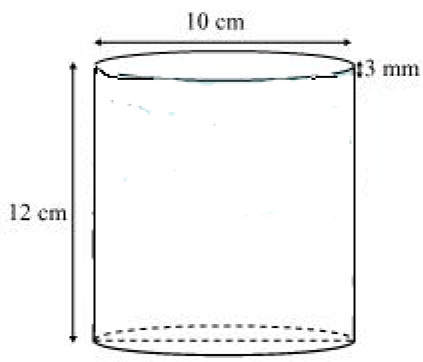
Optional Exercise – 13.5 - 10th Mathematics –Surface area and Volume

Question 1: A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Solution: It can be observed that 1 round of wire will cover 3 mm height of cylinder.



Length of wire required in 1 round = Circumference of base of cylinder= $2\pi r = 2\pi \times 5 = 10\pi$

Number of round = Height of cylinder/Demeter of wire = 12/0.3= 40 round

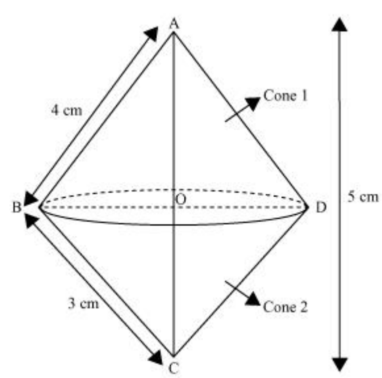
Length of wire in 40 rounds = $40 \times 10\pi$ = 40x10 x3.14=1256cm=12.56m

Radius of wire = 0.3/2 = 0.15cm

Volume of wire = Area of cross-section of wire × Length of wire = $\pi (0.15)^2 \times 1256$ = 3.14 x 0.0225 x 1256 = 88.736 cm^3

Mass = Volume × Density = 88.736 × 8.88 = 787.979 gm

Question 2: A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of p as found appropriate.)



Solution: The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

AC = $\sqrt{(3^2 + 4^2)}$ = 5 cm

Area of ΔABC = ½ x AC x r =1/2 x BC x AB

$\Rightarrow$ 5 x r = 3 x 4 $\Rightarrow$ 12/5=2.4cm

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= 1/3\ \pi\ r^2H\ + /3\ \pi\ r^2h = 1/3\ \pi\ r^2\ (H\ + h)$

$= 1/3 \times 3.14 \times\ 2.4\ \times 2.4 \times\ 5 =\ 30.14\ cm^3$

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi\ r\ L\ + \pi\ r\ l\ = \pi\ r\ (\ L + l\ )$

= 3.14 x 2.4 (4+3) = = 3.14 x 2.4 x7 = 52.75 cm2

Question 3: A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Solution: Volume of cistern = 150 × 120 × 110 = 1980000 cm3

Volume to be filled in cistern = 1980000 – 129600 = 1850400 cm3

Volume of 1 brick = 22.5 × 7.5 × 6.5 = 1096.875

As each brick absorbs one-seventeenth of its volume,

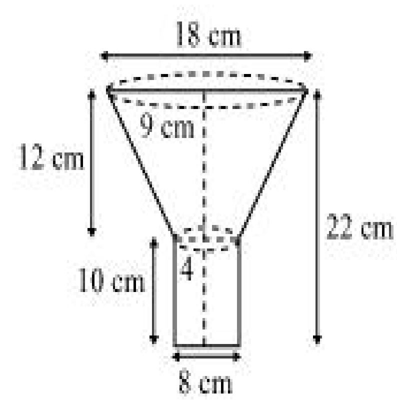
$\Rightarrow$ Volume absorbed by 1 bricks = 1/17 x 1096.875 =64.52

Actual volume of bricks without water = 1096.875 - 64.52 =1032.375

Number of bricks = 1850400/1032.375 =1792.37

Therefore, 1792 bricks were placed in the cistern.

Question 5: An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure).



Solution: Radius (R) of upper circular end of frustum =18/2 = 9 cm

Radius (r) of lower circular end of frustum = Radius of circular end of cylindrical part = 8/2 = 4 cm

Height (H) of frustum = 22 − 10 = 12 cm

Height (h) of cylindrical = 10 cm

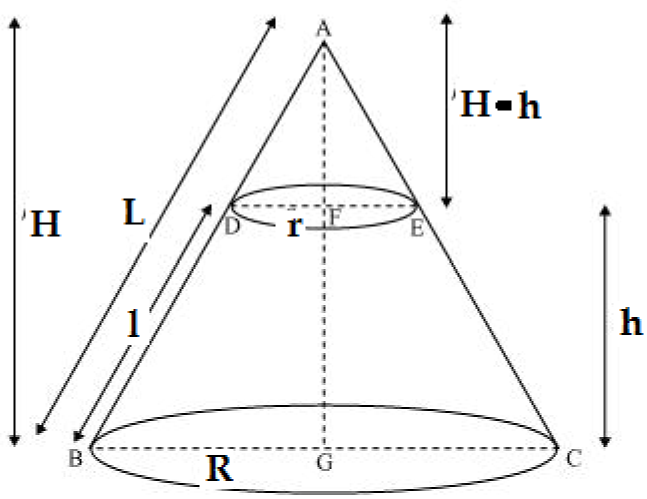
Slant height (l) of frustum $= \sqrt{(R-r)^2 + H^2} = \sqrt{(9-4)^2 + 12^2} = 13$ cm

Area of tin sheet required = CSA of frustum t + CSA of cylindrical

$= \pi (R+r)l + 2\pi rh = \pi [\{(9+4)\times 13\} + \{2\times 4\times 10\}] = 22/7\{169+80\} = 782.57$ cm2

Question 6: Derive the formula for the volume of the frustum of a cone

Solution: Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let R and r be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.



In Δ ABG and Δ ADF, DF||BG

$\angle A = \angle A$ and $\angle AGB = \angle AFD$

$\therefore \Delta ABG \sim \Delta ADF$ (AA similarity)

DF/BG = AF/AG = AD/AB

$\Rightarrow r/R = H-h/H = L-l/L$

$\Rightarrow$ if $r/R = H-h/H \Rightarrow H = (Rh/R-r)$

Volume of frustum of cone = Volume of cone ABC − Volume of cone ADE

$= 1/3\times\pi R^2H - 1/3\times\pi r^2 (H-h)$

$= 1/3\pi\, [R^2H - r^2(H-h)]$

$= 1/3\pi\, [R^2(Rh/R-r) - r^2\{(Rh/R-r)-h\}]$

$= 1/3\pi\, [(R^3h/R-r) - r^2\{(Rh - Rh+rh)/(R-r)\}]$

$= 1/3\pi\, [(R^3h - r^3h)/(R-r)]$

$= 1/3\pi h\, [(R^3 - r^3)/(R-r)]$

$= 1/3\pi h\, [\{(R - r)(R^2 + r^2 - Rr)\}/(R-r)]$

$= 1/3\pi h\, (R^2 + r^2 - R r)$