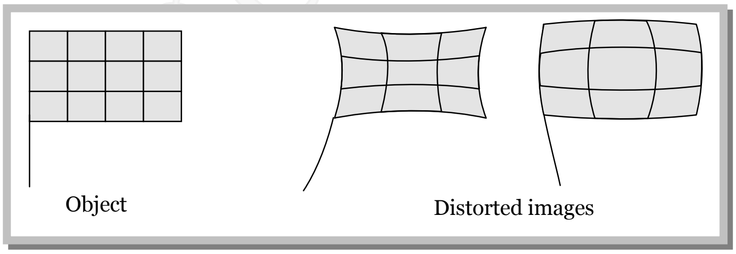


Removal : It can be reduced by properly designing radii of curvature of the lens surfaces. It can also be reduced by appropriate stops placed at appropriate distances from the lens.

(iv) **Curvature :** For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as Curvature.

Removal : Astigmatism or the curvature may be reduced by using proper stops placed at proper locations along the axis.

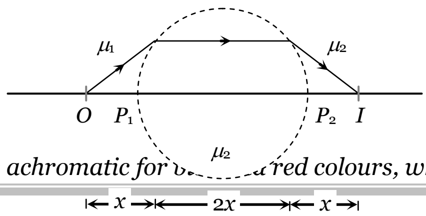
(v) **Distortion :** When extended objects are imaged, different portions of the object are in general at different distances from the axis. The magnification is not the same for all portions of the extended object. As a result a line object is not imaged into a line but into a curve.



(vi) **Astigmatism :** The spreading of image (of a point object placed away from the principal axis) along the principal axis is called Astigmatism.

Concepts

☞ *If a sphere of radius R made of material of refractive index* μ_2 *is placed in a medium of refractive index* μ_1, *Then if the object is placed at a distance* $\left(\frac{\mu_1}{\mu_2 - \mu_1}\right)R$ *from the pole, the real image formed is equidistant from the sphere.*



☞ *The lens doublets used in telescope are achromatic for v… red colours, while these used in camera are achromatic for*

μ_1

violet and green colours. The reason for this is that our eye is most sensitive between blue and red colours, while the photographic plates are most sensitive between violet and green colours.

☞ ***Position of optical centre***

Equiconvex and equiconcave	*Exactly at centre of lens*
Convexo-concave and concavo-convex	*Outside the glass position*
Plano convex and plano concave	*On the pole of curved surface*

☞ ***Composite lens :*** *If a lens is made of several materials then*

Number of images formed = Number of materials used

Here no. of images = 5

Example

Example: 18 A thin lens focal length f_1 and its aperture has diameter *d*. It forms an image of intensity *I*. Now the central part of the aperture upto diameter *d*/2 is blocked by an opaque paper. The focal length and image intensity will change to

(a) $\frac{f}{2}$ and $\frac{I}{2}$ (b) f and $\frac{I}{4}$ (c) $\frac{3f}{4}$ and $\frac{I}{2}$ (d) f and $\frac{3I}{4}$

Solution: (d) Centre part of the aperture up to diameter $\frac{d}{2}$ is blocked *i.e.* $\frac{1}{4}th$ area is blocked $\left(A=\frac{\pi d^2}{4}\right)$. Hence remaining area $A'=\frac{3}{4}A$. Also, we know that intensity ∝ Area ⇒ $\frac{I'}{I}=\frac{A'}{A}=\frac{3}{4}$ ⇒ $I'=\frac{3}{4}I$.

Focal length doesn't depend upon aperture.

Example: 19 The power of a thin convex lens $({}_a\mu_g=1.5)$ is + 5.0 *D*. When it is placed in a liquid of refractive index ${}_a\mu_l$, then it behaves as a concave lens of local length 100 *cm*. The refractive index of the liquid ${}_a\mu_l$ will be

(a) $5/3$ (b) $4/3$ (c) $\sqrt{3}$ (d) $5/4$

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1}$; where ${}_l\mu_g=\frac{\mu_g}{\mu_l}=\frac{1.5}{\mu_l}$ and $f_a=\frac{1}{P}=\frac{1}{5}m=20\,cm$

$$\Rightarrow \frac{-100}{20}=\frac{1.5-1}{\frac{1.5}{\mu_l}-1} \Rightarrow \mu_l=5/3$$

Example: 20 A double convex lens made of a material of refractive index 1.5 and having a focal length of 10 *cm* is immersed in liquid of refractive index 3.0. The lens will behave as **[NCERT 1973]**

(a) Diverging lens of focal length 10 *cm* (b) Diverging lens of focal length 10 / 3 *cm*

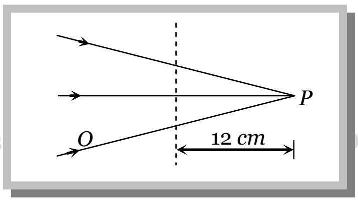
(c) Converging lens of focal length 10 / 3 *cm* (d) Converging lens of focal length 30 *cm*

Solution: (a) By using $\frac{f_l}{f_a}=\frac{{}_a\mu_g-1}{{}_l\mu_g-1} \Rightarrow \frac{f_l}{+10}=\frac{1.5-1}{\frac{1.5}{3}-1} \Rightarrow f_l=-10\,cm$ (*i.e.* diverging lens)

Example: 21 Figure given below shows a beam of light converging at point *P*. When a concave lens of focal length 16 *cm* is introduced in the path of the beam at a place *O* shown by dotted line, the beam converges at a distance *x* from the lens. The value *x* will be equal to

(a) 12 *cm*

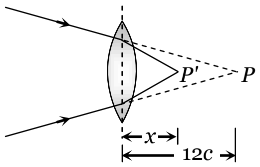
(b) 24 *cm*



(c) 36 *cm*

(d) 48 *cm*

Solution: (d) From the figure shown it is clear that



For lens : $u = 12$ *cm* and $v = x = ?$

By using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\Rightarrow \frac{1}{+16} = \frac{1}{x} - \frac{1}{+12}$ $\Rightarrow x = 48\ cm.$

***Example*: 22** A convex lens of focal length 40 *cm* is an contact with a concave lens of focal length 25 *cm*. The power of combination is

(a) – 1.5 *D* (b) – 6.5 *D* (c) + 6.5 *D* (d) + 6.67 *D*

Solution: (a) By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ $\Rightarrow \frac{1}{F} = \frac{1}{+40} + \frac{1}{-25}$

$\Rightarrow F = -\frac{200}{3} cm$, hence $P = \frac{100}{f(cm)} = \frac{100}{-200/3} = -1.5\ D$

***Example*: 23** A combination of two thin lenses with focal lengths f_1 and f_2 respectively forms an image of distant object at distance 60 *cm* when lenses are in contact. The position of this image shifts by 30 *cm* towards the combination when two lenses are separated by 10 *cm*. The corresponding values of f_1 and f_2 are **[AIIMS 1995]**

(a) 30 *cm*, – 60 *cm* (b) 20 *cm*, – 30 *cm* (c) 15 *cm*, – 20 *cm* (d) 12 *cm*, – 15 *cm*

Solution: (b) Initially $F = 60\ cm$ (Focal length of combination)

Hence by using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60}$ $\Rightarrow \frac{f_1 f_2}{f_1 + f_2}$ ......(i)

Finally by using $\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ where $F' = 30\,cm$ and $d = 10\ cm$ $\Rightarrow$ $\frac{1}{30} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{10}{f_1 f_2}$ ......(ii)

From equations (i) and (ii) $f_1 f_2 = -600$.

From equation (i) $f_1 + f_2 = -10$ .....(iii)

Also,difference of focal lengths can written as $f_1 - f_2 = \sqrt{(f_1 + f_2)^2 - 4 f_1 f_2}$ $\Rightarrow f_1 - f_2 = 50$.(iv)

From (iii) × (iv) $f_1 = 20$ and $f_2 = -30$

***Example*: 24** A thin double convex lens has radii of curvature each of magnitude 40 *cm* and is made of glass with refractive index 1.65. Its focal length is nearly

(a) 20 *cm* (b) 31 *cm* (c) 35 *cm* (d) 50 *cm*

Solution: (b) By using $f = \frac{R}{2(\mu - 1)}$ $\Rightarrow$ $f = \frac{40}{2(1.65 - 1)} = 30.7\,cm \approx 31\,cm.$

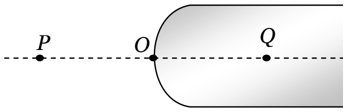
***Example*: 25** A spherical surface of radius of curvature *R* separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object *P* placed in air is found to have a real image *Q* in the glass. The line *PQ* cuts the surface at a point *O* and $PO = OQ$. The distance *PO* is equal to

[MP PMT 1994; Haryana CEE 1996]

(a) $5R$ (b) $3R$ (c) $2R$ (d) $1.5R$

Solution: (a) By using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Where $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -OP$, $v = OQ$



Hence $\frac{1.5}{OQ} - \frac{1}{-OP} = \frac{1.5-1}{(+R)} \Rightarrow \frac{1.5}{OP} + \frac{1}{OP} = \frac{0.5}{R}$

$\Rightarrow OP = 5R$

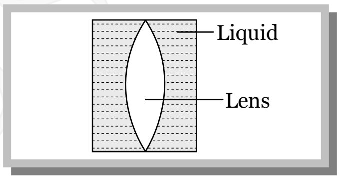
***Example*: 26** The distance between an object and the screen is 100 *cm*. A lens produces an image on the screen when placed at either of the positions 40 *cm* apart. The power of the lens is

(a) $3D$ (b) $5D$ (c) $7D$ (d) $9D$

Solution: (b) By using $f = \frac{D^2 - x^2}{4D} \Rightarrow f = \frac{100^2 - 40^2}{4 \times 100} = 21\,cm$

Hence power $P = \frac{100}{F(cm)} = \frac{100}{21} \approx +5D$

***Example*: 27** Shown in figure here is a convergent lens placed inside a cell filled with a liquid. The lens has focal length +20 *cm* when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, the focal length of the system is



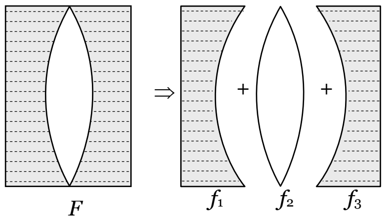
(a) + 80 *cm* (b) – 80 *cm*

(c) – 24 *cm* (d) – 100 *cm*

Solution: (d) Here $\frac{1}{f_1} = (1.6-1)\left(\frac{1}{\infty} - \frac{1}{20}\right) = \frac{-3}{100}$ ....(i)

$\frac{1}{f_2} = (1.5-1)\left(\frac{1}{20} - \frac{1}{-20}\right) = \frac{1}{20}$ ....(ii)

$\frac{1}{f_3} = (1.6-1)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = \frac{-3}{100}$ ....(iii)



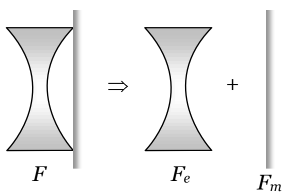
By using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \Rightarrow \frac{1}{F} = \frac{-3}{100} + \frac{1}{20} - \frac{3}{100} \Rightarrow F = -100\,cm$

***Example*: 28** A concave lens of focal length 20 *cm* placed in contact with a plane mirror acts as a **RA 1998]**

(a) Convex mirror of focal length 10 *cm* (b) Concave mirror of focal length 40 *cm*

(c) Concave mirror of focal length 60 *cm* (d) Concave mirror of focal length 10 *cm*

Solution: (a) By using $\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$



Since $f_m = \infty \Rightarrow F = \frac{f_l}{2} = \frac{20}{2} = 10\,cm$

(After silvering concave lens behave as convex mirror)

***Example*: 29** A candle placed 25 *cm* from a lens, forms an image on a screen placed 75 *cm* on the other end of the lens. The focal length and type of the lens should be

(a) + 18.75 *cm* and convex lens (b) – 18.75 *cm* and concave lens

(c) + 20.25 *cm* and convex lens (d) – 20.25 *cm* and concave lens

Solution: (a) In concave lens, image is always formed on the same side of the object. Hence the given lens is a convex lens for which $u = -25\,cm$, $v = 75\,cm$.

By using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ $\Rightarrow$ $\frac{1}{f}=\frac{1}{(+75)}-\frac{1}{(-25)}$ $\Rightarrow$ $f=+18.75\ cm$.

Example: 30 A convex lens forms a real image of an object for its two different positions on a screen. If height of the image in both the cases be 8 *cm* and 2 *cm*, then height of the object is **[KCET (Engg./Med.) 2**

(a) 16 *cm* (b) 8 *cm* (c) 4 *cm* (d) 2 *cm*

Solution: (c) By using $O=\sqrt{I_1 I_2}$ $\Rightarrow$ $O=\sqrt{8\times 2}=4\ cm$

Example: 31 A convex lens produces a real image *m* times the size of the object. What will be the distance of the object from the lens **[JIPMER 2002]**

(a) $\left(\frac{m+1}{m}\right)f$ (b) $(m-1)f$ (c) $\left(\frac{m-1}{m}\right)f$ (d) $\frac{m+1}{f}$

Solution: (a) By using $m=\frac{f}{f+u}$ here $-m=\frac{(+f)}{(+f)+u}$ $\Rightarrow$ $-\frac{1}{m}=\frac{f+u}{f}=1+\frac{u}{f}$ $\Rightarrow u=-\left(\frac{m+1}{m}\right).f$

Example: 32 An air bubble in a glass sphere having 4 *cm* diameter appears 1 *cm* from surface nearest to eye when looked along diameter. If ${}_a\mu_g=1.5$, the distance of bubble from refracting surface is **[CPMT 2002]**

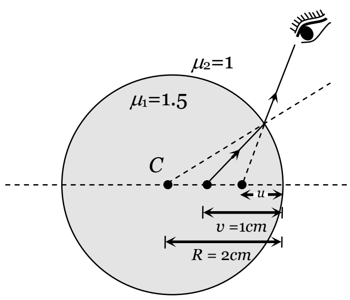
(a) 1.2 *cm* (b) 3.2 *cm* (c) 2.8 *cm* (d) 1.6 *cm*

Solution: (a) By using

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

where $u=?$, $v=-1\ cm$, $\mu_1=1.5$, $\mu_2=1$, $R=-2\ cm$.

$$\frac{1}{-1}-\frac{1.5}{u}=\frac{1-1.5}{(-2)} \Rightarrow u=-\frac{6}{5}=-1.2\ cm.$$

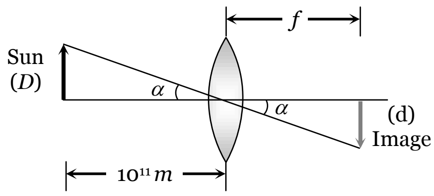


Example: 33 The sun's diameter is $1.4\times 10^{9}\ m$ and its distance from the earth is $10^{11}\ m$. The diameter of its image, formed by a convex lens of focal length 2*m* will be

(a) 0.7 *cm* (b) 1.4 *cm* (c) 2.8 *cm* (d) Zero (*i.e.* point image)

Solution: (c) From figure

$$\frac{D}{d}=\frac{10^{11}}{2} \Rightarrow d=\frac{2\times 1.4\times 10^{9}}{10^{11}}=2.8\ cm.$$



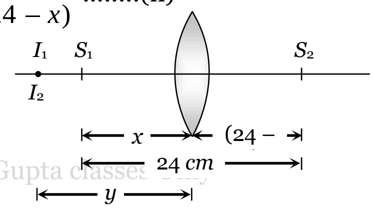
Example: 34 Two point light sources are 24 *cm* apart. Where should a convex lens of focal length 9 *cm* be put in between them from one source so that the images of both the sources are formed at the same place

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

Solution: (a) The given condition will be satisfied only if one source (S_1) placed on one side such that $u<f$ (*i.e.* it lies under the focus). The other source (S_2) is placed on the other side of the lens such that $u>f$ (*i.e.* it lies beyond the focus).

If S_1 is the object for lens then $\frac{1}{f}=\frac{1}{-y}-\frac{1}{-x}$ $\Rightarrow$ $\frac{1}{y}=\frac{1}{x}-\frac{1}{f}$ ....(i)

If S_2 is the object for lens then $\frac{1}{f}=\frac{1}{+y}-\frac{1}{-(24-x)}$ $\Rightarrow$ $\frac{1}{y}=\frac{1}{f}-\frac{1}{(24-x)}$ ....(ii)



From equation (i) and (ii)

$$\frac{1}{x}-\frac{1}{f}=\frac{1}{f}-\frac{1}{(24-x)} \Rightarrow \frac{1}{x}+\frac{1}{(24-x)}=\frac{2}{f}=\frac{2}{9} \Rightarrow x^2-24x+108=0$$

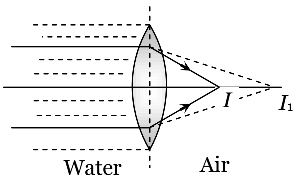
On solving the equation $x = 18\ cm$, $6\ cm$

***Example*: 35** There is an equiconvex glass lens with radius of each face as R and ${}_a\mu_g = 3/2$ and ${}_a\mu_w = 4/3$. If there is water in object space and air in image space, then the focal length is

(a) $2R$ (b) R (c) $3R/2$ (d) R^2

Solution: (c) Consider the refraction of the first surface *i.e.* refraction from rarer medium to denser medium

$$\frac{\mu_2-\mu_1}{R}=\frac{\mu_1}{-u}+\frac{\mu_2}{v_1} \Rightarrow \frac{\left(\frac{3}{2}\right)-\left(\frac{4}{3}\right)}{R}=\frac{\frac{4}{3}}{\infty}+\frac{\frac{3}{2}}{v_1} \Rightarrow v_1 = 9R$$



Now consider the refraction at the second surface of the lens *i.e.* refraction from denser medium to rarer medium

$$\frac{1-\frac{3}{2}}{-R}=-\frac{\frac{3}{2}}{9R}+\frac{1}{v_2} \Rightarrow v_2=\left(\frac{3}{2}\right)R$$

The image will be formed at a distance do $\frac{3}{2}R$. This is equal to the focal length of the lens.

Tricky example: 4

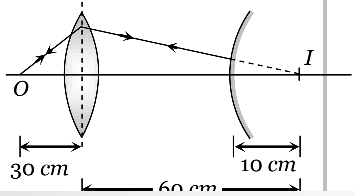
A luminous object is placed at a distance of 30 *cm* from the convex lens of focal length 20 *cm*. On the other side of the lens. At what distance from the lens a convex mirror of radius of curvature 10 *cm* be placed in order to have an upright image of the object coincident with it

[CBSE PMT 1998; JIPMER 2001, 2002]

(a) 12 *cm* (b) 30 *cm* (c) 50 *cm* (d) 60 *cm*

Solution : (c) For lens $u = 30\ cm$, $f = 20\ cm$, hence by using $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{+20}=\frac{1}{v}-\frac{1}{-30} \Rightarrow v = 60\ cm$

The final image will coincide the object, if light ray falls normally on convex mirror as shown. From figure it is seen clear that reparation between lens and mirror is 60 – 10 = 50 *cm*.



Tricky example: 5

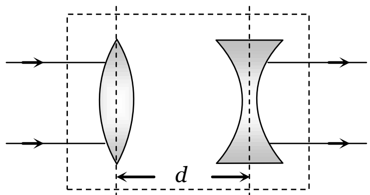
A convex lens of local length 30 *cm* and a concave lens of 10 *cm* focal length are placed so as to have the same axis. If a parallel beam of light falling on convex lens leaves concave lens as a parallel beam, then the distance between two lenses will be

(a) 40 *cm* (b) 30 *cm* (c) 20 *cm* (d) 10 *cm*

Solution : (c) According to figure the combination behaves as plane glass plate (*i.e.*, $F = \infty$)

Hence by using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$

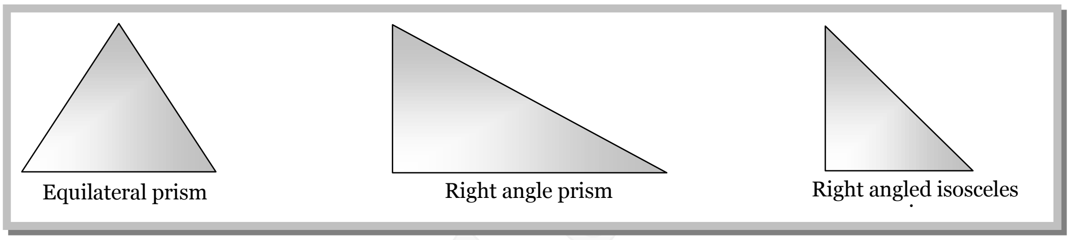
$$\Rightarrow \frac{1}{\infty}=\frac{1}{+30}+\frac{1}{-10}-\frac{d}{(30)(-10)} \Rightarrow d = 20\ cm$$



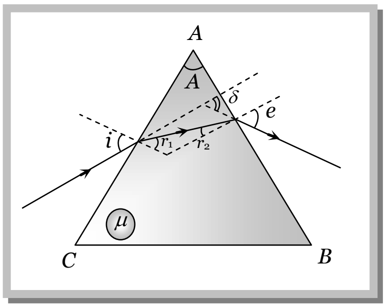
Prism

Prism is a transparent medium bounded by refracting surfaces, such that the incident surface (on which light ray is incidenting) and emergent surface (from which light rays emerges) are plane and non parallel.

Commonly used prism :



(1) **Refraction through a prism**



$A = r_1 + r_2$ and $i + e = A + \delta$

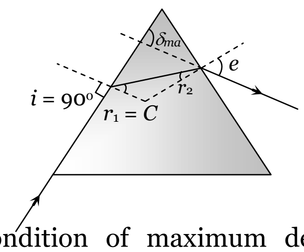
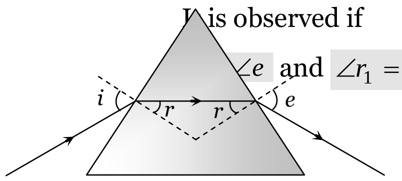
For surface AC $\mu = \frac{\sin i}{\sin r_1}$;

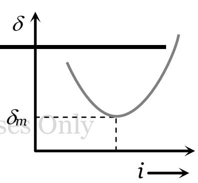
For surface AB $\mu = \frac{\sin r_2}{\sin e}$

i – Angle of incidence, e – Angle of emergence, A – Angle of prism or refracting angle of prism, r_1 and r_2 – Angle of refraction, δ – Angle of

(2) **Deviation through a prism**

For thin prism $\delta = (\mu - 1)A$. Also deviation is different for different colour light *e.g.* $\mu_R < \mu_V$ so $\delta_R < \delta_V$. And $\mu_{\text{Flint}} > \mu_{\text{Crown}}$ so $\delta_F > \delta_C$

Maximum deviation	**Minimum deviation**
 In this condition of maximum deviation $\angle i = 90^o$, $r_1 = C$, $r_2 = A - C$ and from Snell's law on emergent surface	is observed if $\angle e$ and $\angle r_1 = \angle r_2 = r$  (i) Refracted ray inside the prism is parallel to the base of the prism

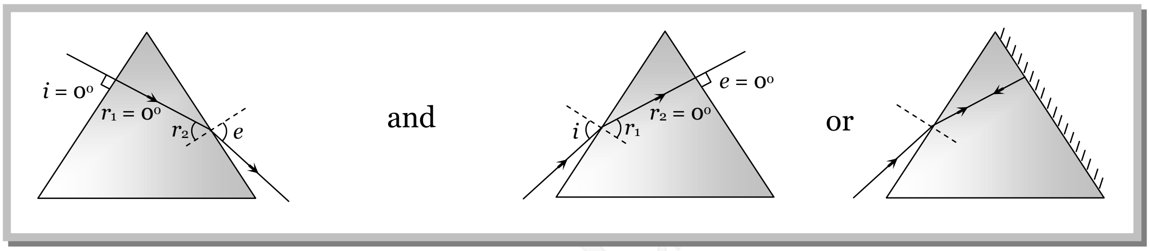


$e = \sin^{-1}\left[\frac{\sin(A-C)}{\sin C}\right]$	(ii) $r = \frac{A}{2}$ and $i = \frac{A+\delta_m}{2}$ (iii) $\mu = \frac{\sin i}{\sin A/2}$ or $\mu = \frac{\sin \frac{A+\delta_m}{2}}{\sin A/2}$

Note : ❑ If $\delta_m = A$ then $\mu = 2\cos A/2$

(3) **Normal incidence on a prism**

If light ray incident normally on any surface of prism as shown

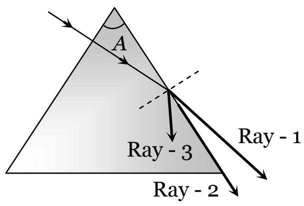


In any of the above case use $\mu = \frac{\sin i}{\sin A}$ and $\delta = i - A$

(4) **Grazing emergence and TIR through a prism**

When a light ray falls on one surface of prism, it is not necessary that it will exit out from the prism. It may or may not be exit out as shown below

Normal incidence



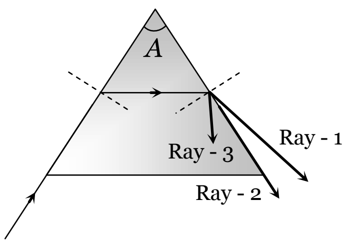
Ray –1 : General emergence
$A < C$ and
$\mu <$ cosec A

Ray – 2: Grazing emergence
$A = C$ and
$\mu =$ cosec A

Ray – 3: TIR
$A > C$ and
$\mu >$ cosec A

A = angle of prism and C = Critical angle for the material of the prism

Grazing incidence



Ray –1 : General emergence
$A < 2C$ and
$\mu <$ cosec $(A/2)$

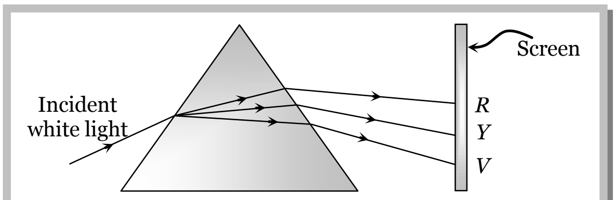
Ray – 2: Grazing emergence
$A = 2C$ and
$\mu =$ cosec $(A/2)$

Ray – 3: TIR
$A > 2C$ and
$\mu >$ cosec $(A/2)$

Note : ❑ For the condition of grazing emergence. Minimum angle of incidence $i_{min} = \sin^{-1}\left[\sqrt{\mu^2 - 1}\,\sin A - \cos A\right]$.

(5) **Dispersion through a prism**

The splitting of white light into it's constituent colours is called dispersion of light.



(i) Angular dispersion (θ) : Angular separation between extreme colours *i.e.* $\boldsymbol{\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A}$. It depends upon μ and A.

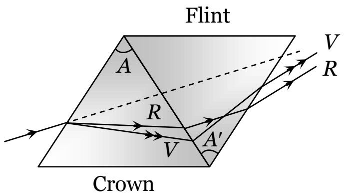
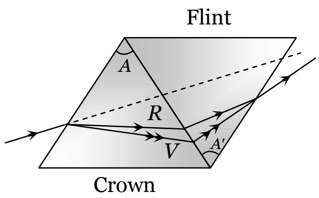
(ii) Dispersive power (ω) : $\omega = \frac{\theta}{\delta_y} = \frac{\mu_V - \mu_R}{\mu_y - 1}$ where $\left\{\mu_y = \frac{\mu_V + \mu_R}{2}\right\}$

⇒ It depends only upon the material of the prism *i.e.* μ and it doesn't depends upon angle of prism A

Note : ❑ Remember $\omega_{\text{Flint}} > \omega_{\text{Crown}}$.

(6) **Combination of prisms**

Two prisms (made of crown and flint material) are combined to get either dispersion only or deviation only.

Dispersion without deviation (chromatic combination)	Deviation without dispersion (Achromatic combination)
	
(i) $\frac{A'}{A} = -\frac{(\mu_y - 1)}{(\mu'_y - 1)}$	(i) $\frac{A'}{A} = -\frac{(\mu_V - \mu_R)}{(\mu'_V - \mu'_R)}$
(ii) $\theta_{net} = \theta\left(1 - \frac{\omega'}{\omega}\right) = (\omega\delta - \omega'\delta')$	(ii) $\delta_{net} = \delta\left(1 - \frac{\omega}{\omega'}\right)$

Scattering of Light

Molecules of a medium after absorbing incoming light radiations, emits them in all direction. This phenomenon is called Scattering.

(1) **According to scientist Rayleigh :** Intensity of scattered light $\propto \frac{1}{\lambda^4}$

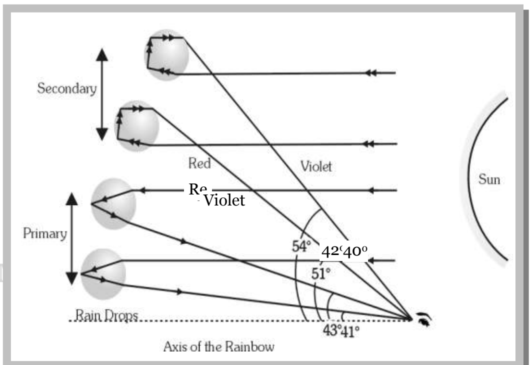
(2) **Some phenomenon based on scattering :** (i) Sky looks blue due to scattering.
(ii) At the time of sunrise or sunset it looks reddish. (iii) Danger signals are made from red.

(3) **Elastic scattering :** When the wavelength of radiation remains unchanged, the scattering is called elastic.

(4) **Inelastic scattering (Raman's effect) :** Under specific condition, light can also suffer inelastic scattering from molecules in which it's wavelength changes.

Rainbow

Rainbow is formed due to the dispersion of light suffering refraction and TIR in the droplets present in the atmosphere.



(1) **Primary rainbow :** (i) Two refraction and one TIR. (ii) Innermost arc is violet and outermost is red. (iii) Subtends an angle of 42^o at the eye of the observer. (iv) More bright

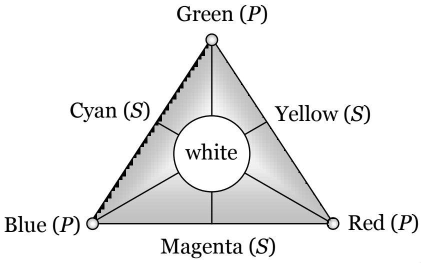
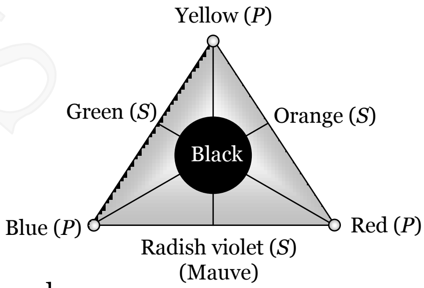
(2) **Secondary rainbow :** (i) Two refraction and two TIR. (ii) Innermost arc is red and outermost is violet.

(iii) It subtends an angle of 52.5^o at the eye. (iv) Comparatively less bright.

Colours

Colour is defined as the sensation received by the eye (cone cells of the eye) due to light coming from object.

(1) **Types of colours**

Spectral colours	**Colours of pigment and dyes**
	
(i) Complementary colours : Green and magenta Blue and yellow Red and cyan	(i) Complementary colours : yellow and mauve Red and green Blue and orange
(ii) Combination : Green + red + blue = White Blue + yellow = White Red + cyan = White Green + magenta = White	(ii) Combination : Yellow + red + blue = Black Blue + orange = Black Red + green = Black Yellow + mauve = Black

(2) **Colours of object :** The perception of a colour by eye depends on the nature of object and the light incident on it.

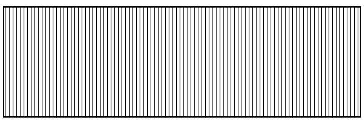
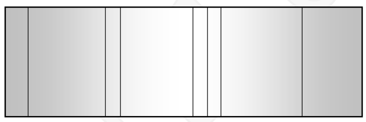
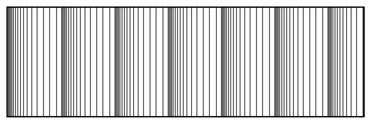
Colours of opaque object	**Colours of transparent object**
(i) Due to selective reflection.	(i) Due to selective transmission.
(ii) A rose appears red in white light because it reflects red colour and absorbs all remaining colours.	(ii) A red glass appears red because it absorbs all colours, except red which it transmits.
(iii) When yellow light falls on a bunch of flowers, then yellow and white flowers looks yellow. Other flowers looks black.	(iii) When we look on objects through a green glass or green filter then green and white objects will appear green while other black.

Note : A hot object will emit light of that colour only which it has observed when it was heated.

Spectrum

The ordered arrangements of radiations according to wavelengths or frequencies is called Spectrum. Spectrum can be divided in two parts (I) Emission spectrum and (II) Absorption spectrum.

(1) **Emission spectrum :** When light emitted by a self luminous object is dispersed by a prism to get the spectrum, the spectrum is called emission spectra.

Continuous emission spectrum	Line emission spectrum	Band emission spectrum
(i) It consists of continuously varying wavelengths in a definite wavelength range.	(i) It consist of distinct bright lines.	(iii) It consist of district bright bands.
(ii) It is produced by solids, liquids and highly compressed gases heated to high temperature.	(ii) It is produced by an excited source in atomic state.	(ii) It is produced by an excited source in molecular state.
(iii) *e.g.* Light from the sun, filament of incandescent bulb, candle flame *etc.*	(iii) *e.g.* Spectrum of excited helium, mercury vapours, sodium vapours or atomic hydrogen.	(iii) *e.g.* Spectra of molecular H_2, CO, NH_3 etc.
		

(2) **Absorption spectrum :** When white light passes through a semi-transparent solid, or liquid or gas, it's spectrum contains certain dark lines or bands, such spectrum is called absorption spectrum (of the substance through which light is passed).

(i) Substances in atomic state produces line absorption spectra. Polyatomic substances such as H_2, CO_2 and $KMnO_4$ produces band absorption spectrum.

(ii) Absorption spectra of sodium vapour have two (yellow lines) wavelengths $D_1(5890\ \mathring{A})$ and $D_2(5896\ \mathring{A})$

Note : ❑ If a substance emits spectral lines at high temperature then it absorbs the same lines at low temperature. This is Kirchoff's law.

(3) **Fraunhoffer's lines :** The central part (photosphere) of the sun is very hot and emits all possible wavelengths of the visible light. However, the outer part (chromosphere) consists of vapours of different elements. When the light emitted from the photosphere passes through the chromosphere, certain wavelengths are absorbed. Hence, in the spectrum of sunlight a large number of dark lines are seen called Fraunhoffer lines.

(i) The prominent lines in the yellow part of the visible spectrum were labelled as D-lines, those in blue part as F-lines and in red part as C-line.

(ii) From the study of Fraunhoffer's lines the presence of various elements in the sun's atmosphere can be identified *e.g.* abundance of hydrogen and helium.

(4) **Spectrometer :** A spectrometer is used for obtaining pure spectrum of a source in laboratory and calculation of μ of material of prism and μ of a transparent liquid.

It consists of three parts : Collimator which provides a parallel beam of light; Prism Table for holding the prism and Telescope for observing the spectrum and making measurements on it.

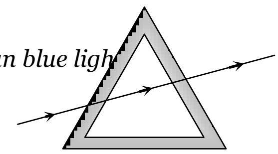
The telescope is first set for parallel rays and then collimator is set for parallel rays. When prism is set in minimum deviation position, the spectrum seen is pure spectrum. Angle of prism (A) and angle of minimum deviation (δ_m) are measured and μ of material of prism is calculated using prism formula. For μ of a transparent liquid, we take a hollow prism with thin glass sides. Fill it with the liquid and measure (δ_m) and A of liquid prism. μ of liquid is calculated using prism formula.

(5) **Direct vision spectroscope :** It is an instrument used to observe pure spectrum. It produces dispersion without deviation with the help of n crown prisms and $(n-1)$ flint prisms alternately arranged in a tabular structure.

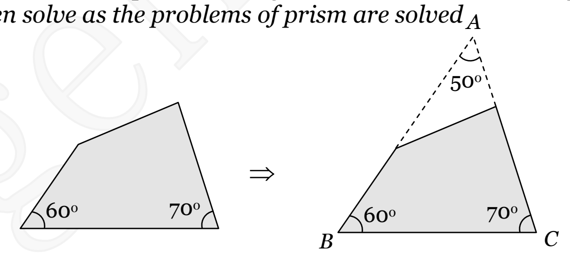
For no deviation $n(\mu-1)A = (n-1)(\mu'-1)A'$.

Concepts

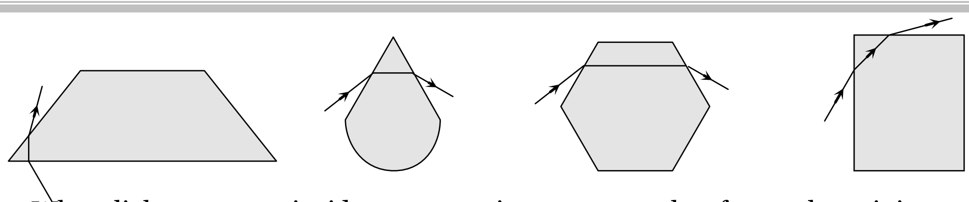
- *When a ray of white light passes through a glass prism red light is deviated less than blue light*
- *For a hollow prism* $A \neq 0$ *but* $\delta = 0$



- *If an opaque coloured object or crystal is crushed to fine powder it will appear white (in sun light) as it will lose it's property of selective reflection.*
- *Our eye is most sensitive to that part at the spectrum which lies between the F line (sky green) one the C-line (red) of hydrogen equal to the refractive index for the D line (yellow) of sodium. Hence for the dispersive power, the following formula is internationally accepted* $\omega = \dfrac{\mu_F - \mu_C}{\mu_D - 1}$
- *Sometimes a part of prism is given and we keep on thinking whether how should we proceed ? To solve such problems first complete the prism then solve as the problems of prism are solved*



- *Some other types of prism*



Example: 36 When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be

(a) 30° (b) 40° (c) 50° (d) 60°

Solution: (d) $\mu = \dfrac{\sin i}{\sin\frac{A}{2}} \Rightarrow \sqrt{2} = \dfrac{\sin 45}{\sin\frac{A}{2}} \Rightarrow \sin\dfrac{A}{2} = \dfrac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \dfrac{1}{2} \Rightarrow \dfrac{A}{2} = 30^o \Rightarrow A = 60^o$

Example: 37 Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism. The angle of prism is $(\cos 41^o = 0.75)$

(a) 62° (b) 41° (c) 82° (d) 31°

Solution: (c) Given $\delta_m = A$, then by using $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \mu = \frac{\sin\frac{A+A}{2}}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$

$\left\{\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}\right\}$

$\Rightarrow 1.5 = 2\cos\frac{A}{2} \Rightarrow 0.75 = \cos\frac{A}{2} \Rightarrow 41^o = \frac{A}{2} \Rightarrow A = 82^o$.

Example: 38 Angle of glass prism is 60° and refractive index of the material of the prism is 1.414,then what will be the angle of incidence, so that ray should pass symmetrically through prism

(a) $38^o 61'$ (b) $35^o 35'$ (c) 45° (d) $53^o 8'$

Solution: (c) incident ray and emergent ray are symmetrical in the cure, when prism is in minimum deviation position.

Hence in this condition

$\mu = \frac{\sin i}{\sin\frac{A}{2}} \Rightarrow \sin i = \mu \sin\left(\frac{A}{2}\right) \Rightarrow \sin i = 1.414 \times \sin 30^o = \frac{1}{\sqrt{2}} \Rightarrow i = 45^o$

Example: 39 A prism $(\mu = 1.5)$ has the refracting angle of 30°. The deviation of a monochromatic ray incident normally on its one surface will be $(\sin 48^o 36' = 0.75)$

(a) $18^o 36'$ (b) $20^o 30'$ (c) 18° (d) $22^o 1'$

Solution: (a) By using $\mu = \frac{\sin i}{\sin A} \Rightarrow 1.5 = \frac{\sin i}{\sin 30} \Rightarrow \sin i = 0.75 \Rightarrow i = 48^o 36'$

Also from $\delta = i - A \Rightarrow \delta = 48^o 36' - 30^o = 18^o 36'$

Example: 40 Angle of a prism is 30° and its refractive index is $\sqrt{2}$ and one of the surface is silvered. At what angle of incidence, a ray should be incident on one surface so that after reflection from the silvered surface, it retraces its path

(a) 30° (b) 60° (c) 45° (d) $\sin^{-1}\sqrt{1.5}$

Solution: (c) This is the case when light ray is falling normally an second surface.

Hence by using $\mu = \frac{\sin i}{\sin A} \Rightarrow \sqrt{2} = \frac{\sin i}{\sin 30^o} \Rightarrow \sin i = \sqrt{2} \times \frac{1}{2} \Rightarrow i = 45^o$

Example: 41 The refracting angle of prism is A and refractive index of material of prism is $\cot\frac{A}{2}$. The angle of minimum deviation is

(a) $180^o - 3A$ (b) $180^o + 2A$ (c) $90^o - A$ (d) $180^o - 2A$

Solution: (d) By using $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \cot\frac{A}{2} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} \Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

$\Rightarrow \sin\left(90 - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right) \Rightarrow 90 - \frac{A}{2} = \frac{A+\delta_m}{2} \Rightarrow \delta_m = 180 - 2A$

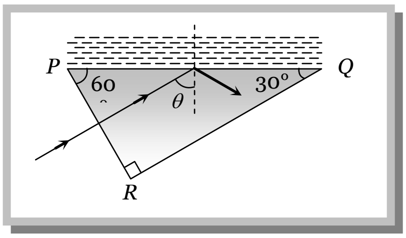
Example: 42 A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to 3/4 of the angle of the prism. The angle of deviation is

(a) 45° (b) 39° (c) 20° (d) 30°

Solution: (d) Given that $A = 60^o$ and $i = e = \frac{3}{4}A = \frac{3}{4} \times 60 = 45^o$

By using $i + e = A + \delta \Rightarrow 45 + 45 = 60 + \delta \Rightarrow \delta = 30^o$

***Example*: 43** *PQR* is a right angled prism with other angles as 60° and 30°. Refractive index of prism is 1.5. *PQ* has a thin layer of liquid. Light falls normally on the face *PR*. For total internal reflection, maximum refractive index of liquid is



(a) 1.4

(b) 1.3

(c) 1.2

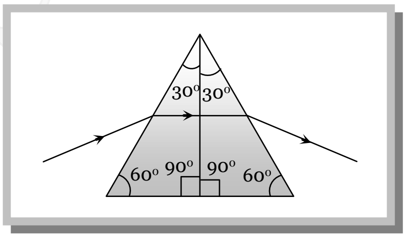
(d) 1.6

Solution: (c) For *TIR* at *PQ* $\theta < C$

From geometry of figure $\theta = 60$ *i.e.* $60 > C \Rightarrow \sin 60 > \sin C$

$\Rightarrow \frac{\sqrt{3}}{2} > \frac{\mu_{Liquid}}{\mu_{Prism}} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times \mu_{Prism} \Rightarrow \mu_{Liquid} < \frac{\sqrt{3}}{2} \times 1.5 \Rightarrow \mu_{Liquid} < 1.3$.

***Example*: 44** Two identical prisms 1 and 2, each will angles of 30°, 60° and 90° are placed in contact as shown in figure. A ray of light passed through the combination in the position of minimum deviation and suffers a deviation of 30°. If the prism 2 is removed, then the angle of deviation of the same ray is **[PMT (Andhra) 1995]**



(a) Equal to 15°

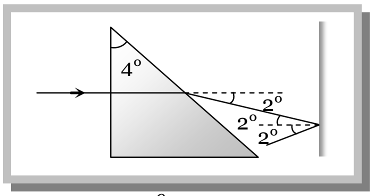
(b) Smaller than 30°

(c) More than 15°

(d) Equal to 30°

Solution: (a) $\delta = (\mu - 1)A$ as *A* is halved, so δ is also halves

***Example*: 45** A prism having an apex angle 4° and refraction index 1.5 is located in front of a vertical plane mirror as shown in figure. Through what total angle is the ray deviated after reflection from the mirror



(a) 176° (b) 4°

(c) 178° (d) 2°

Solution: (c) $\delta_{Prism} = (\mu - 1)A = (1.5 - 1)4^o = 2^o$

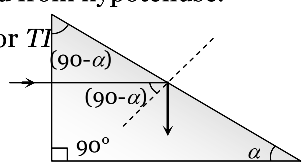
$\therefore \delta_{Total} = \delta_{Prism} + \delta_{Mirror} = (\mu - 1)A + (180 - 2i) = 2^o + (180 - 2 \times 2) = 178^o$

***Example*: 46** A ray of light is incident to the hypotenuse of a right-angled prism after travelling parallel to the base inside the prism. If μ is the refractive index of the material of the prism, the maximum value of the base angle for which light is totally reflected from the hypotenuse is **[EAMCET 2003]**

(a) $\sin^{-1}\left(\frac{1}{\mu}\right)$ (b) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (c) $\sin^{-1}\left(\frac{\mu - 1}{\mu}\right)$ (d) $\cos^{-1}\left(\frac{1}{\mu}\right)$

Solution: (d) If $\alpha =$ maximum value of vase angle for which light is totally reflected from hypotenuse.

$(90 - \alpha) = C =$ minimum value of angle of incidence an hypotenuse for TI



$\sin(90 - \alpha) = \sin C = \frac{1}{\mu} \Rightarrow \alpha = \cos^{-1}\left(\frac{1}{\mu}\right)$

***Example*: 47** If the refractive indices of crown glass for red, yellow and violet colours are 1.5140, 1.5170 and 1.5318 respectively and for flint glass these are 1.6434, 1.6499 and 1.6852 respectively, then the dispersive powers for crown and flint glass are respectively

(a) 0.034 and 0.064 (b) 0.064 and 0.034 (c) 1.00 and 0.064 (d) 0.034 and 1.0

Solution: (a) $\omega_{\text{Crown}} = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.5318 - 1.5140}{(1.5170 - 1)} = 0.034$ and

$\omega_{\text{Flint}} = \frac{\mu_v' - \mu_r'}{\mu_y' - 1} = \frac{1.6852 - 1.6434}{1.6499 - 1} = 0.064$

***Example*: 48** Flint glass prism is joined by a crown glass prism to produce dispersion without deviation. The refractive indices of these for mean rays are 1.602 and 1.500 respectively. Angle of prism of flint prism is 10°, then the angle of prism for crown prism will be

(a) $12^o 2.4'$ (b) $12^o 4'$ (c) 1.24^o (d) 12°

Solution: (a) For dispersion without deviation

$\frac{A_C}{A_F} = \frac{(\mu_F - 1)}{(\mu_C - 1)} \Rightarrow \frac{A}{10} = \frac{(1.602 - 1)}{(1.500 - 1)} \Rightarrow A = 12.04^o = 12^o 2.4'$

Tricky example: 6

An achromatic prism is made by crown glass prism $(A_C = 19^o)$ and flint glass prism $(A_F = 6^o)$. If ${}^C\mu_v = 1.5$ and ${}^F\mu_v = 1.66$, then resultant deviation for red coloured ray will be

(a) 1.04° (b) 5° (c) 0.96° (d) 13.5°

Solution : (d) For achromatic combination $w_C = -w_F \Rightarrow [(\mu_v - \mu_r)A]_C = -[(\mu_v - \mu_r)A]_F$

$\Rightarrow [\mu_r A]_C + [\mu_r A]_F = [\mu_v A]_C + [\mu_v A]_F = 1.5 \times 19 + 6 \times 1.66 = 38.5$

Resultant deviation $\delta = [(\mu_r - 1)A]_C + [(\mu_r - 1)A]_F$

$= [\mu_r A]_C + [\mu_r A]_F - (A_C + A_F) = 38.5 - (19 + 6) = 13.5^o$

Tricky example: 7

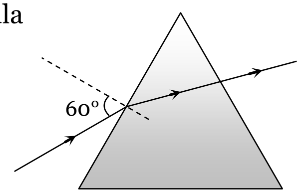
The light is incident at an angle of 60° on a prism of which the refracting angle of prism is 30°. The refractive index of material of prism will be

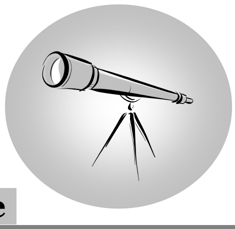
(a) $\sqrt{2}$ (b) $2\sqrt{3}$ (c) 2 (d) $\sqrt{3}$

Solution : (d) By using $i + e = A + \delta \Rightarrow 60 + e = 30 + 30 \Rightarrow e = 0$.

Hence ray will emerge out normally so by using the formula

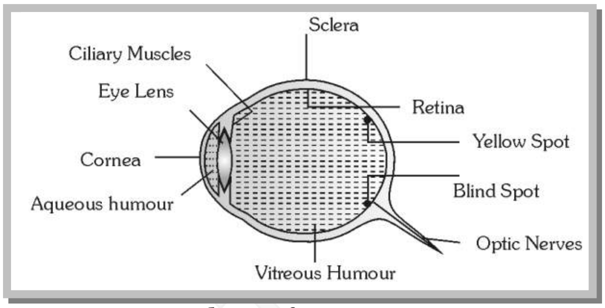
$\mu = \frac{\sin i}{\sin A} = \frac{\sin 60}{\sin 30} = \sqrt{3}$





Optical Instruments

Human Eye



(1) **Eye lens :** Over all behaves as a convex lens of $\mu = 1.437$

(2) **Retina :** Real and inverted image of an object, obtained at retina, brain sense it erect.

(3) **Yellow spot :** It is the most sensitive part, the image formed at yellow spot is brightest.

(4) **Blind spot :** Optic nerves goes to brain through blind spot. It is not sensitive for light.

(5) **Ciliary muscles –** Eye lens is fixed between these muscles. It's both radius of curvature can be changed by applying pressure on it through ciliary muscles.

(6) **Power of accomodation :** The ability of eye to see near objects as well as far objects is called power of accomodation.

Note : ❑ When we look distant objects, the eye is relaxed and it's focal length is largest.

(7) **Range of vision :** For healthy eye it is 25 *cm* (near point) to ∞ (far point).

A normal eye can see the objects clearly, only if they are at a distance greater than 25 *cm*. This distance is called Least distance of distinct vision and is represented by *D*.

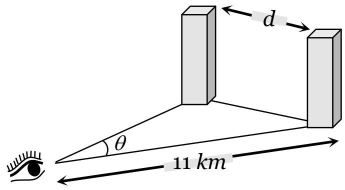
(8) **Persistence of vision :** Is 1/10 *sec*. *i.e.* if time interval between two consecutive light pulses is lesser than 0.1 *sec*., eye cannot distinguish them separately.

(9) **Binocular vision :** The seeing with two eyes is called binocular vision.

(10) **Resolving limit :** The minimum angular displacement between two objects, so that they are just resolved is called resolving limit. For eye it is $1' = \left(\frac{1}{60}\right)^{o}$.

Specific Example

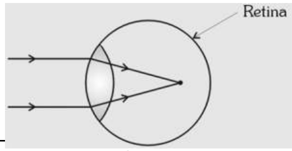
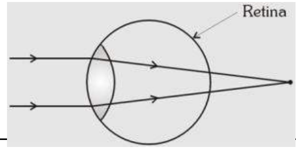
A person wishes to distinguish between two pillars located at a distances of 11 *Km*. What should be the minimum distance between the pillars.



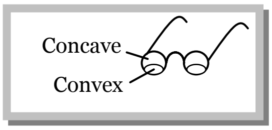
Solution : As the limit of resolution of eye is $\left(\frac{1}{60}\right)^o$

So $\theta > \left(\frac{1}{60}\right)^o$ $\Rightarrow \frac{d}{11\times10^3} > \left(\frac{1}{60}\right)\times\frac{\pi}{180} \Rightarrow d > 3.2\,m$

(11) **Defects in eye**

Myopia (short sightness)	Hypermetropia (long sightness)
(i) Distant objects are not seen clearly but nearer objects are clearly visible.	(i) Distant objects are seen clearly but nearer object are not clearly visible.
(ii) Image formed before the retina. 	(ii) Image formed behind the retina. 
(iii) Far point comes closer.	(iii) Near point moves away
(iv) Reasons : (a) Focal length or radii of curvature of lens reduced or power of lens increases. (b) Distance between eye lens and retina increases.	(iv) Reasons : (a) Focal length or radii of curvature of lens increases or power of lens decreases. (b) Distance between eye lens and retina decreases.
(v) Removal : By using a concave lens of suitable focal length.	(v) Removal : By using a convex lens.
(vi) Focal length : (a) A person can see upto distance $\rightarrow x$ wants to see $\rightarrow \infty$, so focal length of used lens $f = -x$ = – (defected far point) (b) A person can see upto distance $\rightarrow x$ wants to see distance $\rightarrow y$ ($y > x$) so $f = \frac{xy}{x-y}$	(vi) Focal length : (a) A person cannot see before distance $\rightarrow d$ wants to see the object place at distance $\rightarrow D$ so $f = \frac{dD}{d-D}$

Presbyopia : In this defect both near and far objects are not clearly visible. It is an old age disease and it is due to the loosing power of accommodation. It can be removed by using bifocal lens.



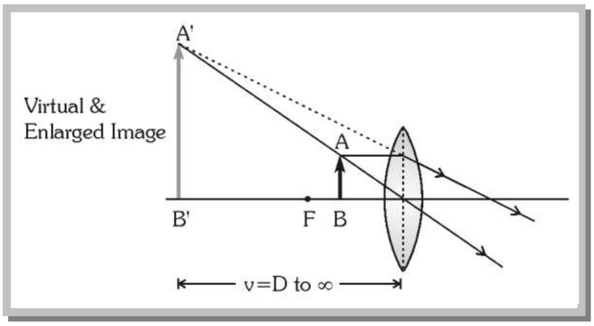
Astigmatism : In this defect eye cannot see horizontal and vertical lines clearly, simultaneously. It is due to imperfect spherical nature of eye lens. This defect can be removed by using cylindrical lens (Torric lenses).

Microscope

It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument } (\beta)}{\text{Visual angle when object is placed at least distance of distinct vision } (\alpha)}$$

(1) **Simple miscroscope**



(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.

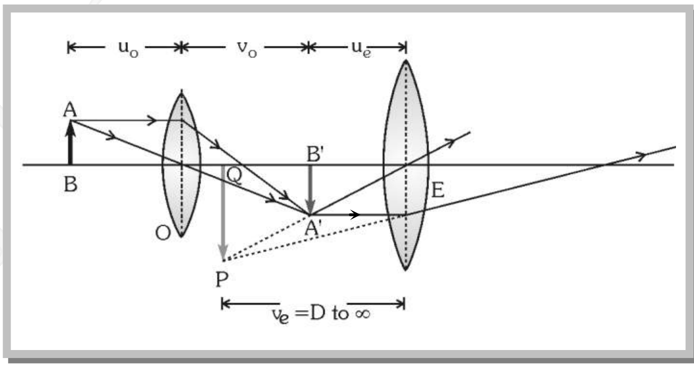
(iii) Magnification's, when final image is formed at D and ∞ (*i.e.* m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{max} \text{ and } m_\infty = \left(\frac{D}{f}\right)_{min}$$

Note : ❑ $m_{max.} - m_{min.} = 1$

❑ If lens is kept at a distance a from the eye then $m_D = 1 + \frac{D-a}{f}$ and $m_\infty = \frac{D-a}{f}$

(2) **Compound microscope**



(i) Consist of two converging lenses called objective and eye lens.

(ii) $f_{eye\,lens} > f_{objective}$ and

$(\text{diameter})_{eye\,lens} > (\text{diameter})_{objective}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (o), v_0 = Distance of image ($A'B'$) formed by objective from objective, u_e = Distance of $A'B'$ from eye lens, v_e = Distance of final image from eye lens, f_0 = Focal length of objective, f_e = Focal length of eye lens.

Magnification : $m_D = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = -\frac{f_0}{(u_0 - f_0)}\left(1 + \frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}\left(1 + \frac{D}{f_e}\right)$

$$m_\infty = -\frac{v_0}{u_0}.\frac{D}{F_e} = \frac{-f_0}{(u_0 - f_0)}\left(\frac{D}{f_e}\right) = -\frac{(v_0 - f_0)}{f_0}.\frac{D}{F_e}$$

Length of the tube (*i.e.* distance between two lenses)

When final image is formed at D ; $L_D = v_0 + u_e = \frac{u_0 f_0}{u_0 - f_0} + \frac{f_e D}{f_e + D}$

When final images is formed at ∞ ; $L_\infty = v_0 + f_e = \frac{u_0 f_0}{u_0 - f_0} + f_e$

(Do not use sign convention while solving the problems)

Note : ❑ $m_\infty = \frac{(L_\infty - f_0 - f_e)D}{f_0 f_e}$

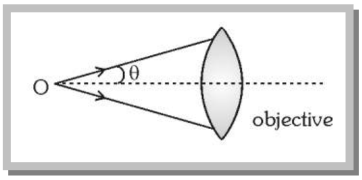
❑ For maximum magnification both f_0 and f_e must be less.

❑ $m = m_{\text{objective}} \times m_{\text{eyelens}}$

❑ If objective and eye lens are interchanged, practically there is no change in magnification.

(3) **Resolving limit and resolving power :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (*RL*) and it's reciprocal is called Resolving power (*RP*)

$$R.L. = \frac{\lambda}{2\mu \sin\theta} \text{ and } R.P. = \frac{2\mu \sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object,

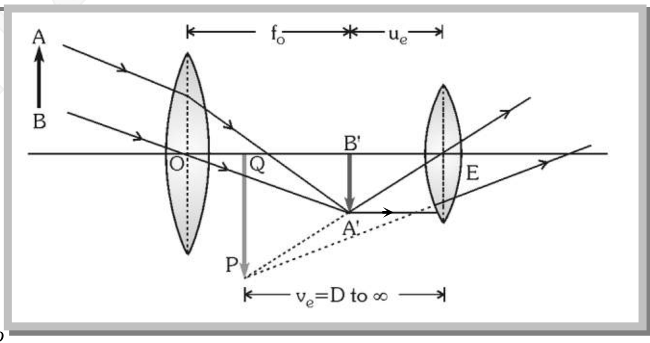
μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

Note : ❑ Electron microscope : electron beam $(\lambda \approx 1 Å)$ is used in it so it's *R.P.* is approx 5000 times more than that of ordinary microscope $(\lambda \approx 5000\ Å)$

Telescope

By telescope distant objects are seen.



(1) **Astronomical telescope**

(i) Used to see heavenly bodies.

(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eyelens}}$.

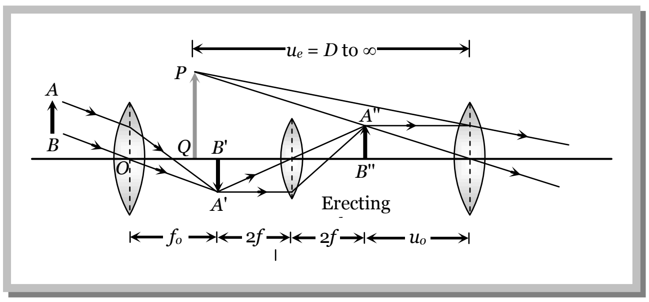
(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.

(v) Magnification : $m_D = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$

(vi) Length : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$

(2) **Terrestrial telescope**



(i) Used to see far off object on the earth.

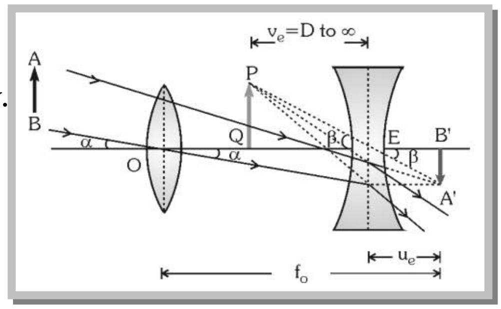
(ii) It consists of three converging lens : objective, eye lens and erecting lens.

(iii) It's final image is virtual erect and smaller.

(iv) Magnification : $m_D = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(v) Length : $L_D = f_0 + 4f + u_e = f_0 + 4f + \dfrac{f_e D}{f_e + D}$ and $L_\infty = f_0 + 4f + f_e$

(3) **Galilean telescope**



(i) It is also a terrestrial telescope but of much smaller field of view.

(ii) Objective is a converging lens while eye lens is diverging lens.

(iii) Magnification : $m_D = \dfrac{f_0}{f_e}\left(1 - \dfrac{f_e}{D}\right)$ and $m_\infty = \dfrac{f_0}{f_e}$

(iv) Length : $L_D = f_0 - u_e$ and $L_\infty = f_0 - f_e$

(4) **Resolving limit and resolving power**

Smallest angular separations ($d\theta$) between two distant objects, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \dfrac{1.22\lambda}{a}$

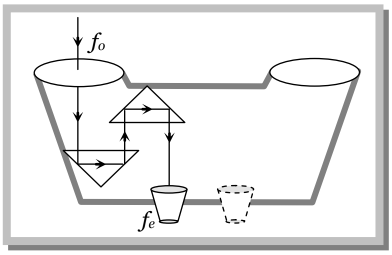
and resolving power $(RP) = \dfrac{1}{d\theta} = \dfrac{a}{1.22\lambda} \Rightarrow R.P. \propto \dfrac{1}{\lambda}$ where a = aperture of objective.

Note :Minimum separation (d) between objects, so they can just resolved by a telescope is –

$d = \dfrac{r}{R.P.}$ where r = distance of objects from telescope.

(5) **Binocular**

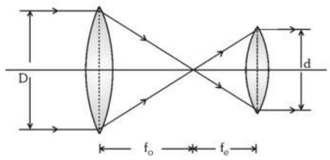
If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, *i.e.*, binocular vision gives proper three-dimensional (3D) image.



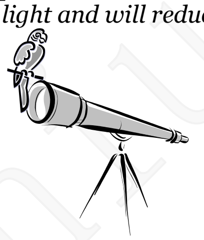
Concepts

- *As magnifying power is negative, the image seen in astronomical telescope is truly inverted, i.e., left is turned right with upside down simultaneously. However, as most of the astronomical objects are symmetrical this inversion does not affect the observations.*
- *Objective and eye lens of a telescope are interchanged, it will not behave as a microscope but object appears very small.*
- *In a telescope, if field and eye lenses are interchanged magnification will change from (f_o / f_e) to (f_e / f_o), i.e., it will change from m to (1/m), i.e., will become ($1/m^2$) times of its initial value.*
- *As magnification for normal setting as (f_o / f_e), so to have large magnification, f_o must be as large as practically possible and f_e small. This is why in a telescope, objective is of large focal length while eye piece of small.*
- *In a telescope, aperture of the field lens is made as large as practically possible to increase its resolving power as resolving power of a telescope $\propto (D/\lambda)^*$. Large aperture of objective also helps in improving the brightness of image by gathering more light from distant object. However, it increases aberrations particularly spherical.*
- *For a telescope with increase in length of the tube, magnification decreases.*
- *In case of a telescope if object and final image are at infinity then :*

$$m = \frac{f_o}{f_e} = \frac{D}{d}$$



- *If we are given four convex lenses having focal lengths $f_1 > f_2 > f_3 > f_4$. For making a good telescope and microscope. We choose the following lenses respectively. Telescope $f_1(o), f_4(e)$ Microscope $f_4(o), f_3(e)$*
- *If a parrot is sitting on the objective of a large telescope and we look towards (or take a photograph)of distant astronomical object (say moon) through it, the parrot will not be seen but the intensity of the image will be slightly reduced as the parrot will act as obstruction to light and will reduce the aperture of the objective.*



Example

***Example*: 1** A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

or

A man can see upto 100 *cm* of the distant object. The power of the lens required to see far objects will be

[MP PMT 1993, 2003]

(a) +0.5 *D* (b) +1.0 *D* (c) +2.0 *D* (d) –1.0 *D*

Solution: (d) f = –(defected far point) = – 100 *cm*. So power of the lens $P = \frac{100}{f} = \frac{100}{-100} = -1D$

***Example*: 2** A man can see clearly up to 3 *metres*. Prescribe a lens for his spectacles so that he can see clearly up to 12 *metres*

[DPMT 2002]

(a) – 3/4 *D* (b) 3 *D* (c) – 1/4 *D* (d) – 4 *D*

Solution: (c) By using $f = \frac{xy}{x-y} \Rightarrow f = \frac{3\times 12}{3-12} = -4m$. Hence power $P = \frac{1}{f} = -\frac{1}{4}D$

***Example*: 3** The diameter of the eye-ball of a normal eye is about 2.5 *cm*. The power of the eye lens varies from

(a) 2 *D* to 10 *D* (b) 40 *D* to 32 *D* (c) 9 *D* to 8 *D* (d) 44 *D* to 40 *D*

Solution: (d) An eye sees distant objects with full relaxation so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-\infty}=\frac{1}{f}$ or

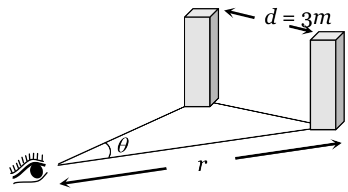
$P=\frac{1}{f}=\frac{1}{25\times10^{-2}}=40D$

An eye sees an object at 25 *cm* with strain so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-25\times10^{-2}}=\frac{1}{f}$ or

$P=\frac{1}{f}=40+4=44D$

***Example*: 4** The resolution limit of eye is 1 *minute*. At a distance of *r* from the eye, two persons stand with a lateral separation of 3 *metre*. For the two persons to be just resolved by the naked eye, *r* should be

(a) 10 *km* (b) 15 *km* (c) 20 *km* (d) 30 *km*



Solution: (a) From figure $\theta=\frac{d}{r}$; where $\theta=1'=\left(\frac{1}{60}\right)^{o}=\left(\frac{1}{60}\right)\times\frac{\pi}{180}rad$

$\Rightarrow 1\times\frac{1}{60}\times\frac{\pi}{180}=\frac{3}{r}\Rightarrow r=10\ km$

***Example*: 5** Two points separated by a distance of 0.1 *mm* can just be resolved in a microscope when a light of wavelength 6000 *Å* is used. If the light of wavelength 4800 *Å* is used this limit of resolution becomes

[UPSEAT 2002]

(a) 0.08 *mm* (b) 0.10 *mm* (c) 0.12 *mm* (d) 0.06 *mm*

Solution: (a) By using resolving limit (R.L.) $\propto\lambda$ $\Rightarrow\frac{(R.L.)_1}{(R.L.)_2}=\frac{\lambda_1}{\lambda_2}\Rightarrow\frac{0.1}{(R.L.)_2}=\frac{6000}{4800}\Rightarrow(R.L.)_2=0.08\,mm$.

***Example*: 6** In a compound microscope, the focal lengths of two lenses are 1.5 *cm* and 6.25 *cm* an object is placed at 2 *cm* form objective and the final image is formed at 25 *cm* from eye lens. The distance between the two lenses is

[EAMCET (Med.) 2000]

(a) 6.00 *cm* (b) 7.75 *cm* (c) 9.25 *cm* (d) 11.00 *cm*

Solution: (d) It is given that $f_o=1.5\ cm$, $f_e=6.25\ cm$, $u_o=2\ cm$

When final image is formed at least distance of distinct vision, length of the tube $L_D=\frac{u_o f_o}{u_o-f_o}+\frac{f_e D}{f_e+D}$

$\Rightarrow L_D=\frac{2\times1.5}{(2-1.5)}+\frac{6.25\times25}{(6.25+25)}=11\,cm$.

***Example*: 7** The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 *cm* and 3.0 *cm* respectively. The distance between the objective and the eye-piece is 15.0 *cm*. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in *cm* of the object and the image produced by the objective measured from the objective lens are respectively **[IIT-JEE 1995]**

(a) 2.4 and 12.0 (b) 2.4 and 15.0 (c) 2.3 and 12.0 (d) 2.3 and 3.0

Solution: (a) Given that $f_o=2\,cm$, $f_e=3\,cm$, $L_\infty=15\,cm$

By using $L_\infty=v_o+f_e\Rightarrow 15=v_o+3\Rightarrow v_o=12\,cm$. Also $\frac{v_o}{u_o}=\frac{v_o-f_o}{f_o}\Rightarrow\frac{12}{u_o}=\frac{12-2}{2}\Rightarrow u_o=2.4\,cm$.

***Example*: 8** The focal lengths of the objective and eye-lens of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(a) 30 *cm* (b) 25 *cm* (c) 15 *cm* (d) 12 *cm*

Solution: (c) Given that $f_o = 1\,cm$, $f_e = 5\,cm$, $m_\infty = 45$

By using $m_\infty = \frac{(L_\infty - f_o - f_e)}{f_o f_e} \Rightarrow 45 = \frac{(L_\infty - 1 - 5)\times 25}{1\times 5} \Rightarrow L_\infty = 15\,cm$

***Example*: 9** If the focal lengths of objective and eye lens of a microscope are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity, then magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

Solution: (b) Given that $f_o = 1.2\,cm$, $f_e = 3\,cm$, $u_o = 1.25\,cm$

By using $m_\infty = -\frac{f_o}{(u_o - f_o)}\cdot\frac{D}{f_e} \Rightarrow m_\infty = -\frac{1.2}{(1.25-1.2)}\times\frac{25}{3} = -200$.

***Example*: 10** The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54*cm*. The focal length of eye lens and objective lens will be respectively **[MP PMT 1991; CPMT 1991**

(a) 6 *cm* and 48 *cm* (b) 48 *cm* and 6 *cm* (c) 8 *cm* and 64 *cm* (d) 64 *cm* and 8 *cm*

Solution: (a) Given that $m_\infty = 8$ and $L_\infty = 54$

By using $|m_\infty| = \frac{f_o}{f_e}$ and $L_\infty = f_o + f_e$ we get $f_o = 6\,cm$ and $f_e = 48\,cm$.

***Example*: 11** If an object subtend angle of 2° at eye when seen through telescope having objective and eyepiece of focal length $f_o = 60\,cm$ and $f_e = 5\,cm$ respectively than angle subtend by image at eye piece will be **[UPSEAT 2001]**

(a) 16° (b) 50° (c) 24° (d) 10°

Solution: (c) By using $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{20} = \frac{60}{5} \Rightarrow \beta = 24°$

***Example*: 12** The focal lengths of the lenses of an astronomical telescope are 50 *cm* and 5 *cm*. The length of the telescope when the image is formed at the least distance of distinct vision is

(a) 45 *cm* (b) 55 *cm* (c) $\frac{275}{6}$ *cm* (d) $\frac{325}{6}$ *cm*

Solution: (d) By using $L_D = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5\times 25}{(5+25)} = \frac{325}{6}\,cm$

***Example*: 13** The diameter of moon is $3.5\times 10^3\,km$ and its distance from the earth is $3.8\times 10^5\,km$. If it is seen through a telescope whose focal length for objective and eye lens are 4 *m* and 10 *cm* respectively, then the angle subtended by the moon on the eye will be approximately

(a) 15° (b) 20° (c) 30° (d) 35°

Solution: (b) The angle subtended by the moon on the objective of telescope $\alpha = \frac{3.5\times 10^3}{3.8\times 10^5} = \frac{3.5}{3.8}\times 10^{-2}\,rad$

Also $m = \frac{f_o}{f_e} = \frac{\beta}{\alpha} \Rightarrow \frac{400}{10} = \frac{\beta}{\alpha} \Rightarrow \beta = 40\,\alpha \Rightarrow \beta = 40\times\frac{3.5\times 10^3}{3.8\times 10^5}\times\frac{180}{\pi} = 20°$

***Example*: 14** A telescope has an objective lens of 10 *cm* diameter and is situated at a distance one *kilometre* from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 *Å*, is of the order of

(a) 0.5 *m* (b) 5 *m* (c) 5 *mm* (d) 5*cm*

Solution: (b) Suppose minimum distance between objects is *x* and their distance from telescope is *r*

So Resolving limit $d\theta = \frac{1.22\lambda}{a} = \frac{x}{r} \Rightarrow x = \frac{1.22\lambda\times r}{a} = \frac{1.22\times(5000\times 10^{-10})\times(1\times 10^3)}{(0-1)} = 6.1\times 10^{-3}m = 6.1\,mm$

Hence, It's order is ≈ 5 *mm*.

Example: 15 A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5 *cm*. Assuming the final image to be at the least distance of distinct vision. The magnification produced by the objective will be

(a) +5 (b) – 5 (c) +6 (d) – 6

Solution (b) Magnification produced by compound microscope $m = m_o \times m_e$

where m_o = ? and $m_e = \left(1 + \frac{D}{f_e}\right) = 1 + \frac{25}{5} = 6 \Rightarrow 30 = -m_o \times 6 \Rightarrow m_o = -5$.

Tricky Example 1 : A man is looking at a small object placed at his least distance of distinct vision. Without changing his position and that of the object he puts a simple microscope of magnifying power 10 *X* and just sees the clear image again. The angular magnification obtained is

(a) 2.5 (b) 10.0 (c) 5.0 (d) 1.0

Solution : (d) Angular magnification $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{I/D}{O/D} = \frac{I}{O}$

Since image and object are at the same position, $\frac{I}{O} = \frac{v}{u} = 1 \Rightarrow$ Angular magnification = 1

Tricky Example 2: A compound microscope is used to enlarge an object kept at a distance 0.03*m* from it's objective which consists of several convex lenses in contact and has focal length 0.02*m*. If a lens of focal length 0.1*m* is removed from the objective, then by what distance the eye-piece of the microscope must be moved to refocus the image

(a) 2.5 *cm* (b) 6 *cm* (c) 15 *cm* (d) 9 *cm*

Solution : (d) If initially the objective (focal length F_o) forms the image at distance v_o then $v_o = \frac{u_o f_o}{u_o - f_o} = \frac{3 \times 2}{3 - 2} = 6\ cm$

Now as in case of lenses in contact $\frac{1}{F_o} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... = \frac{1}{f_1} + \frac{1}{F_o'}$ $\left\{\text{where } \frac{1}{F_o'} = \frac{1}{f_2} + \frac{1}{f_3} + ...\right\}$

So if one of the lens is removed, the focal length of the remaining lens system

$\frac{1}{F_o'} = \frac{1}{F_0} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10} \Rightarrow F_o' = 2.5\ cm$

This lens will form the image of same object at a distance v_o' such that $v_o' = \frac{u_o F_o'}{u_o - F_o'} = \frac{3 \times 2.5}{(3 - 2.5)} = 15\ cm$

So to refocus the image, eye-piece must be moved by the same distance through which the image formed by the objective has shifted *i.e.* 15 – 6 = 9 *cm*.

Assignment

Human eye

80. Near and far points of human eye are **[EAMCET (Med.) 1995; MP PET 2001; Bihar CECE 2004]**

(a) 25 *cm* and infinite (b) 50 *cm* and 100 *cm* (c) 25 *cm* and 50 *cm* (d) 0 *cm* and 25 *cm*

81. A defective eye cannot see close objects clearly because their image is formed **[MP PET 2003]**

(a) On the eye lens (b) Between eye lens and retina

(c) On the retina (d) Beyond retina

82. Retina of eye acts like ..... of camera **[AFMC 2003]**

(a) Shutter (b) Film (c) Lens (d) None of these

83. A person who can see things most clearly at a distance of 10 *cm*. Requires spectacles to enable him to see clearly things at a distance of 30 *cm*. What should be the focal length of the spectacles **[BHU 2003]**

(a) 15 *cm* (concave) (b) 15 *cm* (convex) (c) 10 *cm* (d) 0

84. An astronaut is looking down on earth's surface from a space shuttle at an altitude of $400\,km$. Assuming that the astronaut's pupil diameter is $5\,mm$ and the wavelength of visible light is $500\,nm$. The astronaut will be able to resolve linear object of the size of about **[AIIMS 2003]**

(a) 0.5 *m* (b) 5 *m* (c) 50 *m* (d) 500 *m*

85. A person uses a lens of power + 3*D* to normalise vision. Near point of hypermetropic eye is **[CPMT 2002]**

(a) 1 *m* (b) 1.66 *m* (c) 2 *m* (d) 0.66 *m*

86. The separation between two microscopic particles is measured P_A and P_B by two different lights of wavelength 2000 Å and 3000 Å respectively, then **[AIEEE 2002]**

(a) $P_A > P_B$ (b) $P_A < P_B$ (c) $P_A < 3/2 P_B$ (d) $P_A = P_B$

87. To remove myopia (short sightedness) a lens of power 0.66 *D* is required. The distant point of the eye is approximately **[MP PMT 2001]**

(a) 100 *cm* (b) 150 *cm* (c) 50 *cm* (d) 25 *cm*

88. A person suffering from 'presbyopia' should use **[MP PET 2001]**

(a) A concave lens (b) A convex lens

(c) A bifocal lens whose lower portion is convex (d) A bifocal lens whose upper portion is convex

89. The resolving limit of healthy eye is about **[MP PET 1999; RPMT 1999; AIIMS 2001]**

(a) 1′ (b) 1″ (c) 1° (d) $\frac{1}{60}''$

90. A person uses spectacles of power + 2*D*. He is suffering from **[MP PET 2000]**

(a) Short sightedness or myopia (b) Long sightedness or hypermetropia

(c) Presbyopia (d) Astigmatism

91. The hyper metropia is a **[CBSE PMT 2000]**

(a) Short-side defect (b) Long-side defect

(c) Bad vision due to old age (d) None of these

92. A man cannot see clearly the objects beyond a distance of 20 *cm* from his eyes. To see distant objects clearly he must use which kind of lenses and of what focal length **[MP PMT 2000]**

(a) 100 *cm* convex (b) 100 *cm* concave (c) 20 *cm* convex (d) 20 *cm* concave

93. An eye specialist prescribes spectacles having a combination of convex lens of focal length 40 *cm* in contact with a concave lens of focal length 25 *cm*. The power of this lens combination in diopters is **[IIT 1997 Cancelled; DPMT 2000]**

(a) + 1.5 (b) –1.5 (c) +6.67 (d) –6.67

94. Two parallel pillars are 11 *km* away from an observer. The minimum distance between the pillars so that they can be seen separately will be **[RPET 1997; RPMT 2000]**

(a) 3.2 *m* (b) 20.8 *m* (c) 91.5 *m* (d) 183 *m*

95. A person cannot see objects clearly beyond 2.0 *m*. The power of lens required to correct his vision will be **[MP PMT/PET 1998; JIPMER 2000; KCET (Engg./Med.) 2000]**

(a) + 2.0 *D* (b) – 1.0 *D* (c) + 1.0 *D* (d) – 0.5 *D*

96. When objects at different distances are seen by the eye, which of the following remains constant **[MP PMT 1999]**

(a) The focal length of the eye lens (b) The object distance from the eye lens

(c) The radii of curvature of the eye lens (d) The image distance from the eye lens

97. A person wears glasses of power –2.0 D. The defect of the eye and the far point of the person without the glasses will be **[MP PMT 1999]**

(a) Nearsighted, 50 cm (b) Farsighted, 50 cm (c) Nearsighted, 250 cm (d) Astigmatism, 50 cm

98. A person is suffering from the defect astigmatism. Its main reason is **[MP PMT 1997]**

(a) Distance of the eye lens from retina is increased (b) Distance of the eye lens from retina is decreased

(c) The cornea is not spherical (d) Power of accommodation of the eye is decreased

99. Myopia is due to **[AFMC 1996]**

(a) Elongation of eye ball (b) Irregular change in focal length

(c) Shortening of eye ball (d) Older age

100. Human eye is most sensitive to visible light of the wavelength **[CPMT 1996]**

(a) 6050 $\mathring{A}$ (b) 5500 $\mathring{A}$ (c) 4500 $\mathring{A}$ (d) 7500 $\mathring{A}$

101. Match the List I with the List II from the combinations shown **[ISM Dhanbad 1994]**

(I) Presbiopia (A) Sphero-cylindrical lens

(II) Hypermetropia (B) Convex lens of proper power may be used close to the eye

(III) Astigmatism (C) Concave lens of suitable focal length

(IV) Myopia (D) Convex spectacle lens of suitable focal length

(a) I-A; II-C; III-B; IV-D (b) I-B; II-D; III-C; IV-A (c) I-D; II-B; III-A; IV-C (d) I-D; II-A; III-C; IV-B

102. The human eye has a lens which has a **[MP PET 1994]**

(a) Soft portion at its centre (b) Hard surface

(c) Varying refractive index (d) Constant refractive index

103. A man with defective eyes cannot see distinctly object at the distance more than 60 cm from his eyes. The power of the lens to be used will be **[MP PMT 1994]**

(a) + 60D (b) – 60D (c) – 1.66D (d) $\frac{1}{1.66}D$

104. A person's near point is 50 cm and his far point is 3 m. Power of the lenses he requires for

(i) Reading and (ii) For seeing distant stars are **[MP PMT 1994]**

(a) – 2D and 0.33D (b) 2D and – 0.33D (c) – 2D and 3D (d) 2D and – 3D

105. The focal length of a simple convex lens used as a magnifier is 10 cm. For the image to be formed at a distance of distinct vision (D = 25 cm), the object must be placed away from the lens at a distance of **[CPMT 1991]**

(a) 5 cm (b) 7.14 cm (c) 7.20 cm (d) 16.16 cm

106. A person is suffering from myopic defect. He is able to see clear objects placed at 15 cm. What type and of what focal length of lens he should use to see clearly the object placed 60 cm away **[MP PMT 1991]**

(a) Concave lens of 20 cm focal length (b) Convex lens of 20 cm focal length

(c) Concave lens of 12 cm focal length (d) Convex lens of 12 cm focal length

107. A person can see a thing clearly when it is at a distance of 1 *metre* only. If he wishes to see a distance star, he needs a lens of focal length **[MP PET 1990]**

(a) +100 cm (b) – 100 cm (c) +50 cm (d) –50 cm

108. A man suffering from myopia can read a book placed at 10 cm distance. For reading the book at a distance of 60 cm with relaxed vision, focal length of the lens required will be **[MP PMT 1989]**

(a) 45 cm (b) – 20 cm (c) –12 cm (d) 30 cm

109. A person can see clearly objects at 100 cm distance. If he wants to see objects at 40 cm distance, then the power of the lens he shall require is **[MP PET 1989]**

(a) +1.5 D (b) – 1.5 D (c) +3.0 D (d) –3.0 D

110. If the distance of the far point for a myopia patient is doubled, the focal length of the lens required to cure it will become **[MP PET 1989]**

(a) Half (b) Double

(c) The same but a convex lens (d) The same but a concave lens

111. Image is formed for the short sighted person at **[AFMC 1988]**

(a) Retina (b) Before retina (c) Behind the retina (d) Image is not formed at all

112. A man who cannot see clearly beyond 5 *m* wants to see stars clearly. He should use a lens of focal length **[MP PET/PMT 1988]**

(a) – 100 *metre* (b) + 5 *metre* (c) – 5 *metre* (d) Very large

113. Far point of myopic eye is 250 *cm*, then the focal length of the lens to be used will be **[CPMT 1986; DPMT 2002]**

(a) + 250 *cm* (b) – 250 *cm* (c) + 250/9 *cm* (d) – 250/9 *cm*

114. One can take pictures of objects which are completely invisible to the eye using camera film which are invisible to **[MNR 1985]**

(a) Ultra-violet rays (b) Sodium light (c) Visible light (d) Infra-red rays

115. In human eye the focussing is done by **[CPMT 1983]**

(a) To and fro movement of eye lens (b) To and fro movement of the retina

(c) Change in the convexity of the lens surface (d) Change in the refractive index of the eye fluids

116. The minimum light intensity that can be perceived by the eye is about $10^{-10}\ watt/metre^2$. The number of photons of wavelength $5.6\times10^{-7}\ metre$ that must enter per second the pupil of area $10^{-4}\ metre^2$ for vision, is approximately equal to $(h = 6.6\times10^{-34}\ joule-\text{sec})$ **[NCERT 1982]**

(a) 3×10^2 *photons* (b) 3×10^6 *photons* (c) 3×10^4 *photons* (d) 3×10^5 *photons*

117. A far sighted man who has lost his spectacles, reads a book by looking through a small hole (3-4 *mm*) in a sheet of paper. The reason will be **[CPMT 1977]**

(a) Because the hole produces an image of the letters at a longer distance

(b) Because in doing so, the focal length of the eye lens is effectively increased

(c) Because in doing so, the focal length of the eye lens is effectively decreased

(d) None of these

118. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is

(a) Always strained in looking at an object (b) Strained for objects at large distances only

(c) Strained for objects at short distances only (d) Unstrained for all distances

119. The focal length of a normal eye-lens is about

(a) 1 *mm* (b) 2 *cm* (c) 25 *cm* (d) 1

120. The distance of the eye-lens from the retina is *x*. For normal eye, the maximum focal length of the eye-lens is

(a) $= x$ (b) $< x$ (c) $> x$ (d) $= 2x$

121. A man wearing glasses of focal length +1*m* can clearly see beyond 1*m*

(a) If he is farsighted (b) If he is nearsighted (c) If his vision is normal (d) In each of these cases

122. The near point of a person is 50 *cm* and the far point is 1.5 *m*. The spectacles required for reading purpose and for seeing distance are respectively

(a) $+2D, -\left(\frac{2}{3}\right)D$ (b) $+\left(\frac{2}{3}\right)D-2D$ (c) $-2D, +\left(\frac{2}{3}\right)D$ (d) $-\left(\frac{2}{3}\right)D+2D$

123. A man, wearing glasses of power +2D can read clearly a book placed at a distance of 40 *cm* from the eye. The power of the lens required so that he can read at 25 *cm* from the eye is

(a) +4.5 D (b) +4.0 D (c) +3.5 D (d) +3.0 D

124. A person can see clearly between 1 *m* and 2*m*. His corrective lenses should be

(a) Bifocals with power –0.5D and additional +3.5D (b) Bifocals with power –1.0D and additional +3.0 D

(c) Concave with power 1.0 D (d) Convex with power 0.5 D

125. While reading the book a man keeps the page at a distance of 2.5 *cm* from his eye. He wants to read the book by holding the page at 25 *cm*. What is the nature of spectacles one should advice him to use to completely cure his eye sight

(a) Convex lens of focal length 25 *cm* (b) Concave lens of focal length 25 *cm*

(c) Convex lens of focal length 2.5 *cm* (d) Concave lens of focal length 2.5 *cm*

126. The blades of a rotating fan can not be distinguished from each other due to

(a) Parallex (b) Power of accommodation (c) Persistence of vision (d) Binocular vision

127. Aperture of the human eye is 2 *mm*. Assuming the mean wavelength of light to be 5000 Å, the angular resolution limit of the eye is nearly

(a) 2 *minutes* (b) 1 *minute* (c) 0.5 *minute* (d) 1.5 *minutes*

128. If there had been one eye of the man, then

(a) Image of the object would have been inverted (b) Visible region would have decreased

(c) Image would have not been seen three dimensional (d) (b) and (c) both

129. A man can see the object between 15*cm* and 30*cm*. He uses the lens to see the far objects. Then due to the lens used, the near point will be at

(a) $\frac{10}{3}$ *cm* (b) 30 *cm* (c) 15 *cm* (d) $\frac{100}{3}$ *cm*

130. A presbyopic patient has near point as 30 *cm* and far point as 40 *cm*. The dioptric power for the corrective lens for seeing distant objects is

(a) 40 D (b) 4 D (c) 2.5 D (d) 0.25 D

131. A man swimming under clear water is unable to see clearly because

(a) The size of the aperture decreases (b) The size of the aperture increases

(c) The focal length of eye lens increases (d) The focal length of eye lens decreases

132. The distance between retina and eye-lens in a normal eye is 2.0 *cm*. The accommodated power of eye lens range from

(a) 45 D to 50 D (b) 50 D to 54 D (c) 10 D to 16 D (d) 5 D to 8 D

133. If the eye is taken as a spherical ball of radius 1 *cm*, the range of accommodated focal length of eye-lens is

(a) 1.85 *cm* to 2.0 *cm* (b) 1.0 *cm* to 2.8 *cm* (c) 1.56 *cm* to 2.5 *cm* (d) 1.6 *cm* to 2.0 *cm*

134. A person cannot read printed matter within 100 *cm* from his eye. The power of the correcting lens required to read at 20 *cm* from his eye if the distance between the eye lens and the correcting lens is 2 *cm* is

(a) 4.8 D (b) 1.25 D (c) 4.25 D (d) 4.55 D

135. A student having –1.5 D spectacles uses a lens of focal length 5 *cm* as a simple microscope to read minute scale divisions in the laboratory. The least distance of distinct vision without glasses is 20 *cm* for the student. The maximum magnifying power he gets with spectacles on is

(a) 6 (b) 9 (c) 5 (d) 4

Microscope

136. In a compound microscope the object of f_o and eyepiece of f_e are placed at distance L such that L equals **[Kerala PMT 2004]**

(a) $f_o + f_e$ (b) $f_o - f_e$

(c) Much greater than f_o or f_e (d) Need not depend either value of focal lengths

137. In a simple microscope, if the final image is located at infinity then its magnifying power is **[CPMT 1985; MP PMT 2004]**

(a) $\frac{25}{f}$ (b) $\frac{D}{25}$ (c) $\frac{f}{25}$ (d) $\frac{f}{D^{+1}}$

138. In a simple microscope, if the final image is located at 25 *cm* from the eye placed close to the lens, then the magnifying power is

[BVP 2003]

(a) $\frac{25}{f}$ (b) $1+\frac{25}{f}$ (c) $\frac{f}{25}$ (d) $\frac{f}{25}+1$

139. The maximum magnification that can be obtained with a convex lens of focal length 2.5 *cm* is (the least distance of distinct vision is 25 *cm*) **[MP PET 2003]**

(a) 10 (b) 0.1 (c) 62.5 (d) 11

140. In a compound microscope, the intermediate image is **[IIT-JEE (Screening) 2000; AIEEE 2003]**

(a) Virtual, erect and magnified (b) Real, erect and magnified

(c) Real, inverted and magnified (d) Virtual, erect and reduced

141. A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is **[Kerala PMT 2002]**

(a) 10 (b) 20 (c) 50 (d) 25

142. Wavelength of light used in an optical instrument are $\lambda_1 = 4000\ Å$ and $\lambda_2 = 5000\ Å$, then ratio of their respective resolving power (corresponding to λ_1 and λ_2) is **[AIEEE 2002]**

(a) 16 : 25 (b) 9 : 1 (c) 4 : 5 (d) 5 : 4

143. The angular magnification of a simple microscope can be increased by increasing **[Orissa JEE 2002]**

(a) Focal length of lens (b) Size of object (c) Aperture of lens (d) Power of lens

144. The magnification produced by the objective lens and the eye lens of a compound microscope are 25 and 6 respectively. The magnifying power of this microscope is **[Manipal MEE 1995; DPMT 2002]**

(a) 19 (b) 31 (c) 150 (d) $\sqrt{150}$

145. The length of the compound microscope is 14 *cm*. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 *cm*, then the object distance for objective lens will be **[Pb. PMT 2002]**

(a) 1.8 *cm* (b) 1.5 *cm* (c) 2.1 *cm* (d) 2.4 *cm*

146. The magnifying power of a simple microscope is 6. The focal length of its lens in *metres* will be, if least distance of distinct vision is 25 *cm* **[MP PMT 2001]**

(a) 0.05 (b) 0.06 (c) 0.25 (d) 0.12

147. Relative difference of focal lengths of objective and eye lens in the microscope and telescope is given as

[MH CET (Med.) 2001]

(a) It is equal in both (b) It is more in telescope (c) It is more in microscope (d) It may be more in any one

148. Three objective focal lengths (f_o) and two eye piece focal lengths (f_e) are available for a compound microscope. By combining these two, the magnification of microscope will be maximum when **[RPMT 2001]**

(a) $f_o = f_e$ (b) $f_o >> f_e$ (c) f_o and f_e both are small (d) $f_o >> f_e$

149. If the red light is replaced by blue light illuminating the object in a microscope the resolving power of the microscope

[DCE 2001]

(a) Decreases (b) Increases (c) Gets halved (d) Remains unchanged

150. In case of a simple microscope, the object is placed at **[UPSEAT 2000]**

(a) Focus f of the convex lens (b) A position between f and $2f$ (c) Beyond $2f$ (d) Between the lens and f

151. In a compound microscope cross-wires are fixed at the point **[EAMCET (Engg.) 2000]**

(a) Where the image is formed by the objective (b) Where the image is formed by the eye-piece

(c) Where the focal point of the objective lies (d) Where the focal point of the eye-piece lies

152. The length of the tube of a microscope is 10 *cm*. The focal lengths of the objective and eye lenses are 0.5 *cm* and 1.0 *cm*. The magnifying power of the microscope is about **[MP PMT 2000]**

(a) 5 (b) 23 (c) 166 (d) 500

153. Least distance of distinct vision is 25 *cm*. Magnifying power of simple microscope of focal length 5 *cm* is **[EAMCET (Engg.) 1995; Pb. PMT 1999]**

(a) 1/5 (b) 5 (c) 1/6 (d) 6

154. The objective of a compound microscope is essentially **[SCRA 1998]**

(a) A concave lens of small focal length and small aperture (b) Convex lens of small focal length and large aperture

(c) Convex lens of large focal length and large aperture (d) Convex lens of small focal length and small aperture

155. For relaxed eye, the magnifying power of a microscope is **[CBSE PMT 1998]**

(a) $-\frac{v_o}{u_o}\times\frac{D}{f_e}$ (b) $-\frac{v_o}{u_o}\times\frac{f_e}{D}$ (c) $\frac{u_o}{v_o}\times\frac{D}{f_e}$ (d) $\frac{u_o}{v_o}\times\left(-\frac{D}{f_e}\right)$

156. A person using a lens as a simple microscope sees an **[AIIMS 1998]**

(a) Inverted virtual image (b) Inverted real magnified image

(c) Upright virtual image (d) Upright real magnified image

157. The focal length of the objective lens of a compound microscope is **[CPMT 1985; MNR 1986; MP PET 1997]**

(a) Equal to the focal length of its eye piece (b) Less than the focal length of eye piece

(c) Greater than the focal length of eye piece (d) Any of the above three

158. To produce magnified erect image of a far object, we will be required along with a convex lens, is **[MNR 1983; MP PAT 1996]**

(a) Another convex lens (b) Concave lens (c) A plane mirror (d) A concave mirror

159. An object placed 10 *cm* in front of a lens has an image 20 *cm* behind the lens. What is the power of the lens (in *dioptres*) **[MP PMT 1995]**

(a) 1.5 (b) 3.0 (c) – 15.0 (d) +15.0

160. Resolving power of a microscope depends upon **[MP PET 1995]**

(a) The focal length and aperture of the eye lens (b) The focal lengths of the objective and the eye lens

(c) The apertures of the objective and the eye lens (d) The wavelength of light illuminating the object

161. If the focal length of the objective lens is increased then **[MP PMT 1994]**

(a) Magnifying power of microscope will increase but that of telescope will decrease

(b) Magnifying power of microscope and telescope both will increase

(c) Magnifying power of microscope and telescope both will decrease

(d) Magnifying power of microscope will decrease but that of telescope will increase

162. If in compound microscope m_1 and m_2 be the linear magnification of the objective lens and eye lens respectively, then magnifying power of the compound microscope will be **[CPMT 1985; KCET 1994]**

(a) $m_1 - m_2$ (b) $\sqrt{m_1 + m_2}$ (c) $(m_1 + m_2)/2$ (d) $m_1 \times m_2$

163. The magnifying power of a microscope with an objective of 5 *mm* focal length is 400. The length of its tube is 20 *cm*. Then the focal length of the eye-piece is **[MP PMT 1991]**

(a) 200 *cm* (b) 160 *cm* (c) 2.5 *cm* (d) 0.1 *cm*

164. In a compound microscope, if the objective produces an image I_o and the eye piece produces an image I_e, then **[MP PET 1990]**

(a) I_o is virtual but I_e is real (b) I_o is real but I_e is virtual (c) I_o and I_e are both real (d) I_o and I_e are both virtual

165. In an electron microscope if the potential is increased from 20 *kV* to 80 *kV*, the resolving power of the microscope will change from *R* to **[CPMT 1988, 89]**

(a) $R/4$ (b) $4R$ (c) $2R$ (d) $R/2$

166. When the length of a microscope tube increases, its magnifying power **[MNR 1986]**

(a) Decreases (b) Increases (c) Does not change (d) May decrease or increase

167. An electron microscope is superior to an optical microscope in **[CPMT 1984]**

(a) Having better resolving power (b) Being easy to handle

(c) Low cost (d) Quickness of observation

168. In a compound microscope magnification will be large, if the focal length of the eye piece is **[CPMT 1984]**

(a) Large (b) Smaller (c) Equal to that of objective (d) Less than that of objective

169. An electron microscope gives better resolution than optical microscope because **[CPMT 1982]**

(a) Electrons are abundant (b) Electrons can be focused nicely

(c) Effective wavelength of electron is small (d) None of these

170. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5*X* before his eyes. The angular magnification achieved is

(a) 5 (b) 2.5 (c) 1 (d) 0.2

171. The focal length of the objective of a compound microscope is f_0 and its distance from the eyepiece is *L*. The object is placed at a distance *u* from the objective. For proper working of the instrument

(a) $L < u$ (b) $L > u$ (c) $f_0 < L < 2f_0$ (d) $L > 2f_0$

172. Find the maximum magnifying power of a compound microscope having a 25 *diopter* lens as the objective, a 5 *diopter* lens as the eyepiece and the separation 30 *cm* between the two lenses. The least distance for clear vision is 25 *cm*

(a) 8.4 (b) 7.4 (c) 9.4 (d) 10.4

173. The focal length of the objective and the eye-piece of a microscope are 2 *cm* and 5 *cm* respectively and the distance between them is 30 *cm*. If the image seen by the eye is 25 *cm* from the eye-piece, the distance of the object from the objective is

(a) 0.8 *cm* (b) 2.3 *cm* (c) 0.4 *cm* (d) 1.2 *cm*

174. The focal length of objective and eye-piece of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for relaxed eye is 45, then length of the tube is

(a) 6 *cm* (b) 9 *cm* (c) 12 *cm* (d) 15 *cm*

175. A microscope has an objective of focal length 1.5 *cm* and an eye-piece of focal length 2.5 *cm*. If the distance between objective and eye-piece is 25 *cm*. What is the approximate value of magnification produced for relaxed eye is

(a) 75 (b) 110 (c) 140 (d) 25

176. The magnifying power of a microscope is generally marked as 10*X*, 100 *X*, etc. These markings are for a normal relaxed eye. A microscope marked 10*X* is used by an old man having his near point at 40 *cm*. The magnifying power of the microscope for the old man with his eyes completely relaxed is

(a) 10 (b) 18 (c) 12 (d) 16

177. If the focal length of objective and eye lens are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is

(a) 150 (b) 200 (c) 250 (d) 400

178. A compound microscope is adjusted for viewing the distant image of an object, the distance of the object from the object glass is now slightly increased, what re-adjustment of the instrument would be necessary for obtaining a distant image again

(a) Objective should be moved away from the eye-piece (b) Eye-piece should be moved towards the objective

(c) Both should be moved towards each other (d) Both should be moved away from each other

179. When the object is self-luminous, the resolving power of a microscope is given by the expression

(a) $\frac{2\mu \sin\theta}{\lambda}$ (b) $\frac{\mu \sin\theta}{\lambda}$ (c) $\frac{2\mu \cos\theta}{\lambda}$ (d) $\frac{2\mu}{\lambda}$

180. In a compound microscope, maximum magnification is obtained when the final image

(a) Is formed at infinity (b) Is formed at the least of distinct vision

(c) Coincides with the object (d) Coincides with the objective lens

181. How should people wearing spectacles work with a microscope

(a) They should keep on wearing their spectacles

(b) They should take off their spectacles

(c) They may keep on wearing or take off their spectacles, It makes no difference

(d) They cannot use a microscope at all

Telescope

182. The focal length of the objective and eyepiece of an astronomical telescope for normal adjustments are 50 *cm* and 5 *cm*. The length of the telescope should be **[MP PMT 2004]**

(a) 50 *cm* (b) 55 *cm* (c) 60 *cm* (d) 45 *cm*

183. The resolving power of an astronomical telescope is 0.2 *seconds*. If the central half portion of the objective lens is covered, the resolving power will be **[MP PMT 2004]**

(a) 0.1 *sec* (b) 0.2 *sec* (c) 1.0 *sec* (d) 0.6 *sec*

184. If F_o and F_e are the focal length of the objective and eye-piece respectively of a telescope, then its magnifying power will be

[CPMT 1977, 82, 97, 99, 2003; SCRA 1994; KCET (Engg./Med.) 1999; Pb. PMT 2000; BHU 2001; BCECE 2003, 2004]

(a) $F_o + F_e$ (b) $F_o \times F_e$ (c) F_o / F_e (d) $\frac{1}{2}(F_o + F_e)$

185. The length of an astronomical telescope for normal vision (relaxed eye) (f_o = focal length of objective lens and f_e = focal length of eye lens) is **[EAMCET (Med.) 1995; MP PAT 1996; CPMT 1999; BVP 2003]**

(a) $f_o \times f_e$ (b) $\frac{f_o}{f_e}$ (c) $f_o + f_e$ (d) $f_o - f_e$

186. A telescope of diameter 2*m* uses light of wavelength 5000 Å for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is **[MP PET 2003]**

(a) 4×10^{-4} *rad* (b) 0.25×10^{-6} *rad* (c) 0.31×10^{-6} *rad* (d) 5.0×10^{-3} *rad*

187. The aperture of the objective lens of a telescope is made large so as to **[AIEEE 2003; KCET 2003]**

(a) Increase the magnifying power of the telescope (b) Increase the resolving power of the telescope

(c) Make image aberration less (d) Focus on distant objects

188. The distance of the moon from earth is 3.8×10^5 *km*. The eye is most sensitive to light of wavelength 5500 Å. The separation of two points on the moon that can be resolved by a 500 *cm* telescope will be **[AMU (Med.) 2002]**

(a) 51 *m* (b) 60 *m* (c) 70 *m* (d) All of the above

189. To increase both the resolving power and magnifying power of a telescope **[Kerala PET 2002; KCET (Engg.) 2002]**

(a) Both the focal length and aperture of the objective has to be increased

(b) The focal length of the objective has to be increased

(c) The aperture of the objective has to be increased

(d) The wavelength of light has to be decreased

190. The focal lengths of the objective and eye lenses of a telescope are respectively 200*cm* and 5*cm*. The maximum magnifying power of the telescope will be **[MP PMT/PET 1998; JIPMER 2001, 2002]**

(a) – 40 (b) – 48 (c) – 60 (d) – 100

191. A telescope has an objective of focal length 50 *cm* and an eye piece of focal length 5 *cm*. The least distance of distinct vision is 25 *cm*. The telescope is focussed for distinct vision on a scale 200 *cm* away. The separation between the objective and the eye-piece is **[Kerala PET 2002]**

(a) 75 *cm* (b) 60 *cm* (c) 71 *cm* (d) 74 *cm*

192. In a laboratory four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8*cm* respectively are available. Two of these lenses form a telescope of length 10*cm* and magnifying power 4. The objective and eye lenses are **[MP PMT 2001]**

(a) L_2, L_3 (b) L_1, L_4 (c) L_3, L_2 (d) L_4, L_1

193. Four lenses of focal length + 15 *cm*, + 20 *cm*, + 150 *cm* and + 250 *cm* are available for making an astronomical telescope. To produce the largest magnification, the focal length of the eye-piece should be **[CPMT 2001; AIIMS 2001]**

(a) + 15 *cm* (b) + 20 *cm* (c) + 150 *cm* (d) + 250 *cm*

194. In a terrestrial telescope, the focal length of objective is 90 *cm*, of inverting lens is 5 *cm* and of eye lens is 6 *cm*. If the final image is at 30 *cm*, then the magnification will be **[DPMT 2001]**

(a) 21 (b) 12 (c) 18 (d) 15

195. The focal lengths of the objective and the eyepiece of an astronomical telescope are 20 *cm* and 5 *cm* respectively. If the final image is formed at a distance of 30 *cm* from the eye piece, find the separation between the lenses for distinct vision **[BHU (Med.) 2000]**

(a) 32.4 *cm* (b) 42.3 *cm* (c) 24.3 *cm* (d) 30.24 *cm*

196. Resolving power of reflecting type telescope increases with **[DPMT 2000]**

(a) Decrease in wavelength of incident light (b) Increase in wavelength of incident light

(c) Increase in diameter of objective lens (d) None of these

197. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eye-piece of focal length 2*cm* **[IIT-JEE 1992; Roorkee 2000]**

(a) The distance between the objective and the eye-piece is 16.02 *m*

(b) The angular magnification of the planet is 800

(c) The image of the planet is inverted

(d) All of the above

198. The astronomical telescope consists of objective and eye-piece. The focal length of the objective is **[AIIMS 1998; BHU 2000]**

(a) Equal to that of the eye-piece (b) Greater than that of the eye-piece

(c) Shorter than that of the eye-piece (d) Five times shorter than that of the eye-piece

199. The diameter of the objective of a telescope is a, the magnifying power is m and wavelength of light is λ. The resolving power of the telescope is **[MP PMT 2000]**

(a) $(1.22\lambda)/a$ (b) $(1.22a)/\lambda$ (c) $\lambda m/(1.22a)$ (d) $a/(1.22\lambda m)$

200. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 *cm* and final image is formed at infinity. The focal lengths of the objective and eyepiece are respectively **[IIT-JEE 1989; MP PET 1995; JIPMER 2000]**

(a) 20 *cm*, 16 *cm* (b) 50 *cm*, 10 *cm* (c) 30 *cm*, 6 *cm* (d) 45 *cm*, –9 *cm*

201. A photograph of the moon was taken with telescope. Later on, it was found that a housefly was sitting on the objective lens of the telescope. In photograph **[NCERT 1970; MP PET 1999]**

(a) The image of housefly will be reduced (b) There is a reduction in the intensity of the image

(c) There is an increase in the intensity of the image (d) The image of the housefly will be enlarged

202. The magnifying power of a telescope is M. If the focal length of eye piece is doubled, then the magnifying power will become **[Haryana CEET 1998]**

(a) $2M$ (b) $M/2$ (c) $\sqrt{2}M$ (d) $3M$

203. The minimum magnifying power of a telescope is M. If the focal length of its eyelens is halved, the magnifying power will become **[MP PMT/PET 1998]**

(a) $M/2$ (b) $2M$ (c) $3M$ (d) $4M$

204. The final image in an astronomical telescope is **[EAMCET (Engg.) 1998]**

(a) Real and errect (b) Virtual and inverted (c) Real and inverted (d) Virtual and errect

205. The astronomical telescope has two lenses of focal powers 0.5 D and 20 D. Its magnifying power will be **[CPMT 1997]**

(a) 40 (b) 10 (c) 100 (d) 35

206. An astronomical telescope of ten-fold angular magnification has a length of 44 *cm*. The focal length of the objective is **[CBSE PMT 19**

(a) 4 *cm* (b) 40 *cm* (c) 44 *cm* (d) 440 *cm*

207. A telescope consisting of an objective of focal length 100 *cm* and a single eyes lens of focal length 10 *cm* is focussed on a distant object in such a way that parallel rays emerge from the eye lens. If the object subtends an angle of 2° at the objective, the angular width of the image is **[JIPMER 1997]**

(a) 20° (b) 1/6° (c) 10° (d) 24°

208. When diameter of the aperture of the objective of an astronomical telescope is increased, its **[MP PMT 1997]**

(a) Magnifying power is increased and resolving power is decreased

(b) Magnifying power and resolving power both are increased

(c) Magnifying power remains the same but resolving power is increased

(d) Magnifying power and resolving power both are decreased

209. The focal length of objective and eye-piece of a telescope are 100 *cm* and 5 *cm* respectively. Final image is formed at least distance of distinct vision. The magnification of telescope is **[RPET 1997]**

(a) 20 (b) 24 (c) 30 (d) 36

210. A simple telescope, consisting of an objective of focal length 60 *cm* and single eye lens of focal length 5 *cm* is focussed on a distant object in such a way that parallel rays comes out from the eye lens. If the object subtends an angle 2° at the objective, the angular width of the image **[CPMT 1979; NCERT 1980; MP PET 1992; JIPMER 1997]**

(a) 10° (b) 24° (c) 50° (d) 1/6°

211. The diameter of the objective of the telescope is 0.1 *metre* and wavelength of light is 6000 Å. Its resolving power would be approximately **[MP PET 1997]**

(a) 7.32×10^{-6} *radian* (b) 1.36×10^{6} *radian* (c) 7.32×10^{-5} *radian* (d) 1.36×10^{5} *radian*

212. A Gallilean telescope has objective and eye-piece of focal lengths 200 *cm* and 2 *cm* respectively. The magnifying power of the telescope for normal vision is **[MP PMT 1996]**

(a) 90 (b) 100 (c) 108 (d) 198

213. All of the following statements are correct except **[Manipal MEE 1995]**

(a) The total focal length of an astronomical telescope is the sum of the focal lengths of its two lenses

(b) The image formed by the astronomical telescope is always erect because the effect of the combination of the two lenses its divergent

(c) The magnification of an astronomical telescope can be increased by decreasing the focal length of the eye-piece

(d) The magnifying power of the refracting type of astronomical telescope is the ratio of the focal length of the objective to that of the eye-piece

214. The length of a telescope is 36 *cm*. The focal length of its lenses can be **[Bihar MEE 1995]**

(a) 30 *cm*, 6 *cm* (b) – 30 *cm*, – 6 *cm* (c) – 30 *cm*, – 6 *cm* (d) – 30 *cm*, 6 *cm*

215. The diameter of the objective lens of telescope is 5.0 *m* and wavelength of light is 6000 Å. The limit of resolution of this telescope will be **[MP PMT 1994]**

(a) 0.03 *sec* (b) 3.03 *sec* (c) 0.06 *sec* (d) 0.15 *sec*

216. If tube length of astronomical telescope is 105 *cm* and magnifying power is 20 for normal setting, calculate the focal length of objective **[AFMC 1994]**

(a) 100 *cm* (b) 10 *cm* (c) 20 *cm* (d) 25 *cm*

217. Radio telescope is used to see **[AFMC 1994]**

(a) Distant start and planets (b) Sun and to measure its temperature

(c) Stars and to measures diameters (d) None of these

218. Four lenses with focal lens ± 15 *cm* and ± 150 *cm* are being placed for used as a telescopic objective. The focal length of the lens which produces the largest magnification with a given eye-piece is **[CBSE PMT 1994]**

(a) –15 *cm* (b) +150 *cm* (c) –150 *cm* (d) +15 *cm*

219. The image of a star (effectively a point source) is made by convergent lens of focal length 50 *cm* and diameter of aperture 5.0 *cm*. If the lens is ideal, and the effective wavelength in image formation is taken as $5 \times 10^{-5}\ cm$, the diameter of the image formed will be nearest to **[NSEP 1994]**

(a) Zero (b) $10^{-6}\ cm$ (c) $10^{-5}\ cm$ (d) $10^{-3}\ cm$

220. To increase the magnifying power of telescope (f_o = focal length of the objective and f_e = focal length of the eye lens) **[MP PET/PMT 1988; MP PMT 1992, 94]**

(a) f_o should be large and f_e should be small (b) f_o should be small and f_e should be large

(c) f_o and f_e both should be large (d) f_o and f_e both should be small

221. The limit of resolution of a 100 *cm* telescope $(\lambda = 5.5 \times 10^{-7}\ m)$ is **[BHU 1993]**

(a) 0.14" (b) 0.3" (c) 1′ (d) 1"

222. In a reflecting astronomical telescope, if the objective (a spherical mirror) is replaced by a parabolic mirror of the same focal length and aperture, then **[IIT-JEE 1993]**

(a) The final image will be erect (b) The larger image will be obtained

(c) The telescope will gather more light (d) Spherical aberration will be absent

223. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 *m* and an eyepiece of focal length 2 *cm* **[IIT-JEE 1993]**

(a) The distance between the objective and the eyepiece is 16.02 *m*

(b) The angular magnification of the planet is 800

(c) The image of the planet is inverted

(d) The objective is larger than the eyepiece

224. The average distance between the earth and moon is 38.6×10^4 *km*. The minimum separation between the two points on the surface of the moon that can be resolved by a telescope whose objective lens has a diameter of 5 *m* with λ = 6000 Å is **[MP PMT 1993]**

(a) 5.65 *m* (b) 28.25 *m* (c) 11.30 *m* (d) 56.51 *m*

225. The focal length of the objective and eye piece of a telescope are respectively 60 *cm* and 10 *cm*. The magnitude of the magnifying power when the image is formed at infinity is **[MP PET 1991]**

(a) 50 (b) 6 (c) 70 (d) 5

226. The focal length of an objective of a telescope is 3 *metre* and diameter 15 *cm*. Assuming for a normal eye, the diameter of the pupil is 3 *mm* for its complete use, the focal length of eye piece must be **[MP PET 1989]**

(a) 6 *cm* (b) 6.3 *cm* (c) 20 *cm* (d) 60 *cm*

227. An opera glass (Gallilean telescope) measures 9 *cm* from the objective to the eyepiece. The focal length of the objective is 15 *cm*. Its magnifying power is **[DPMT 1988]**

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

228. The focal length of objective and eye lens of a astronomical telescope are respectively 2 *m* and 5 *cm*. Final image is formed at (i) least distance of distinct vision (ii) infinity. The magnifying power in both cases will be **[MP PMT/PET 1988]**

(a) – 48, – 40 (b) – 40, – 48 (c) – 40, 48 (d) – 48, 40

229. An optical device that enables an observer to see over or around opaque objects, is called **[CPMT 1986]**

(a) Microscope (b) Telescope (c) Periscope (d) Hydrometer

230. The magnifying power of a telescope can be increased by **[CPMT 1979]**

(a) Increasing focal length of the system (b) Fitting eye piece of high power

(c) Fitting eye piece of low power (d) Increasing the distance of objects

231. An achromatic telescope objective is to be made by combining the lenses of flint and crown glasses. This proper choice is **[CPMT 1977]**

(a) Convergent of crown and divergent of flint (b) Divergent of crown and convergent of flint

(c) Both divergent (d) Both convergent

232. An observer looks at a tree of height 15 *m* with a telescope of magnifying power 10. To him, the tree appears **[CPMT 1975]**

(a) 10 times taller (b) 15 times taller (c) 10 times nearer (d) 15 times nearer

233. The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 *m*. The magnification when the image is formed at least distance of distinct vision (D = 25 *cm*) is

(a) 14 (b) 6 (c) 16 (d) 18

234. The objective of a telescope has a focal length of 1.2 *m*. it is used to view a 10.0 *m* tall tower 2 *km* away. What is the height of the image of the tower formed by the objective

(a) 2 *mm* (b) 4 *mm* (c) 6 *mm* (d) 8 *mm*

235. A giant telescope in an observatory has an objective of focal length 19 *m* and an eye-piece of focal length 1.0 *cm*. In normal adjustment, the telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective? The diameter of the moon is 3.5×10^6 *m* and the radius of the lunar orbit round the earth is 3.8×10^8 *m*

(a) 10 *cm* (b) 12.5 *cm* (c) 15 *cm* (d) 17.5 *cm*

236. The aperture of the largest telescope in the world is ≈ 5 *metre*. If the separation between the moon and the earth is $\approx 4 \times 10^5$ *km* and the wavelength of the visible light is $\cong 5000$ Å, then the minimum separation between the objects on the surface of the moon which can be just resolved is

(a) 1 *metre* approximately (b) 10 *metre* approximately (c) 50 *metre* approximately (d) 200 *metre* approximately

237. In Galileo's telescope, magnifying power for normal vision is 20 and power of eye-piece is –20 *D*. Distance between the objective and eye-piece should be

(a) 90 *cm* (b) 95 *cm* (c) 100 *cm* (d) 105 *cm*

238. The least resolve angle by a telescope using objective of aperture 5 *m* and light of wavelength = 4000 *A.U.* is nearly

(a) $\frac{1}{50}^{\circ}$ (b) $\frac{1}{50}$ *sec* (c) $\frac{1}{50}$ *minute* (d) $\frac{1}{500}$ *sec*

239. The limit of resolution of a 10 *cm* telescope for visible light of wavelength 6000 Å is approximately

(a) 0.1 *s* or arc (b) 30° (c) $\left(\frac{1}{6}\right)^{o}$ (d) None of these

240. An eye-piece of a telescope with a magnification of 100 has a power of 20 diopters. The object of this telescope has a power of

(a) 2 *diopters* (b) 0.2 *diopters* (c) 2000 *diopters* (d) 20 *diopters*

241. The Yerkes Observatory telescope has a large telescope with objective of diameter of about 1 *m*. Assuming wavelength of light to be $6\times10^{-7} m$, the angular distance θ between two stars which can just be resolved is

(a) $(7.3\times10^{-7})^{\circ}$ (b) 7.3×10^{-7} *rad* (c) $\frac{1}{40}$ of a second (d) None of these

242. A Galilean telescope measures 9 *cm* from the objective to the eye-piece. The focal length of the objective is 15 *cm*. Its magnifying power is

(a) 2.5 (b) 2/5 (c) 5/3 (d) 0.4

243. For seeing a cricket match, we prefer binoculars to the terrestrial telescope, because

(a) Binoculars give three-dimensional view (b) Terrestrial telescope gives inverted image

(c) To avoid chromatic aberration (d) To have larger magnification

244. A simple two lens telescope has an objective of focal length 50 *cm* and an eye-piece of 2.5 *cm*. The telescope is pointed at an object at a very large distance which subtends at an angle of 1 *milliradian* on the naked eye. The eye piece is adjusted so that the final virtual image is formed at infinity. The size of the real image formed by the objective is

(a) 5 *mm* (b) 1 *mm* (c) 0.5 *mm* (d) 0.1 *mm*

245. The objective of a telescope, after focussing for infinity is taken out and a slit of length *L* is placed in its position. A sharp image of the slit is formed by the eye-piece at a certain distance from it on the other side. The length of this image is *l*, then magnification of telescope is

(a) $\frac{l}{2L}$ (b) $\frac{2L}{l}$ (c) $\frac{l}{L}$ (d) $\frac{L}{l}$

246. An astronomical telescope in normal adjustment receives light from a distant source *S*. The tube length is now decreased slightly

(a) A virtual image of *S* will be formed at a finite distance

(b) No image will be formed

(c) A small, real image of *S* will be formed behind the eye-piece, close to it

(d) A large, real image of *S* will be formed behind the eye-piece, far away from it

247. A telescope consisting of object glass of power + 2 *D* and eye-glass of power + 20 *D* is focussed on an object 1*m* from the object glass. The final image is seen with completely relaxed eye. The magnifying power of the telescope is

(a) 20 (b) 41 (c) 24 (d) 49.2

248. An astronomical telescope and a Galilean telescope use identical objective lenses. They have the same magnification, when both are in normal adjustment. The eye-piece of the astronomical telescope has a focal length *f*

(a) The tube lengths of the two telescopes differ by *f* (b) The tube lengths of the two telescopes differ by 2*f*

(c) The Galilean telescope has a shorter tube length (d) The Galilean telescope has a longer tube length

79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98

a	**d**	**b**	**a**	**c**	**a**	**b**	**b**	**c**	**a**	**b**	**b**	**d**	**b**	**a**	**d**	**d**	**a**	**c**	**a**
99	**100**	**101**	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**
b	**c**	**c**	**c**	**b**	**b**	**c**	**b**	**c**	**a**	**b**	**b**	**c**	**b**	**d**	**c**	**c**	**a**	**a**	**b**
119	**120**	**121**	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**
a	**d**	**a**	**c**	**a**	**d**	**c**	**b**	**d**	**b**	**c**	**c**	**b**	**a**	**d**	**a**	**c**	**a**	**b**	**d**
139	**140**	**141**	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**
c	**b**	**d**	**d**	**c**	**a**	**a**	**b**	**c**	**b**	**d**	**a**	**d**	**d**	**d**	**a**	**c**	**b**	**b**	**d**
159	**160**	**161**	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**
d	**d**	**d**	**c**	**b**	**c**	**a**	**a**	**b**	**c**	**c**	**b,d**	**a**	**b**	**d**	**c**	**d**	**b**	**b**	**a**
179	**180**	**181**	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**
b	**b**	**b**	**c**	**c**	**c**	**c**	**b**	**a**	**a**	**b**	**c**	**d**	**a**	**c**	**c**	**a, c**	**d**	**b**	**d**
199	**200**	**201**	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**
c	**b**	**b**	**b**	**b**	**a**	**b**	**a**	**c**	**b**	**b**	**d**	**b**	**b**	**a**	**a**	**a**	**a**	**b**	**d**
219	**220**	**221**	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**
a	**a**	**d**	**a**	**d**	**b**	**a**	**a**	**a**	**c**	**b**	**a**	**c**	**a**	**c**	**d**	**c**	**b**	**b**	**a**
239	**240**	**241**	**242**	**243**	**244**	**245**	**246**	**247**											
b	**b**	**a**	**a**	**c**	**d**	**a**	**b**	**b, c**											