

REAL NUMBERS

1. Important Concepts/ Result

Theorem 1.1 (Fundamental Theorem of Arithmetic): Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

HCF: Product of the smallest power of each common prime factor in the numbers.

LCM: Product of the greatest power of each prime factor, involved in the numbers.

Theorem 1.2: Let p be a prime number. If p divides a^2 , then p divides a , where a is a positive integer.

Proof: Let the prime factorisation of a be as follows:

$a = p_1 p_2 \dots p_n$, where p_1, p_2, \dots, p_n are primes, not necessarily distinct. Therefore,
 $a^2 = (p_1 p_2 \dots p_n)(p_1 p_2 \dots p_n) = p^2 p^2 \dots p^2 n$. Now, we are given that p divides a^2 .

Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2 .

However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are p_1, p_2, \dots, p_n . So p is one of p_1, p_2, \dots, p_n .

Now, since $a = p_1 p_2 \dots p_n$, and p divides a .

We are now ready to give a proof that $\sqrt{2}$ is irrational. The proof is based on a technique called 'proof by contradiction'.

Theorem 1.3: $\sqrt{2}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{2}$ is rational.

So, we can find integers r and s ($\neq 0$) such that $\sqrt{2} = \frac{r}{s}$. Suppose r and s have a common factor other than 1.

Then, we divide by the common factor to get $\sqrt{2} = \frac{a}{b}$ where a and b are co-prime.

So, $b\sqrt{2} = a$.

Squaring on both sides and rearranging, we get $2b^2 = a^2$. Therefore, 2 divides a^2 .

Now, by Theorem 1.3, it follows that 2 divides a .

So, we can write $a = 2c$ for some integer c .

Substituting for a , we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$.

This means that 2 divides b^2 , and so 2 divides b (again using Theorem 1.3 with $p = 2$). Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.

2. Some Illustrations/ Examples

i) MCQs

1. What is the HCF of the least prime number and the least composite number

- (a) 1 (b) 2 (c) 3 (d) 4

Solution: (b) Least prime number = 2,

Least composite number = 4,

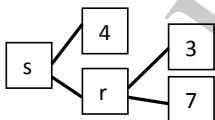
HCF = 2

2. If two positive integers p and q are written as $p = x^5 y^2$, $q = x^3 y^3$, where x, y are prime no's then HCF (p, q)

is : (a) xy (b) $x^2 y^2$ (c) $x^3 y^2$ (d) $x^5 y^3$

Solution: (c) $p = x^5 y^2$, $q = x^3 y^3$, HCF = $x^3 y^2$

3. The values of r and s in the given figure is



(a) $r = 10, s = 14$

(b) $r = 21, s = 84$

(c) $r = 21, s = 25$

(d) $r = 10, s = 40$

Solution: (b) $r = 3 \times 7 = 21$, $s = 4 \times r = 4 \times 21 = 84$,

4. The sum of exponents of prime factors in the prime factorisation of 196 is

- (a) 3 (b) 4 (c) 5 (d) 2

Solution: (b) 4, $196 = 22 \times 72$, Sum = $2 + 2 = 4$

5. The prime factorisation of 96 is:

- (a) $2^5 \times 3$ (b) 2^6 (c) $2^4 \times 3$ (d) $2^4 \times 32$

Solution: (a) $2^5 \times 3$, The prime factorisation of 96 is: $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

ii) Short answer type questions

1. Show that $6 - \sqrt{2}$ is irrational

Solution: Let $6 - \sqrt{2}$ is rational. Then, $6 - \sqrt{2} = \frac{a}{b}$ (where a, b are co-prime integers and $b \neq 0$)

$$6 - \frac{a}{b} = \sqrt{2} \Rightarrow \frac{6b - a}{b} = \sqrt{2}$$

Since a and b are integers, we get $6 - \frac{a}{b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational. Hence our assumption $6 - \sqrt{2}$ is rational is wrong. So $6 - \sqrt{2}$ is irrational.

2. Find the HCF and LCM of 12, 14 and 16 using prime factorisation method.

Solution: $12 = 2 \times 2 \times 3 = 2^2 \times 3^1$, $14 = 2 \times 7 = 2^1 \times 7^1$, $16 = 2 \times 2 \times 2 \times 2 = 2^4$

H.C.F (12, 14, 16) = $2^1 = 2$, LCM (12, 14, 16) = $2^4 \times 3^1 \times 7^1 = 16 \times 21 = 336$

3. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

Solution: $117 > 65$, $117 = 65 \times 1 + 52$, $65 = 52 \times 1 + 13$, $52 = 13 \times 4 + 0$, HCF(65,117) = 13

According to the given, $65m - 117 = 13$, $65m = 117 + 13$, $65m = 130$, $m = 130/65 = 2$

3. Questions for Practice:

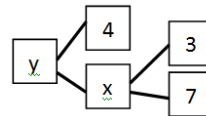
i) MCQs

1. The LCM of two numbers is 2079 and their HCF is 27. If one of the number is 297. The other number is

- (a) 190 (b) 189 (c) 163 (d) 199

2. The values of x and y in the given below figure are:

- (a) $x = 25, y = 75$ (b) $x = 20, y = 80$ (c) $x = 21, y = 84$ (d) $x = 23, y = 92$



3. 5050 as product of its prime factors is.

- (a) $2 \times 5^2 \times 101$ (b) $2 \times 5 \times 5 \times 5 \times 101$ (c) $2 \times 5^3 \times 100$ (d) 2×5^2

4. The expression of 2658 as a product of its prime factors.

- (a) $2 \times 3 \times 443$ (b) $2 \times 4 \times 443$ (c) $2 \times 3 \times 333$ (d) $2 \times 2 \times 443$

5. The ratio between the LCM and HCF of 5, 15 and 20.

- (a) 1:12 (b) 12 : 11 (c) 14 : 1 (d) 12 : 1

6. The HCF of two numbers *a* and *b* is 5 and their LCM is 200. The product *ab* is:

- (a) 1001 (b) 1000 (c) 100 (d) 2000

7. The product of two numbers is 1050 and their HCF is 25. Their LCM is:

- (a) 24 (b) 42 (c) 44 (d) 40

8. The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, the other number is.

- (a) 84 (b) 90 (c) 81 (d) 91

9. The least number that is divisible by all the numbers from 1 to 5 is:

- (a) 70 (b) 60 (c) 80 (d) 90

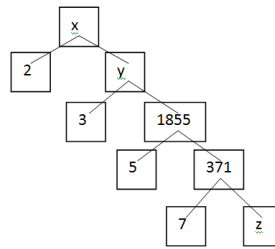
10. **ASSERTION:** 5 is an example of a rational number.

REASON: The square root of all positive integers is irrational numbers.

- (a) Both assertion (A) and reason (R) are true and assertion reason R is the correct explanation of assertion A.
 (b) Both assertion A and reason R are true but reason R is not the correct explanation of assertion A
 (c) Assertion A is true but reason R is false. (d) Assertion A is false but reason R is true.

ii) Short answer type questions(5)

1. Complete the following factor tree and the composite number x.



- Find the least positive integer divisible by first five natural numbers.
- Find the HCF of the numbers: $k, 2k, 3k, 4k$ and $5k$, where k is any positive integer.
- Explain why $11 \times 7 \times 5 \times 3 + 3$ is a composite number.
- The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, find the other number.

ANSWERS

i) MCQs

- (b) 189 2 (c) $x=21, y=84$ 3. (a) $2 \times 5^2 \times 101$ 4. (a) $2 \times 3 \times 443$ 5 (d) $12 : 1$
- (b) 1000 7 (b) 42 8 (d) 91 9.(b) 60 10.(c)

ii) Short answer type questions

- $z=53, x= 11130, y= 5565,$
- LCM= 60 3. HCF= K
- $11 \times 7 \times 5 \times 3$ can be expressed as a product of primes. Therefore, it is a composite numbers.
- Other number = 81

TEST PAPER 1: REAL NUMBERS (CLASS -X)M.M. 20

Section A (Each question carries 1 mark)

- Q.1 Which of the following is a real number?
 (a) 23 (b) 1.234 (c) $\frac{22}{7}$ (d) all the above
- Q.2 If p and Q are two consecutive natural numbers then HCF (a ,b) is
 (a) 1 (b) 2 (c) 3 (d) 4

Section B (Each question carries 2 marks)

- Q.3 Find the prime factorization of the denominator of the rational number 2.345.
- Q.4 Find the LCM and HCF of 24 and 56 by Prime factorization method.
- Q.5 Prove that $7 + \sqrt{5}$ is irrational.

Section C (Each question carries 3 marks)

- Q.6 Check whether 3^n never ends with the digit 0 for any natural number n.
- Q.7 Prove that $\sqrt{7}$ is irrational.
- Q.8 Three alarm clocks ring at intervals of 6, 10, and 14 minutes respectively. If they start ringing together, after how much time will they ring together?
- Q.9 Given that $\text{HCF}(25, 45) = 5$, find their LCM.

TEST PAPER 2 REAL NUMBERS (CLASS -X-)M.M. 30

- Q.1 The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively is
 (a) 13 (b) 65 (c) 875 (d) 1750
- Q.2 If two positive integers p and q can be expressed as $p = a^3b^2$ and $q = ab$ where a and b are prime numbers then LCM (p , q) is
 (a) ab (b) a^2b^2 (c) a^3b^2 (d) ab^2

Q.3 **A (Assertion):** If the product of two numbers is 5780 and their HCF is 17, then their LCM is 340 &

R (Reason): HCF is always a factor of LCM.

(a) Both Assertion (A) and Reason (R) are true & Reason (R) is the correct explanation of Assertion (A)

(b) Both Assertion (A) & Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A)

(c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true

Q.4 If the product of two numbers is 34560 and their HCF IS 24 then find their LCM.

Q.5 Given that $\text{LCM}(91, 26) = 182$. Find the HCF OF 91 and 26.

Q.6 Is it possible for the HCF & LCM of two numbers to be 18 and 378 respectively? Justify your answer.

Q.7 The LCM of two numbers is 9 times their HCF. The sum of LCM and HCF is 500. Find their HCF.

Q.8 Find the least number which is divisible by all the numbers from 1 to 10.

Q.9 Four bells toll together at 9:00 am. They toll after 6,8,10 and 12 seconds respectively. How many times will they toll together again in next 3 hours.

Q.10 What will be the least possible number of planks, if three pieces of timber 42 m, 49 m, and 63 m long have to be divided into planks of the same length.

Q.11 Prove that $\sqrt{3} + 7$ is an irrational.

Q. 12 Check whether 4^n can end with the digit 0 for any natural number n.

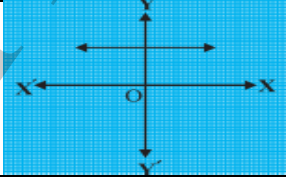
Q 13 There is a circular path around a sports field. Shyama takes 18 minutes to drive one round of the field, while Sunny takes 12 minutes for the same. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?

POLYNOMIALS

Important Concepts/ Result:

1. Polynomial: If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \dots$ a_n are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, ($a_n \neq 0$) is called a polynomial in x .
2. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
3. Polynomial in the form $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$ is called a quadratic polynomial in variable x .
4. A polynomial can have at most the same number of zeros as the degree of the polynomial
5. A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.
6. The graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like or open downwards like depending on whether $a > 0$ or $a < 0$. (These curves are called parabolas.)
7. Relationship between Zeroes and Coefficients of a Polynomial $ax^2 + bx + c$, where α, β are the zeroes of the polynomial
 - (i) Sum of zeroes = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$, $\alpha + \beta = \frac{-b}{a}$
 - (ii) Product of zeroes = $\frac{\text{constant term}}{\text{coefficient of } x^2}$, $\alpha\beta = \frac{c}{a}$
8. Relationship between Zeroes and Coefficients of a Polynomial $ax^3 + bx^2 + cx + d$, where α, β, γ are the zeroes of the polynomial
 - (i) Sum of zeroes = $\frac{-\text{coefficient of } x^2}{\text{coefficient of } x^3}$, $\alpha + \beta + \gamma = \frac{-b}{a}$
 - (ii) Sum of Product of zeroes = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^3}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
 - (iii) Product of zeroes = $\frac{\text{constant term}}{\text{coefficient of } x^3}$, $\alpha\beta\gamma = \frac{-d}{a}$
9. Quadratic polynomial whose zeroes are α and β is $Kx^2 - (\alpha + \beta)x + \alpha\beta$ where k is any real number.

II. Some illustrations/Examples (with solution).

1	The graphs of $y = p(x)$ are given in Fig. , for some polynomials $p(x)$. Find the number of zeroes of $p(x)$. a) 0 b) 1 c) 2 d) 3	
2	The degree of zero polynomial is a) 0 b) 1 c) 2 d) not defined	
3	If -1 is a zero of the polynomial $x^2 - 7x - 8$, then the other zero is (a) 6 (b) 8 (c) -8 (d) 1	
4	Find a quadratic polynomial whose sum and product of zeroes are $\frac{1}{4}$ and -1 respectively. a) $x^2 - \frac{1}{4}x + \frac{1}{4}$ b) $x^2 - \frac{3}{4}x - \frac{1}{4}$ c) $x^2 - \frac{1}{4}x - 1$ d) $x^2 - \frac{1}{4}x - \frac{1}{4}$	
5	Assertion(A): Quadratic polynomial, whose zeroes are 1 and -1 is $x^2 - 1$. Reason(R): Quadratic polynomial whose zeroes are α and β is $k[x^2 - (\alpha + \beta)x + \alpha\beta]$ (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	

	(b) Both Assertion (A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.
6	Find the zeroes of the following quadratic polynomials $6x^2 - 3 - 7x$.
7	Find the zeroes of the following quadratic polynomials $4u^2 + 8u$.
8	If the product of zeros of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeros.
9	Find a quadratic polynomial, whose zeroes are -1 and $\frac{1}{3}$.
10	Find the zeroes of the quadratic polynomial $t^2 - 5$, and verify the relationship between the zeroes and the coefficients.

Answers: 1(a). 2(b). 3(b). 4(c). 5(a), 6) $\frac{3}{2}, \frac{-1}{3}$. 7) 0, -2, 8) a = 3/2, 4 9) $x^2 + \frac{2}{3}x - \frac{1}{3}$,
10) Zeroes are $\sqrt{5}, -\sqrt{5}$, Verification: $\alpha + \beta = \frac{-b}{a} = 0$ and $\alpha\beta = \frac{c}{a} = -5$

Practice Questions: Number of questions should be as mentioned in the table:

1	The graph of the quadratic equation $ax^2 + bx + c$ is an upward open parabola if a) $a > 0$ b) $a < 0$ c) $a = 0$ d) $a = -1$
2	The degree of constant polynomial is a) 0 b) 1 c) 2 d) 3
3	If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then find the value of k. (a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$
4	Find a quadratic polynomial whose sum and product of zeroes are -3 and 2 respectively. a) $x^2 + 3x + 2$. b) $x^2 - x - 6$ c) $2x^2 + 2x - 2$ d) $x^2 + 3x - 2$
5	Find a quadratic polynomial, the sum and product of whose zeroes are $\frac{1}{4}$ and -1 respectively. a) $x^2 - \frac{1}{4}x + \frac{1}{4}$ b) $x^2 - \frac{3}{4}x - \frac{1}{4}$ c) $x^2 - \frac{1}{4}x - 1$ d) $x^2 - \frac{1}{4}x - \frac{1}{4}$
6	Find the quadratic polynomial whose sum and product of zeroes are 0 and $\sqrt{5}$ respectively. a) $x^2 - \sqrt{5}x$ b) $x^2 - 6$ c) $x^2 - \sqrt{5}$, d) $x^2 + \sqrt{5}$
7	If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic equation $2x^2 - x + 8k$, then the value of k is a) 4 b) $\frac{1}{4}$ c) $\frac{-1}{4}$ d) 2
8	The number of polynomial having zeroes -2 and 5 is (a) 1 (b) 2 (c) 3 (d) more than 3
9	Find the zeroes of the following quadratic polynomials $t^2 - 15$. (a) $\sqrt{15}, \sqrt{15}$ (b) $-\sqrt{15}, \sqrt{15}$ (c) $-\sqrt{15}, -\sqrt{15}$ (d) $\sqrt{5}, \sqrt{3}$
10	Assertion(A): quadratic polynomial whose sum and product of zeroes are -3 and 2 respectively is $x^2 + 3x + 2$

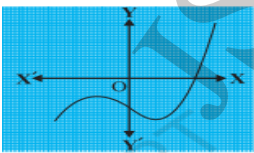
	<p>Reason(R): Quadratic polynomial whose zeroes are α and β is $k[x^2 - (\alpha + \beta)x + \alpha\beta]$</p> <p>(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).</p> <p>(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).</p> <p>(c) Assertion (A) is true but Reason (R) is false.</p> <p>(d) Assertion (A) is false but Reason (R) is true.</p>
11	Find a quadratic polynomial, whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$
12	If α and β are the zeroes of the polynomial $21x^2 - x - 2$, find the quadratic polynomial whose zeroes are 2α and 2β .
13	Form a quadratic polynomial one of whose zeroes is $2 + \sqrt{5}$ and sum of zero is 4.
14	For what value of k, the number -4 is a zero of polynomial $x^2 - x - (2k + 2)$.
15	Find a quadratic polynomial whose sum and product of zeroes are $\sqrt{3}$ and 1 respectively.
16	Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.
17	Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.
18	Find the zeroes of the quadratic polynomial $x^2 + 5x + 6$, and verify the relationship between the zeroes and the coefficients.
19	Find the zeroes of the polynomial $x^2 - 4$ and verify the relationship between the zeroes and the coefficients.
20	If α and β are zeros of $x^2 - x - 2$, find a polynomial whose zeros are $(2\alpha + 1)$ and $(2\beta + 1)$.

IV. ANSWER

1(a) 2(b) 3(c) 4(a) 5(c) 6(d) 7(b) 8(d) 9(b) 10(a)

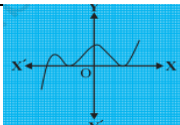
11). $x^2 - 4x + 11$ 2) $21x^2 - 2x - 8$ 3) $x^2 - 4x - 11$ 4) $9x^2 - 4x + 1$ 5) $x^2 - 4x + 1$ 6) -2, -5 Verification: $\alpha + \beta = \frac{-b}{a} = -7$ and $\alpha\beta = \frac{c}{a} = 10$ 7) $\sqrt{3}, -\sqrt{3}$ Verification: $\alpha + \beta = \frac{-b}{a} = 0$ and $\alpha\beta = \frac{c}{a} = -3$ 8) -2, -3 Verification: $\alpha + \beta = \frac{-b}{a} = -5$ and $\alpha\beta = \frac{c}{a} = 6$ 9) 2, -2 Verification: $\alpha + \beta = \frac{-b}{a} = 0$ and $\alpha\beta = \frac{c}{a} = -4$ 10) $x^2 - 4x - 5$

Test-1 (20 marks)

S.Nos.	Questions.	Marks
1	<p>The graphs of $y = p(x)$ are given in Fig. for some polynomials $p(x)$. Find the number of zeroes of $p(x)$.</p> 	2
2	If α and β are zeros of $2x^2 - 7x + 3$, then find the value of $\alpha^2 + \beta^2$.	2
3	If α and β are zeros of $x^2 - 5x + 6$, then find the value of $\alpha + \beta - \alpha\beta$	2

4	Find a quadratic polynomial, whose zeroes are 4 and -5.	2
5	Find the zeroes of the quadratic polynomial $3x^2 - x - 4$, and verify the relationship between the zeroes and the coefficients.	3
6	If α and β are zeroes of $2x^2 - 4x + 5$, then find the value of $\alpha^3 + \beta^3$	3
7	If Zeroes of a quadratic polynomial $x^2 - (a+1)x + b$ are 2 and -3, then Find the value of a and b.	3
8	If α and β are the zeroes of the polynomial $6x^2 - 7x + 2$, then find the quadratic polynomial whose zeroes are α^{-1} and β^{-1} .	3

Test-2 (30marks)

S.Nos.	Questions.	Marks
1	The graphs of $y = p(x)$ are given in Fig. for some polynomials $p(x)$. Find the number of zeroes of $p(x)$ 	2
2	Find a quadratic polynomial, whose zeroes are α and β .	2
3	Find the sum of zeroes of a quadratic polynomial $x^2 - 3x + 2$.	2
4	Find the product of zeroes of a quadratic polynomial $x^2 - x + 6$.	2
5	If one zero of the quadratic polynomial $x^2 + 4x + k$ is 2, then find the value of k.	2
6	If the sum of the zeros of the polynomial $p(x) = 2x^2 + 3kx - 5$ is 6, then find the value of k.	2
7	If zeros of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then find the values of a and b.	3
8	If α and β are the zeros of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m.	3
9	If the sum of squares of zeros of the polynomial $x^2 - 8x + k$ is 40, find the value of k.	3
10	Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.	3
11	Find the zeroes of the polynomial $x^2 - 8x$ and verify the relationship between the zeroes and the coefficients.	3
12	If one zero of the zeroes of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other, find the value of a.	3

Formatted: Font: Bold, Font color: Text 1, Complex Script Font: Bold

Formatted: Centered

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

KEY POINTS:

The general form for a pair of linear equations in two variables x and y .

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1 \neq 0, b_1 \neq 0, a_2 \neq 0, b_2 \neq 0$.

Consistency of system of linear equations in two variables

A pair of values of the variables x and y satisfying each one of the equation is called a **solution of the system**.

Consistent System: A system of linear equations is said to consistent if it has at least one solution.

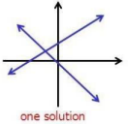
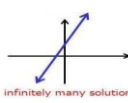
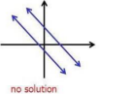
In-consistent System: A system of linear equation is said to be in-consistent if it has no solution.

Algebraic methods of solving a pair of linear equations:

(i) Substitution method

(ii) Elimination Method

Solution by Graphical Representation

Ratio comparison	Graphical Representation	Algebraic Interpretation	Consistent/Inconsistent
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$		Exactly one solution (unique Sol)	Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$		Infinite solution	Dependent(consistent)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$		No solution	Inconsistent

SOLVED EXAMPLES

Ques1. Determine, whether the system of equations is **consistent or in-consistent**.

$$x + y = 14 \text{ and } x - y = 4$$

Solution: Rearrange the given equation like general form: $x + y - 14 = 0$ and $x - y - 4 = 0$

By comparing we get, $a_1 = 1, b_1 = 1, c_1 = -14, a_2 = 1, b_2 = -1, c_2 = -4$

$$\frac{a_1}{a_2} = \frac{1}{1} \quad \frac{b_1}{b_2} = \frac{1}{-1} \quad \frac{c_1}{c_2} = \frac{-14}{-4}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Therefore, system of equations has only one sol

Hence, the system of equations is **consistent**

Ques-2 Solve the following system of equation by **substitution method**: $x + y = 14$ and $x - y = 4$

SOLUTION: Equations are $x + y = 14$ (i) $x - y = 4$ (ii)

from equation (ii) find x in term of y $x = y + 4$ (iii)

substitute this x in (i), we get $(y + 4) + y = 14,$ or $2y + 4 = 14,$ or $2y = 14 - 4$

or $2y = 10$ or $y = 10/2,$ or $y = 5$

Now from equation (iii) we find the value of x with the help of $(y = 5) x = 5 + 4 = 9$

Thus, $x = 9$ and $y = 5$

Ques3 Solve the following system of equation by **elimination method**: $2x + y = 24$ and $x - 2y = 7$

Solution: Given equations are

$$2x + y = 24 \dots\dots\dots (i), \quad x - 2y = 7 \dots\dots\dots(ii)$$

Here, if we multiply by 2 in equation (i) and adding with equation (ii), we get

$$4x + 2y = 48 \dots\dots\dots (i) \quad , x - 2y = 7 \dots\dots(ii), \quad 5x = 55, \quad \text{Or } x = 55/5, \quad \text{Or } x = 11$$

Now, in equation (i) putting the value of x to find y

$$2 \times 11 + y = 24 \text{ Or } 22 + y = 24 \text{ Or } y = 24 - 22 \text{ Or } y = 2, \quad \text{Thus, } x = 11 \text{ and } y = 2$$

Ques 4 To determine the value of **k** (unknown) for which the given system of linear equations has **infinitely many solutions**. $2x + 3y = 5$ and $4x + ky = 10$

Solution: Rearrange the given equation in general form $2x + 3y - 5 = 0$ and $4x + ky - 10 = 0$

By comparing we get, $\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{k}, \frac{c_1}{c_2} = \frac{-5}{-10}$

For infinitely many solutions, we must have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}, \frac{2}{4} = \frac{3}{k} = \frac{-5}{-10}$ Or $k = 2 \times 3, \quad \text{Or } k = 6$

Hence, the given system of equations will have infinitely many solutions, if $k = 6$

Ques5. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units the area increases by 67 square units. Find the dimensions of the rectangle

Solution. Let the length of the rectangle be x units and breadth be y units.

ATQ, $(x - 5)(y + 3) = xy - 9,$ $(x + 3)(y + 2) = xy + 67,$ Solve for x and y.

Ques6 One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital?

Solution: Let amount of money with first person = Rs. x

Let amount of money with second person = Rs. Y

ATQ, $x + 100 = 2(y - 100),$ Also, $y + 10 = 6(x - 10),$ Solve for x and y.

PRACTICE QUESTIONS

MCQ


Q1	The value of k for which $kx + 2y = 5$ and $3x + y = 1$ have unique solution, is: (a) $k = -1$ (b) $k \neq 6$ (c) $k = 6$ (d) $k = 2$	
Q2	The graph of an equation $y = -3$ is a line which will be: (a) parallel to x-axis (b) parallel to y-axis (c) passing through origin (d) on x-axis	
Q3	The pair of equations $3x + 2y = 5, 2x + 3y = 7$ has: (a) no solution (b) one solution (c) many solutions (d) two solutions	
Q4	If $(6, k)$ is a solution of the equation $3x + y = 22$ then, the value of k is: (a) -4 (b) 4 (c) 3 (d) -3	
Q5	ASSERTION & REASON Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A) (C) Assertion (A) is true but reason (R) is false (d) Assertion (A) is false but reason (R) is true. Assertion: The value of k for which the system of linear equations $kx - y = 2$ and	

	$6x-2y=3$ has a unique solution is 3. Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$	
--	--	--

SHORT ANSWER TYPE QUESTIONS

Q6	Find the value of a so that the point (2,9) lies on the line represented by $ax-3y=5$	
Q7	Find the value of k so that the lines $2x-3y = 9$ and $kx - 9y = 18$ will be parallel.	
Q8	Determine the value of k (unknown) for which the following system of equations has infinitely many solutions . $2x + 3y = 5$ and $4x + ky = 10$	
Q9	Solve the following system of equation by elimination method $2x + y = 24$ and $x - 2y = 7$	
Q10	Solve the following system of equation by substitution method: $x + y = 14$ and $x - y = 4$	
Q11	The difference between two numbers is 26 and one number is three times the other. Find them.	
Q12	Find the point of intersection of line $-3x+7y=3$ with x-axis	
Q13	Find the fraction which becomes to $2/3$ when the numerator is increased by 2 and equal to $4/7$ when the denominator is increased by 4.	
Q14	For what value of k will the equation $x + 5y - 7 = 0$ and $4x + 20y + k = 0$ represent coincident lines?	
Q15	Do the equations $y = x$ and $y = x + 3$ represent parallel lines?	

LONG ANSWER TYPE QUESTIONS

Q16	Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less, if one student is less in a row there would be 3 rows more. Find the number of the students in the class	Ans60
Q17	The sum of digits of a two-digit number is 9. If 27 is subtracted from the number, the digits are reversed. find the number	Ans63
Q18	For what values of a and b, will the following pair of linear equations have infinitely many solutions: $x + 2y = 1$, $(a - b)x + (a + b)y = a + b - 2$	
Q19	The perimeter of a rectangle is 44 cm. Its length exceeds twice its breadth by 4cm. Find the length and breadth of the rectangle	
Q20	There are two classrooms A and B. If 15 students are sent from A to B, then the number of students in each classroom will be the same. If 5 students are sent from B to A, then the number of students in A will be double the number of students in B. Represent this information algebraically.	
Q21	<p style="text-align: center;">CASE STUDY QUESTIONS</p> <p>Dipesh bought 3 notebooks and 2 pens for Rs. 80. Lokesh also bought the same types of notebooks and pens as Dipesh. He paid 110 for 4 notebooks and 3 pens</p> 	
i)	<p>Let the cost of one notebook be x and that of pen be y . Which of the following set describe the given problem?</p> <p>(a) $2x + 3y = 80$ and $3x + 4y = 110$</p> <p>(b) $3x + 2y = 80$ and $4x + 3y = 110$</p>	1

ii) iii) iv)	<p>(c) $2x + 3y = 80$ and $4x + 3y = 110$ (d) $3x + 2y = 80$ and $3x + 4y = 110$ What is the exact cost of the notebook? (a) Rs 10 (b) Rs 20 (c) Rs 16 (d) Rs 24</p> <p>What is the exact cost of the pen? (a) Rs 10 (b) Rs 20 (c) Rs 16 (d) Rs 24</p> <p>What is the total cost if they will purchase the same type of 15 notebooks and 12 pens. (a) Rs 410 (b) Rs 200 (c) Rs 420 (d) Rs 240</p>	1 1 2													
Q22	<p>Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m</p>														
	<p>Based on the above information, answer the following questions:</p> <ol style="list-style-type: none"> Form the pair of linear equations in two variables from this situation. Find the length of the outer boundary of the layout. Find the area of each bedroom and kitchen in the layout. Find the area of living room in the layout. Find the cost of laying tiles in kitchen at the rate of Rs. 50 per sq.m 														
Q23	<p>It is common that Governments revise travel fares from time to time based on various factors such as inflation (a general increase in prices and fall in the purchasing value of money) on different types of vehicles like auto, Rickshaws, taxis, Radio cab etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations</p> <table border="1" data-bbox="148 1301 1056 1480"> <thead> <tr> <th>Name of the city</th> <th>Distance travelled (Km)</th> <th>Amount paid (Rs.)</th> </tr> </thead> <tbody> <tr> <td rowspan="2">City A</td> <td>10</td> <td>75</td> </tr> <tr> <td>15</td> <td>110</td> </tr> <tr> <td rowspan="2">City B</td> <td>8</td> <td>91</td> </tr> <tr> <td>14</td> <td>145</td> </tr> </tbody> </table> <p>Refer situation 1</p> <p>1.If the fixed charges of auto rickshaw be Rs x and the running charges be Rs y km/hr, the pair of linear equations representing the situation is</p> <p>a) $x + 10y = 110, x + 15y = 75$ b) $x + 10y = 75, x + 15y = 110$ c) $10x + y = 110, 15x + y = 75$ d) $10x + y = 75, 15x + y = 110$</p> <p>2. A person travels a distance of 50km in City A. The amount he has to pay is</p> <p>a) Rs.155 b) Rs.255 c) Rs.355 d) Rs.450</p>	Name of the city	Distance travelled (Km)	Amount paid (Rs.)	City A	10	75	15	110	City B	8	91	14	145	
Name of the city	Distance travelled (Km)	Amount paid (Rs.)													
City A	10	75													
	15	110													
City B	8	91													
	14	145													

3. What will a person have to pay for travelling a distance of 30km in City B? a) Rs.185 b) Rs.289 c) Rs.275 d) Rs.30	
--	--

ANSWERS

MCQ	Answer	Short Ans Ques	Answers	Long Ans Ques	Answers	Case Study Ques	Answers
Q1	b	Q6	a=16	Q16	60	Q21	i) a ii) b iii) a iv) c v) a
Q2	a	Q7	K=6	Q17	63	Q22	i) $x+y=13$ 2 $x+y=19$ ii) 54 cm iii) $x=6$ $y=7$ iv) 75 sqm v) Rs 1750
Q3	b	Q8	K=6	Q18	a=3, b=1	Q23	i) a ii) d iii) b
Q4	b	Q9	$x=11, y=2$	Q19	Length=16, Breadth=6		
Q5	a	Q10	$x=9, y=5$	Q20	SecA=75 students, SecB=45 students		
		Q11	$x=39, y=13$				
		Q12	$x=-1, y=0$				
		Q13	$\frac{28}{45}$				
		Q14	$k=-28$				
		Q15	Yes parallel lines				

TEST-1 Mathematics- Class-X M.M.:20

SECTION-A

Q1	The solutions of the equation $2x - y - 5 = 0$ are: (a) $x=2, y=-1$ (b) $x=2, y=1$ (c) $x=1, y=-1$ (d) $x=-2, y=1$	1
Q2	The system of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution when: (a) $k=0$ (b) $k \neq 0$ (c) $k=3$ (d) $k \neq 3$	1
Q3	Point (4, 3) lies on the line: (a) $3x + 7y = 27$ (b) $7x + 2y = 47$ (c) $3x + 4y = 24$ (d) $5x + 4y = 1$	1
Q4	The graph of an equation $y = -3$ is a line which will be: (a) parallel to x-axis (b) parallel to y-axis (c) passing through origin (d) on x-axis	1

SECTION-B

Q5	If the lines represented by the pair of linear equations $2x + 5y = 3$, $2(k+2)y + (k+1)x = 2k$ are coincident then, find the value of k.	2
Q6	If $x = a$, $y = b$ is the solution of the pair of equations $x - y = 2$ and $x + y = 4$, then the respective values of a and b are	2
Q7	Solve the pair of linear equations $x + y = 4$ and $2x - 3y = 3$ algebraically using	2

	elimination method	
--	--------------------	--

SECTION-C

Q8	The perimeter of a rectangle is 44 cm. Its length exceeds twice its breadth by 4 cm. Find the length and breadth of the rectangle.	3
Q9	The sum of the digits of a two-digit number is 8. If 18 is added to the number, then the digits interchange their places. Find the number.	3

SECTION-D

Q10	A company placed two orders from two different shops. The first order was for 13 desktops and 4 laptops and the total cost came out to be Rs 487000. The second order was for 6 desktops and 2 laptops and the total cost came out to be Rs 232000. What is the cost of one desktop and of one laptop?	4
-----	--	---

PRACTICE TEST-2, Class-X M.M.:30

SECTION-A

Q1	If $2x + 3y = 0$, $4x - 3y = 0$ then, $x + y$ equals: (a) 0 (b) -1 (c) 1 (d) 2	1
Q2	The pair of equations $3x + 4y = 18$; $4x + \frac{16}{3}y = 24$ has: (a) no solution (b) unique solution (c) infinitely many solutions (d) can't say	1
Q3	Determine the value of k (unknown) for which the given system of equations has infinitely many solutions $5x + 2y = k$ and $10x + 4y = 6$ a) $k = -3$, b) $k \neq 3$ c) $k = 3$ d) $k = 2$	1
Q4	The values of x and y if $23x + 35y = 209$ and $35x + 23y = 197$, are : (a) 3 and 4 (b) 4 and 5 (c) -3 and 2 (d) 5 and 3	1
Q5	ASSERTION & REASON Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false (d) Assertion (A) is false but reason (R) is true. Assertion : If a pair of lines are coincident, then we say that pair is consistent and it has a unique solution. Reason (R) : If a pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations. (a) Both (A) and (R) are true and (R) is the correct explanation of (A) (b) Both (A) and (R) are true but (R) is not the correct explanation of (A) (c) (A) is true but (R) is false (d) (A) is false but (R) is true	1

SECTION-B

Q6		2
----	--	---

	If (6, k) is a solution of the equation $3x + y = 22$ then, find the value of k .	
Q7	If 1 is added in numerator and denominator both, then a fraction changes to 4. If 1 is subtracted from the numerator and denominator both, the fraction changes to 7. Find numerator of the fraction.	2
Q8	Find the value of y obtained on solving the equations $2x + y = 2x - y = \sqrt{8}$.	2
Q9	Half the perimeter of a rectangular garden, whose length is 4 m more than its width is 36 m. Find the dimensions of the garden .	2
Q10	Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current . (a) 4 km/h, 4 km/h (b) 6 km/h, 4 km/h (c) 6 km/h, 6 km/h (d) 4 km/h, 6 km	2
Q11	The coordinates of the point where x-axis and the line $\frac{x}{2} + \frac{y}{3} = 1$ intersect, are: (a) (0, 3) (b) (3, 2) (c) (2, 0) (d) (0, 2)	2

SECTION-C

Q12	A father's age is three times the sum of ages of his two children. After 5 years, his age will be two times the sum of their ages. Find the present age of the father. OR For which values of a and b, will the following pair of linear equations have infinitely many solutions? $x + 2y = 1$; $(a - b)x + (a + b)y = a + b - 2$	3
Q13	There are two classrooms A and B. If 15 students are sent from A to B, then the number of students in each classroom will be the same. If 5 students are sent from B to A, then the number of students in A will be double the number of students in B. Represent this information algebraically	3
Q14	There are two points on a highway A and B. They are 70 km apart. An auto starts from A and another auto starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other they meet in 1 hour. Find how fast the speed of two autos .	3

SECTION-D

Q15	CASE STUDY Mr. RK Agarwal is owner of a famous amusement park in Delhi. Generally he	4
-----	--	---

does not go to park and it is managed by team of staff. The ticket charge for the park is Rs 150 for children and Rs 400 for adults. One day Mr Agrawal decided to random check the park and went there. When he checked the cash counter, he found that 480 tickets were sold and Rs 134500 was collected



(i) Let the number of children visited be x and the number of adults visited be y . Which of the following is the correct system of equation that model the problem?

- (a) $x + y = 480$ and $3x + 8y = 2690$
- (b) $x + 2y = 480$ and $3x + 4y = 2690$
- (c) $x + y = 480$ and $3x + 4y = 2690$
- (d) $x + 2y = 480$ and $3x + 8y = 2690$

(ii) How many children attended?

- (a) 250
- (b) 500
- (c) 230
- (d) 460

(iii) How many adults attended?

- (a) 250
- (b) 500
- (c) 230
- (d) 460

(iv) How much amount collected if 300 children and 350 adults attended?

- (a) Rs 225400
- (b) Rs 154000
- (c) Rs 112500
- (d) Rs 185000

OR

One day total attended children and adults together is 750 and the total amount collected is Rs 212500. What are the number of children and adults attended ?

- (a) (700, 800)
- (b) (350, 400)
- (c) (800, 700)
- (d) (400, 350)

ANSWERS OF TEST-1

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
a	d	c	a	K=3	X=3, y=1	X=3, y=1	L=16 B=6	X=3 y=5	Desktop= Rs 23000 Laptop=Rs47000

ANSWERS OF TEST-2

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
a	c	c	a	d	K=4	Numerator=15	Y=0	L=20 , B=16	Speed of rowing =6km/hr Speed of current =4km/hr

Q11	Q12	Q13	Q14	Q15(i)	Q15(ii)	Q15(iii)	Q15(iv)		
(2,0)	Fathers age=45 yrs	secA=75 secB=45	AutoA=40 km/hr AutoB=30 km/hr	a	c	a	d		

JSUNIL TUTORIAL

QUADRATIC EQUATIONS

I. Important Concepts / Results

A quadratic polynomial of the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are real numbers, is called a quadratic equation when $ax^2 + bx + c = 0$.

Here a and b are the coefficients of x^2 and x respectively and ' c ' is a constant term.

Any value is a solution of a quadratic equation if and only if it satisfies the quadratic equation.

Quadratic formula: The roots, i.e., α and β of a quadratic equation $ax^2 + bx + c = 0$ are given

$$\text{by } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } b^2 - 4ac \geq 0 \text{ or } x = \frac{-b \pm \sqrt{D}}{2a}$$

Here, the value $b^2 - 4ac = D$ is known as the discriminant and is generally denoted by D . ' D ' helps us to determine the nature of roots for a given quadratic equation. Thus $D = b^2 - 4ac$.

The rules are: If $D = 0 \Rightarrow$ The roots are Real and Equal. If $D > 0 \Rightarrow$ The two roots are Real and Unequal.

If $D < 0 \Rightarrow$ No Real roots exist.

If α and β are the roots of the quadratic equation, then Quadratic equation is:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \text{ Or } x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

Examples

i) MCQ

1) 1. Which of the following is not a quadratic equation

(a) $x^2 + 3x - 5 = 0$ (b) $x^2 + x^3 + 2 = 0$ (c) $3 + x + x^2 = 0$ (d) $x^2 - 9 = 0$

Ans:- (b)

:Reason: Since it has degree 3.

2) The polynomial equation $x(x + 1) + 8 = (x + 2)(x - 2)$ is

(a) linear equation (b) quadratic equation (c) cubic equation

(d) bi-quadratic equation

Ans:- (a)

Explanation: We have $x(x + 1) + 8 = (x + 2)(x - 2)$

$$\Rightarrow x^2 + x + 8 = x^2 - 4$$

$$\Rightarrow x^2 + x + 8 - x^2 + 4 = 0$$

$$\Rightarrow x + 12 = 0, \text{ which is a linear equation.}$$

3) The roots of the quadratic equation $6x^2 - x - 2 = 0$ are

(a) $\frac{2}{3}, \frac{1}{2}$

(b) $-\frac{2}{3}, \frac{1}{2}$

(c) $\frac{2}{3}, -\frac{1}{2}$

(d) $-\frac{2}{3}, -\frac{1}{2}$

Ans:- (c)

Explanation: Reason: We have $6x^2 - x - 2 = 0$

$$\Rightarrow 6x^2 + 3x - 4x - 2 = 0$$

$$\Rightarrow 3x(2x + 1) - 2(2x + 1) = 0$$

$$\Rightarrow (2x + 1)(3x - 2) = 0$$

$$\Rightarrow 2x + 1 = 0 \text{ or } 3x - 2 = 0$$

$$\therefore x = -\frac{1}{2}, x = \frac{2}{3}$$

4) If one root of the quadratic equation $2x^2 + kx - 6 = 0$ is 2, the value of k is

(a) 1 (b) -1 (c) 2 (d) -2

Ans: (b)

Reason: Since $x = 2$ is a root of the equation $2x^2 + kx - 6 = 0$

$$\therefore 2(2)^2 + k(2) - 6 = 0$$

$$\Rightarrow 8 + 2k - 6 = 0$$

$$\Rightarrow 2k = -2$$

$$\therefore k = -1$$

5) The roots of the equation $7x^2 + x - 1 = 0$ are

- (a) real and distinct (b) real and equal (c) not real (d) none of these

Answer: (a)

Reason: Here $a = 2$, $b = 1$, $c = -1$

$$\therefore D = b^2 - 4ac = (1)^2 - 4 \times 2 \times (-1) = 1 + 8 = 9 > 0$$

\therefore Roots of the given equation are real and distinct.

Short Answer type Questions:-

1) If the equation $12x^2 + 4kx + 3 = 0$ has real and equal roots, find the value of k

Solution: Here $a = 12$, $b = 4k$, $c = 3$

Since the given equation has real and equal roots

$$\therefore b^2 - 4ac = 0$$

$$\Rightarrow (4k)^2 - 4 \times 12 \times 3 = 0$$

$$\Rightarrow 16k^2 - 144 = 0$$

$$\Rightarrow k^2 = 9$$

$$\Rightarrow k = \pm 3$$

2) Find the roots of the quadratic equation $2x^2 + x - 6 = 0$

Solution: $x^2 - 5x + 2x - 10 = 0$

$$\Rightarrow x(x - 5) + 2(x - 5) = 0$$

$$\Rightarrow (x - 5)(x + 2) = 0$$

Therefore, $x - 5 = 0$ or $x + 2 = 0$

$$\Rightarrow x = 5 \text{ or } x = -2$$

3) Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let us say the two consecutive positive integers are x and $x + 1$.

Therefore, as per the given questions,

$$x^2 + (x + 1)^2 = 365$$

$$\Rightarrow x^2 + x^2 + 1 + 2x = 365$$

$$\Rightarrow 2x^2 + 2x - 364 = 0$$

$$\Rightarrow x^2 + x - 182 = 0$$

$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow x(x + 14) - 13(x + 14) = 0$$

$$\Rightarrow (x + 14)(x - 13) = 0$$

Thus, either, $x + 14 = 0$ or $x - 13 = 0$,

$$\Rightarrow x = -14 \text{ or } x = 13$$

Since the integers are positive, x can be 13 only.

$$\therefore x + 1 = 13 + 1 = 14$$

Therefore, two consecutive positive integers will be 13 and 14.

III. Questions for Practice

i) MCQ

1) The quadratic equation $x^2 + x - 182 = 0$ has degree

- (a) 0 (b) 1 (c) 2 (d) 3

- 2) The equation $(x - 2)^2 + 1 = 2x - 3$ is a
 (a) linear equation (b) quadratic equation (c) cubic equation
 (d) bi-quadratic equation
- 3) The quadratic equation whose roots are 1 and
 (a) $2x^2 + x - 1 = 0$ (b) $2x^2 - x - 1 = 0$ (c) $2x^2 + x + 1 = 0$ (d) $2x^2 - x + 1 = 0$
- 4) The quadratic equation whose one rational root is $3 + \sqrt{2}$ is
 (a) $x^2 - 7x + 5 = 0$ (b) $x^2 + 7x + 6 = 0$ (c) $x^2 - 7x + 6 = 0$ (d) $x^2 - 6x + 7 = 0$
- 5) The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is
 (a) $\pm\sqrt{6}$ (b) ± 4 (c) $\pm 3\sqrt{2}$ (d) $\pm 2\sqrt{6}$
- 6) The roots of the quadratic equation $2x^2 - 2\sqrt{2}x + 1 = 0$ are

- (a) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $\sqrt{2}, \sqrt{2}$
 (c) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}, \frac{1}{\sqrt{2}}$

- 7) The roots of the quadratic equation $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$ is

- (a) a, b (b) -a, b (c) a, -b (d) -a, -b
- 8) The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then
 (a) $2a = b + c$ (b) $2c = a + b$ (c) $b = a + c$ (d) $2b = a + c$
- 9) The sum of the squares of two consecutive natural numbers is 313. The numbers are
 (a) 12, 13 (b) 13, 14 (c) 11, 12 (d) 14, 15
- 10) **Assertion:** If one root of the quadratic equation $6x^2 - x - k = 0$ is $2/3$, then the value of k is 2.
Reason: The quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ has almost two roots.

Directions:

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
 (c) If Assertion is correct but Reason is incorrect.
 (d) If Assertion is incorrect but Reason is correct.

i) Short Answer Type Questions:-

- 1) Find the roots of the quadratic equation $2x^2 - x + \frac{1}{8} = 0$
 2) Find two numbers whose sum is 27 and product is 182.
 3) Solve : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
 4) Find the roots of the quadratic equation $2x^2 - x + 4 = 0$ by using Quadratic formula
 5) Find the roots of the following equations:

$$x - \frac{1}{x} = 3, x \neq 0$$

Answer

Test-1 Quadratic Equations: M.M.: 20

I Choose the correct answer:- 4 x 1=4 marks

The quadratic equation $x^2 + x - 182 = 0$ has degree

- (a) 0 (b) 1 (c) 2 (d) 3

2) The equation $(x - 2)^2 + 1 = 2x - 3$ is a

- (a) linear equation (b) quadratic equation (c) cubic equation
(d) bi-quadratic equation

3) The equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is

- (a) $\pm\sqrt{6}$ (b) ± 4 (c) $\pm 3\sqrt{2}$ (d) $\pm 2\sqrt{6}$

4) The sum of the squares of two consecutive natural numbers is 313. The numbers are

- (a) 12, 13 (b) 13,14 (c) 11,12 (d) 14,15

II. Answer the following : 5 X 2 = 10 Marks

5) Solve : $\sqrt{2x^2 + 7x + 5}\sqrt{2} = 0$

6) Find the roots of the quadratic equation $2x^2 - x + 4 = 0$ by using Quadratic formula

7) Find the roots of the quadratic equation $2x^2 - x + \frac{1}{8} = 0$

8) Find two numbers whose sum is 27 and product is 182.

9) If the equation $12x^2 + 4kx + 3 = 0$ has real and equal roots, find the value of k

III. Answer the following:- 2 x 3 = 6 Marks

10) Find the roots of the following equations:

11) Find two consecutive positive integers, the sum of whose reciprocals is $x - \frac{1}{x} = 3, x \neq 0$ is 365.

12) A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Test-2 Quadratic Equations M.M. : 30

I Choose the correct answer:- 5 x 1 = 5marks

1) If one root of equation $4x^2 - 2x + k - 4 = 0$ is reciprocal of the other. The value of k is:

- (a) -8 (b) 8 (c) -4 (d) 4

2) The roots of quadratic equation $2x^2 + x + 4 = 0$ are:

- (a) Positive and negative (b) Both Positive (c) Both Negative
(d) No real roots

MCQ		Short Answer	
1	c	1	$\frac{1}{4}$ and $\frac{1}{4}$
2	b	2	13 and 14
3	b	3	$-\frac{5}{\sqrt{2}}, -\sqrt{2}$
4	d	4	$\frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$
5	d	5	$\frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$
6	c		
7	d		
8	d		
9	a		
10	a		

3) If $\frac{1}{2}$ is a root of the quadratic equation $x^2 - mx - \frac{5}{4} = 0$, then value of m is:

- (a) 2 (b) -2 (c) -3 (d) 3

4) Which one of the following is not a quadratic equation?

- (a) $(x + 2)^2 = 2(x + 3)$ (b) $x^2 + 3x = (-1)(1 - 3x)^2$
(c) $(x + 2)(x - 1) = x^2 - 2x - 3$ (d) $x^3 - x^2 + 2x + 1 = (x + 1)^3$

5) A quadratic equation $ax^2 + bx + c = 0$ has no real roots, if

- (a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac < 0$ (d) $b^2 - ac < 0$

II. Answer the following : **5 X 2 = 10 Marks**

- 1) Find the roots of the quadratic equation: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
- 2) Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.
- 3) If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.
- 4) The product of two consecutive positive integers is 306. Find the integers.
- 5) If the sum of a number and its reciprocal is $\frac{10}{3}$, find the number.

III. Answer the following:- **5 x 3 = 15Marks**

- 1) If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.
- 2) A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed.
- 3) The hypotenuse of a right triangle is 1 m less than twice the shortest side. If the third side is 1 m more than the shortest side, find the sides of the triangle.32

ARITHMETIC PROGRESSIONS

KEY POINTS

1. An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference.

The general form of an AP is $a, a + d, a + 2d, a + 3d, \dots$

2. A given list of numbers $a_1, a_2, a_3, a_4, a_5, \dots$ is an AP, if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \dots$, give the same value, i.e., $a_{k+1} - a_k$ is the same for different values of k .

3. In an AP with first term a and common difference d , the n th term (or the general term) is given by $a_n = a + (n - 1)d$.

4. The sum of the first n terms of an AP is given by $S_n = \frac{n}{2}[2a + (n - 1)d]$

5. If l is the last term of the finite AP, say the n th term, then the sum of all terms of the AP is given by $S_n = \frac{n}{2}(a + l)$

6. If a, b, c are in AP, then $b = \frac{a+c}{2}$ and b is called the arithmetic mean of a and c

MCQS

1. The sum of the first five multiples of 3 is:

(a) 45 (b) 55 (c) 65 (d) 75

Answer: (a) 45

Explanation: The first five multiples of 3 is 3, 6, 9, 12 and 15

$$a=3 \text{ and } d=3n=5$$

$$\text{Sum, } S_n = n/2[2a+(n-1)d]$$

$$S_5 = 5/2[2(3)+(5-1)3]=5/2[6+12]=5/2[18]=5 \times 9= 45$$

2. The 10th term of the AP: 5, 8, 11, 14, ... is

(a) 32 (b) 35 (c) 38 (d) 185

Answer: (a) 32

Explanation:

Given AP: 5, 8, 11, 14, ...

First term = $a = 5$

Common difference = $d = 8 - 5 = 3$

n th term of an AP = $a_n = a + (n - 1)d$

Now, 10th term = $a_{10} = a + (10 - 1)d$

$$= 5 + 9(3) = 5 + 27 = 32$$

3. In an AP, if $d = -4, n = 7, a_n = 4$, then a is

(a) 6 (b) 7 (c) 20 (d) 28

Answer: (d) 28

Solution; Given, $d = -4, n = 7, a_n = 4$

We know that, $a_n = a + (n - 1)d$

$$4 = a + (7 - 1)(-4)$$

$$4 = a + 6(-4)$$

$$4 = a - 24$$

$$\Rightarrow a = 4 + 24 = 28$$

4. 30th term of the A.P: 10, 7, 4, ..., is

- (a) 97 (b) 77 (c) -77 (d) -87

Answer: (c) -77

Explanation: Given, A.P. = 10, 7, 4, ...

First term, $a = 10$, Common difference, $d = a_2 - a_1 = 7 - 10 = -3$

As we know, for an A.P., $a_n = a + (n-1)d$

Putting the values;

$$a_{30} = 10 + (30-1)(-3)$$

$$a_{30} = 10 + (29)(-3)$$

$$a_{30} = 10 - 87 = -77$$

5. The missing terms in AP: __, 13, __, 3 are:

- (a) 11 and 9 (b) 17 and 9 (c) 18 and 8 (d) 18 and 9

Answer: (c)

Explanation: $a_2 = 13$ and $a_4 = 3$

The n th term of an AP;

$$a_n = a + (n-1)d$$

$$a_2 = a + (2-1)d$$

$$13 = a + d \dots\dots\dots (i)$$

$$a_4 = a + (4-1)d$$

$$3 = a + 3d \dots\dots\dots (ii)$$

Subtracting equation (i) from (ii), we get,

$$-10 = 2d$$

$$d = -5$$

$$13 = a + (-5)$$

$$a_3 = 18 + (3-1)(-5) = 18 + 2(-5) = 18 - 10 = 8 \text{ (third term).}$$

CASE STUDY :Q. 1. India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year

Based on the above information, answer the following questions:

(i) Find the production during first year.

OR

(i) Find the production during 8th year.

(ii) Find the production during first 3 years.

(iii) In which year, the production is ₹ 29,200.

OR

(iii) Find the difference of the production during 7th year and 4th year.

Sol. (i) 5000

Explanation: $a_6 = 16,000$

$$a + (n + 1)d = 16,000$$

$$a + (6 - 1)d = 16,000$$

$$a + 5d = 16,000 \dots(i)$$

$$a_9 = 22,600$$

$$a + (n - 1)d = 22,600$$

$$a + (9 - 1)d = 22,600$$

$$a + 8d = 22,600 \dots(ii)$$



Solving equation (i) and (ii) $a + 5d = 16,000$

$$a + 8d = 22,600 \text{ ----}$$

$$3d = -6,600$$

$$d = 2,200$$

Now, putting $d = 2,200$ in equation (i)

$$a + 5d = 16,000$$

$$a + 5 \times 2,200 = 16,000$$

$$a + 11,000 = 16,000$$

$$a = 5,000$$

OR

(i) Sol. Production during 8th year is $(a + 7d) = 5000 + 2(2200) = 20400$

(II) Sol. Production during first 3 year = $5000 + 7200 + 9400 = 21600$

(III) Sol. $N = 12$

Explanation: $a_n = 29,200$

$$a + (n - 1)d = 29,200$$

$$(x - 1)2,900 = 29,200 - 5,000$$

$$2,200n - 2,200 = 24,200$$

$$2200n = 26,400 \quad n = 26\,400 / 2\,200, n = 12$$

(IV) Sol. Difference = $18200 - 11600 = 6600$

SHORT ANSWER TYPE

Q.1: Which term of the AP: 21, 18, 15, ... is -81 ? Also, is any term 0? Give reason for your answer.

Solution. $a = 21, d = 18 - 21 = -3$.

Let -81 be the n th term.

$$\text{Now, } n\text{th term, } -81 = 21 + (n - 1) \cdot (-3)$$

$$-81 = 21 - 3n + 3$$

$$-81 = 24 - 3n$$

$$3n = 24 + 81$$

$n = 35$ Therefore, the 35th term is -81 and the eighth term is 0.

Q.2: The fourth term of an AP is zero. Prove that its 25th term is triple its 11th term.

Ans: Given that $a_4 = 0$

$$\Rightarrow a + 3d = 0$$

$$\Rightarrow a = -3d$$

...(1)

We have to prove that $a_{25} = 3a_{11}$

$$\Rightarrow a + 24d = 3(a + 10d)$$

$$\Rightarrow a + 24d = 3a + 30d$$

From equation (1),

$$\Rightarrow -3d + 24d = 3(-3d) + 30d$$

$$\Rightarrow 21d = 21d$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

3. How many terms of the arithmetic progression 45, 39, 33, must be taken so that their sum is 180? Explain the double answer

Solution: Let the number of terms be n

$$\text{First term (a)} = 45$$

$$\text{Common difference (d)} = 39 - 45 = -6$$

We know that,

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow 180 = \frac{n}{2} [90 + (n-1)(-6)]$$

$$\Rightarrow n^2 - 16n + 60 = 0$$

$$\Rightarrow (n-10)(n-6) = 0$$

$$\Rightarrow n-10=0 \text{ or } n-6=0$$

4. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Solution: Let the four consecutive terms of A.P. be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$.

By given conditions $a - 3d + a - d + d + a + 3d = 32 \Rightarrow 4a = 32$

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}, \quad d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the numbers are 2, 6, 10 and 14 or 14, 10, 6 and 2

QUESTIONS FOR PRACTICE

MCQs

1. A sequence is an ordered list of numbers. A sequence of numbers such that the difference between the consecutive terms is constant is said to be an arithmetic progression (A.P.).

On the basis of above information, answer the following questions.

Which of the following sequence is an A.P.?

- (a) 10, 24, 39, 52, ... (b) 11, 24, 39, 52, ... (c) 10, 24, 38, 52, ... (d) 10, 38, 52, 66, ...

2. If x , y and z are in A.P., then

- (a) $x + z = y$ (b) $x - z = y$ (c) $x + z = 2y$ (d) None of these

3. If the n th term of the A.P. $-1, 4, 9, 14, \dots$ is 129. Find the value of n .

- (a) 24 (b) 50 (c) 13 (d) 27

4. The list of numbers $-10, -6, -2, 2, \dots$ is

- (a) an AP with $d = -16$ (b) an AP with $d = 4$ (c) an AP with $d = -4$ (d) not an AP

5. In an Arithmetic Progression, if $a = 28$, $d = -4$, $n = 7$, then a_n is:

- (a) 4 (b) 5 (c) 3 (d) 7

6. If $k-1$, $k+3$ and $3k-1$ are in AP, then find the value of k

- (a) 4 (b) 5 (c) 3 (d) 7

7. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

- (a) 8 (b) -8 (c) -4 (d) 4

8. The famous mathematician associated with finding the sum of the first 100 natural numbers is

- (a) Pythagoras (b) Newton (c) Gauss (d) Euclid

9. If the n th term ($n > 1$) of an A.P. is smaller than the first term, then nature of its common difference d is

- (a) $d > 0$ (b) $d < 0$ (c) $d = 0$ (d) Can't be determined

10. **Assertion:** Sum of natural number from 1 to 100 is 5050

Reason: Sum of n natural number is $= n \left(\frac{n+1}{2} \right)$

- a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
 b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
 c.) Assertion is true but the reason is false.
 d.) Both assertion and reason are false.

CASE STUDY QUESTIONS: CASE STUDY-1

The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

- If the first circular row has 30 seats, how many seats will be there in the 10th row?
- For 1500 seats in the auditorium, how many rows need to be there?



OR

- If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?
 3. If there were 17 rows in the auditorium, how many seats will be there in the middle row?

CASE STUDY 2

In a class the teacher asks every student to write an example of A.P. Two friends Geeta And Madhuri writes their progressions as $-5, -2, 1, 4, \dots$ and $187, 184, 181, \dots$ respectively. Now, the teacher asks various students of the class the following questions on these two progressions. Help students to find the answers of the questions.



1. Find the sum of common difference of the two progressions.
2. Find the 34th term of the progression written by Madhuri.

OR

- Find the 19th term of the progression written by Geeta.
 3. Find the sum of first 10 terms of the progression written by Geeta.

OR

Which term of the two progressions will have the same value?

CASE STUDY 3

Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, answer the following:

- (i) The amount paid by him in 30th instalment is
- (ii) What is the amount paid by him in the 30 instalments? What amount does he still have to pay after 30th instalment?
- (iii) If total instalments are 40 then what is the amount paid in the last instalment?

OR

The ratio of the 1st instalment to the last instalment is



SHORT ANSWER TYPE QUESTIONS

1. If the 3rd and the 9th terms of an AP are 4 and -8, respectively, then which term of this AP is zero.
2. How many multiples of 4 lie between 10 and 250?
- Q.3: The sum of 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
4. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.
5. Find the sum of the first 22 terms of an AP in which $d = 7$ and 22nd term is 149.
6. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of the first two terms? What is the second term? Similarly find the 3rd, the 10th and the n th terms.
7. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.
8. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.
9. How many number of multiples of 4 lie between 10 and 250?
10. What is the 20th term from the last term of the A.P. 3, 8, 13, ..., 253?

ANSWERS TO PRACTICE QUESTIONS

MCQS

1 C 2 C 3 D 4 B 5 A 6 A 7 A 8 C 9 B 10 A

CASE STUDY 1: 1 120 seats in the tenth row, 2 $n = 15$ OR number of rows = 750, 3 110 seats

CASE STUDY 2: 1. 0, 2. 88 OR 49, 3. $S_{10} = 85$ OR $n = 33$,

CASE STUDY 3: (i) 3900, (ii) 73500, **44500**, (iii) 4900 OR 10:49

SHORT ANSWER QUESTIONS

- 5th term of the given AP is 0
- There are 60 multiples of 4 between 10 and 250.
- The first three terms of this AP are -13 , -8 , and -3 .
- Number of terms, $n = 16$ and common difference $d = 40/15 = 8/3$.
- 1661, 6. First term = 3 Sum of first two terms = $S_2 = 4$ Second term, $a_2 = 1$
- 3rd, 10th, and n th terms are -1 , -15 , and $5 - 2n$ respectively.
- The value of each of the prizes was ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, and ₹40.
- Sum of first 16 terms is 20, 9. $n = 60$, 10. $a_{20} = 158$

TEST-2 (20 MARKS)

- Find the value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an A.P
- How many two digits numbers are divisible by 3 ?
- Find the tenth term of the sequence: $\sqrt{2}, \sqrt{8}, \sqrt{18}, \dots$
- How many terms of an A.P. 9, 17, 25, must be taken to give a sum of 636 ?

Section B

- The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.
- Find the middle term of the A.P. 213, 205, 197, 37.
- In an A.P. of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P

Q. 8. If S_n , the sum of first n terms of an A.P. is given by $S_n = 3n^2 - 4n$. Find the n th term.

ANSWERS: Q. 1. $x = 6$., Q. 2. 30, two digit numbers divisible by 3. Q. 3. $\sqrt{100}$

Q. 4. $n = 12$, Q. 5. common difference, $d = 3$, Q. 6. middle term will be 125

Q. 7. A.P. is 3, 7, 11, Q. 8. $a_n = 6n - 7$

TEST-2 M.M.: 30

- In an A.P, $a_{m+n} + a_{m-n}$ is equal to
(a) 0 (b) 1 (c) $2a_m$ (d) a_m
- The first and last term of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be (a) 5 (b) 6 (c) 7 (d) 8
- Sum of first n natural number is
(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n(n+1)(2n+1)}{2}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$
- If 7th and 13th terms of an A.P. are 34 and 64 respectively, then its 18th term is
(a) 87 (b) 88 (c) 89 (d) 90

SECTION B

- Sum of all natural numbers lying between 250 and 1000 which are exactly divisible by 3 is
- If n th terms of the APs 63, 65, 67 ... and 3, 10, 17, are equal, then n is

7. The number of terms of an A.P. 3, 7, 11, 15... to be taken so that the sum is 406 is

8. Find the number of terms in each of the AP 7, 13, 19..., 205

SECTION C

9. Check whether -150 is a term of the AP: 11, 8, 5, 2...

10. Find the sum of first n odd natural numbers.

11. The sum of 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

12. How many terms of the AP: 24, 21, 18, ... must be taken so that their sum is 78?

13. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Q.14: The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.

ANSWERS: 1. C, 2. B, 3. B, 4. C, 5. 156375, 6. 13, 7. 14, 8. 34, 9. -150 cannot be term in AP., 10. n^2 , 11. First three terms of this AP are -13, -8, and -3

12. $n = 4$ or 13. Both values of n are admissible. So, the number of terms is either 4 or 13.

13. Number of terms, $n = 16$, Common difference $d = 40/15 = 8/3$, 14. $S_{16} = 20$

JSUNIL TUTORIAL

TRIANGLE

IMPORTANT CONCEPTS

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
6. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
7. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
9. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).

SOME ILLUSTRATIONS/EXAMPLES

MCQ

1. Two squares are similar, if their sides are
 - a) Equal
 - b) Proportional
 - c) Both a and b
 - d) None of the above
2. All congruent figures are
 - a) Similar
 - b) Not similar
 - c) Both a and b
 - d) None of the above
3. All similar figures are
 - a) congruent
 - b) Not congruent
 - c) Both a and b
 - d) None of the above
4. All circles are
 - a) congruent
 - b) Similar
 - c) Both b
 - d) None of the above
5. Which of the following is not a similarity criterion in triangles
 - a) AAA
 - b) SAS
 - c) SSS
 - d) RHS

CASE STUDY BASED QUESTION

If $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then

- a) If $\angle A = 60^\circ$, and $\angle B = 40^\circ$, then $\angle F$ is
- b) In $\triangle ABC$, $AD = 7\text{cm}$, $BC = 10\text{cm}$, $DE = 3.5\text{cm}$, find EF
- c) If $AB = 8\text{cm}$, perimeter $\triangle ABC = 28\text{cm}$ and $DE = 4\text{cm}$, then what is the perimeter of $\triangle DEF$

SOLUTION a) $\angle A + \angle B + \angle C = 180^\circ \therefore \angle C = 80^\circ \therefore \angle F = \angle C = 80^\circ$

b) $\frac{7}{3.5} = \frac{10}{EF} \therefore EF = 5\text{cm}$

c) Perimeter $\triangle ABC$: perimeter of $\triangle DEF = AB : DE = 8 : 4 = 2 : 1 \therefore$ perimeter of $\triangle DEF = 14\text{cm}$

SHORT ANSWER TYPE QUESTIONS

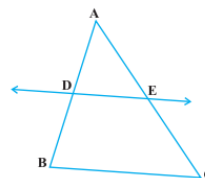
1. Given a $\triangle ABC$ and $DE \parallel BC$. If $AD = 2\text{cm}$, $DB = 3\text{cm}$ then find $AE : EC$

Solution: By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$ hence $\frac{2\text{cm}}{3\text{cm}} = \frac{AE}{EC}$ or $AE : EC = 2 : 3$

2. Given a $\triangle ABC$ and $DE \parallel BC$. If $AD = 1.5\text{cm}$, $DB = 4.5\text{cm}$, $AE = 2\text{cm}$ then find EC

Solution: By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$ hence $\frac{1.5\text{cm}}{4.5\text{cm}} = \frac{2\text{cm}}{EC} \therefore EC = 6\text{cm}$

3. In $\triangle ABC$ and $\triangle PQR$, $\angle A = 60^\circ$, $\angle B = 30^\circ$, $\angle Q = 30^\circ$, $\angle R = 90^\circ$. Are the triangles similar? Why?



By ASP, $\angle A + \angle B + \angle C = 180^\circ$ hence $\angle C = 90^\circ \therefore \angle B = \angle Q$ and $\angle C = \angle R$

$\triangle ABC \sim \triangle PQR$ (By AA Similarity criterion)

LONG ANSWER TYPE QUESTIONS.

1. R and S are points on the sides DE and EF respectively of a $\triangle DEF$

Such that $ER = 5$ cm, $RD = 2.5$ cm, $SE = 1.5$ cm and $FS = 3.5$ cm.

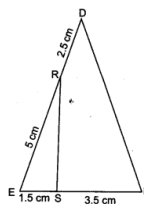
Find whether $RS \parallel DF$ or not.

Solution: Construction: Join RS

RE = 5 cm , , RD = 2.5 cm , ES = 1.5 cm , SF= 3.5 cm

$$\frac{RE}{RD} = \frac{5}{2.5} = \frac{2}{1} , \frac{SF}{ES} = \frac{3.5}{1.5} = \frac{7}{3}$$

$\frac{RE}{RD} \neq \frac{SF}{ES}$ Hence RS is not parallel to DF.



2 In the figure, ABCD is a parallelogram and E divides BC in the ratio 1 : 3. DB and AE intersect at F. Show that $DF = 4FB$ and $AF = 4FE$

Given; ABCD is a parallelogram and $BE : EC :: 1 : 3$

To prove: $DF = 4 FB$ AND $AF = 4FE$

Proof: In $\triangle ADF$ and $\triangle EBF$

$\angle ADF = \angle EBF$ (alternate angles)

$\angle AFD = \angle EFB$ (V O A)

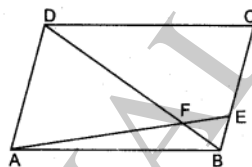
$\angle ADF \sim \angle EBF$ (A A Similarity)

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{AD}{BE} \dots\dots\dots (1)$$

$$\therefore \frac{BE}{CE} = \frac{1}{3} , \text{ hence } EC = 3 BE$$

$$BC = BE + CE = BE + 3BE = 4 BE$$

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{4}{1} \text{ WE GET } DF = 4 BF \text{ and } AF = 4FE$$



QUESTIONS FOR PRACTICE

MCQ

- 1 All equilateral triangles are
 - a) Similar b) Congruent c) Both a and b d) None of the above
- 2 Which of the following is not a congruence criterion in triangle.
 - a) AAA b) SAS c) SSS d) RHS
- 3 Given a $\triangle ABC$, D and E lie on the sides AB & AC such that $DE \parallel BC$, $AD = 8$ cm , $AB = 12$ cm, $AE = 12$ cm , then the length of CE is
 - a) 6 cm b) 18 cm c) 9 cm d) 15 cm
- 4 The perimeters of two similar triangles ADC and PQR are 60cm and 36 cm respectively. If $PQ = 9$ cm, then AB is
 - a) 6cm b) 10cm c) 15 cm d) 24cm

5 ASSERTION REASON QUESTION

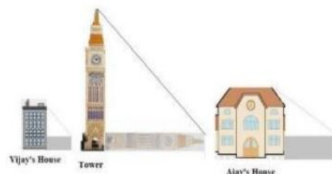
Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

Assertion (A): If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

Reason (R): In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

CASE STUDY 1:



Vijay is trying to find the average height of a tower near his house.

He is using the properties of similar triangles. The height of Vijay's house is 20m when Vijay's house casts a shadow 10m long on the ground. At the same time, the tower casts a shadow 50m long on the ground and the house of Ajay casts 20m shadow on the ground.

1. What is the height of the tower?
2. What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12m?
3. What is the height of Ajay's house?

CASE STUDY 2- SCALE FACTOR

A scale drawing of an object is the same shape as the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. The ratio of two corresponding sides in similar figures is called the scale factor.

Scale factor = length in image / corresponding length in object

If one shape can become another using resizing, then the shapes are similar. Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn. In the photograph below showing the side view of a train engine, the scale factor is 1:200.

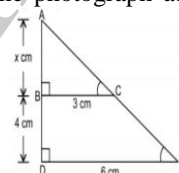


This means that a length of 1 cm on the photograph above corresponds to a length of 200cm or 2 m, of the actual engine. The scale can also be written as the ratio of two lengths.

Q1. If the length of the model is 11cm, then the overall length of the engine in the photograph above, including the couplings (mechanism used to connect) is:

Q2. What is the actual width of the door if the width of the door in photograph is 0.35cm?

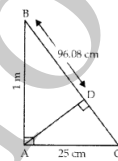
Q3. The length of AB in the given figure:



CASE STUDY 3

Q1. If $\triangle ABC \sim \triangle DEF$, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then

- a) If $\angle A = 50^\circ$, and $\angle B = 30^\circ$, then $\angle F$ is
- b) In $\triangle ABC$, $AD = 7\text{ cm}$, $BC = 10\text{ cm}$, $DE = 2.1\text{ cm}$, find EF
- c) If $AB = 5\text{ cm}$, perimeter $\triangle ABC = 20\text{ cm}$ and $DE = 4\text{ cm}$, then what is the perimeter of $\triangle DEF$



SHORT ANSWER TYPE QUESTIONS

1. In the given figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. If $AC = 25\text{ cm}$, $AB = 1\text{ m}$ and $BD = 96.08\text{ cm}$, then find the value of AD .
2. $\triangle ABC$ is such that $AB = 3\text{ cm}$, $BC = 2\text{ cm}$ and $CA = 2.5\text{ cm}$. If $\triangle DEF \sim \triangle ABC$ and $FE = 4\text{ cm}$, then find the perimeter of $\triangle DEF$.
3. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5\text{ cm}$, $AC = 8\text{ cm}$ and $DF = 7.5\text{ cm}$. Then, find: DE , $\angle F$.
4. It is given that $\triangle DEF \sim \triangle RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?
5. A and B are respectively the points on the sides PQ and PR of $\triangle PQR$ such that $PQ = 12.5\text{ cm}$, $PA = 5\text{ cm}$, $BR = 6\text{ cm}$ and $PB = 4\text{ cm}$. Is $AB \parallel QR$? Give reasons for your answer.

LONG ANSWER TYPE QUESTIONS

1. In Fig 6.13, OB is the perpendicular bisector of the line segment DE, $FA \perp OB$ and FE intersects OB at the point C.

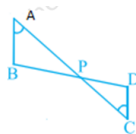
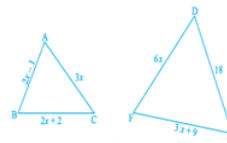
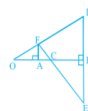
Prove that $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$.

2. In the given Fig., if $\triangle ABC \sim \triangle DEF$ and their sides are of Length (in cm)

$AB = 2x - 1$, $BC = 2x + 2$, $AC = 3x$, $DF = 6x$, $DE = 18$,

$FE = 3x + 9$, then find the lengths of the sides of each triangle

3. In the given Fig., if $\angle A = \angle C$, $AB = 6\text{ cm}$, $BP = 15\text{ cm}$,



AP = 12cm, CP = 4 cm, then find the lengths of PD and CD.

4. It is given that $\Delta ABC \sim \Delta EDF$ such that AB = 5 cm,

AC = 7 cm, DF = 15 cm & DE = 12 cm. Find the lengths of the remaining sides of the triangles.

5. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

TEST:1 MM20

NOTE: Q1 to 4 are of 3 marks each, Q5 to 6 are of 4 marks each

1 Given, ΔABC and $DE \parallel BC$. If AD=5cm, DB=7cm then find AE: EC

2 Given a ΔABC , D and E lie on the sides AB & AC such that $DE \parallel BC$,

AD= 8cm, AB=10cm AE= 6cm, then find the length of CE

3 ΔABC is such that AB = 5 cm, BC = 2 cm and CA = 4 cm. If $\Delta DEF \sim \Delta ABC$ and FE = 4 cm, then find the perimeter of ΔDEF

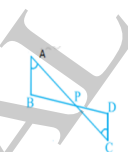
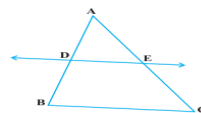
4 It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 50^\circ$, $\angle C = 60^\circ$, AB = 5 cm, AC = 8 cm and

DF = 10 cm. Then find: DE, $\angle F$

5 It is given that $\Delta DEF \sim \Delta CBA$. Is it true to say that $\angle D = \angle C$ and $\angle F = \angle B$? Why?

6. In the given Fig., if $\angle A = \angle C$, AB = 8 cm, BP = 16 cm,

AP = 14 cm and CP = 7 cm, then find the lengths of PD and CD



TEST-2 M.M. 30

NOTE: Q1--4 are of 2 marks each, Q5 & 6 are of 3 marks each, Q7 & 8 are of 4 marks each

1 Given a ΔABC , D and E lie on the sides AB & AC such that

$DE \parallel BC$, AD=3cm, AB=10cm AE= 6cm, then find the length of AC.

2 If AC and BD intersect at P, AP=6cm, BP=3cm, PD=5cm DC=2.5cm, $\angle APB = 50^\circ$, $\angle D = 30^\circ$ then find $\angle PBA$.

3 A and B are respectively the points on the sides PQ and PR of a triangle PQR such that PQ=12.5cm, PA=5cm, BR=6cm, PB=4cm. Is $AB \parallel QR$. Give reasons for your answer.

4. D is a point on the side QR of a ΔPQR such that $PD \perp QR$. Is $\Delta PQR \sim \Delta PRD$? Why?

5. It is given that $\Delta ABC \sim \Delta EDF$ such that AB = 6 cm, AC = 9 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.

6 Given a ΔABC , D and E lie on the sides AC & BC such that $DE \parallel AB$ and $CE = x$, $DA = 3x + 19$, $BE = 3x + 4$, $CD = x + 3$. Find x?

7. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

8 Legs (sides other than the hypotenuse) of a right triangle are of 16cm and 8cm length. Find the length of the side of the largest square that can be inscribed in the triangle?

ANSWERS

MCQ 1) a 2) a 3) a 4) c 5) d

SHORT ANSWER QUESTIONS

1) 24.02m 2) 15 cm 3) DE=12cm, $\angle F = 100^\circ$ 4) False because, if two triangles are similar, then their corresponding angles are equal. $\therefore \angle D = \angle R$, $\angle E = \angle P$ and $\angle F = \angle Q$

5) By converse of basic proportionality theorem, $AB \parallel QR$

CASE STUDY BASED QUESTIONS: 1(i) 100m. (ii) 60m. (iii) 40m. 2(i) 22m. (ii) 0.7m. (iii) 4cm. 3(i) 100° (ii) 3cm (iii) 16cm

LONG ANSWER QUESTIONS: 2) AB=9, BC=12, AC=15, DE=18, EF=24, DF=30

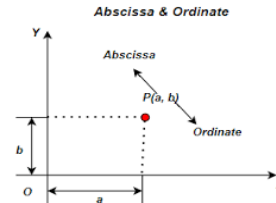
3) MPD=5cm, CD= 2cm 4) BC=6.25cm, EF=16.8cm

COORDINATE GEOMETRY

IMPORTANT CONCEPTS

A pair of numbers locate points on a plane called the **coordinates**. The distance of a point from the y-axis is known as **abscissa** or x-coordinate. The distance of a point from the x-axis is called **ordinate** or y-coordinate.

Example: Consider a point P(3, 2), where 3 is the abscissa, and 2 is the ordinate. 3 represents the distance of point P from the y-axis, and 2 represents the distance of point P from the x-axis.



Distance Formula

Distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where d is the distance between the points (x_1, y_1) and (x_2, y_2) .

Section Formula

If the point P(x, y) **divides** the line segment joining A(x_1, y_1) and B(x_2, y_2) **internally** in the **ratio m:n**, then, the coordinates of P are given by the **section formula** as:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

Midpoint formula:

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points A(x_1, y_1), and B(x_2, y_2) are:

$$P(x, y) = \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$$

PLEASE KEEP IN MIND

✔ To check whether the three points form an isosceles triangle or an equilateral triangle, find out the distance between all the three points and if the two sides or three sides are same, we can conclude the answer respectively.

✔ To calculate the area of any quadrilateral, divide it into two triangles then find the area of individual triangle and add them.

✔ To check whether the three points A, B and C are collinear either show $AB + BC = AC$ or you can prove it by calculating the area of a triangle formed by these three points is zero.

✔ Please try to remember all the properties of quadrilaterals and triangles, for the questions which ask to check whether the points form any geometrical shape or not.

SOME EXAMPLES (With Solutions)

MCQs

1. AOB is a triangle whose three vertices are A(0, 3), O(0, 0) and B(3, 0). The length of its diagonal is:

- (a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

Ans (c) $\sqrt{34}$

2. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is:

- (a) 5 (b) 12 (c) 11 (d) 4

Ans b) 12

3. The distance between the point P(1, 4) and Q(4, 0) is

- (a) 4 (b) 5 (c) 6 (d) $3\sqrt{3}$

Ans b) 5

4. If the distance between the points (8,p) and (4,3) is 5 units, then value of p is
 (a) 6 (c) both (a) and (b)
 (b) 0 (d) none of these

Ans (c) both (a) and (b)

5. If the origin is the mid-point of the line segment joined by the points (2,3) and (x,y), then the value of (x,y) is
 (a) (2, 3) (c) (-2,-3)
 (b) (-2,3) (d) (2,-3)

Ans (d) (2,-3)

CASE STUDY BASED QUESTIONS

Q6. In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th distance AD on the eighth line and posts a red flag.

1. Find the position of green flag
 a) (2,25) b) (2,0.25) c) (25,2) d) (0, -25)

Answer: a) (2,25)

2. Find the position of red flag
 a) (8,0) b) (20,8) c) (8,20) d) (8,0.2)

Answer: c) (8,20)

3. What is the distance between both the flags?
 a) $\sqrt{41}$ b) $\sqrt{11}$ c) $\sqrt{61}$ d) $\sqrt{51}$

Answer: c) $\sqrt{61}$

4. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

- a) (5, 22.5) b) (10,22) c) (2,8.5) d) (2.5,20)

Answer: a) (5, 22.5)

5. If Joy has to post a flag at one-fourth distance from green flag, in the line segment joining the green and red flags, then where should he post his flag?

- a) (3.5,24) b) (0.5,12.5) c) (2.25,8.5) d) (25,20)

Answer: a) (3.5,24)

SHORT ANSWER TYPE QUESTIONS

Q6. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is O(2,3) and B (1,-4). **Ans: . A = (3,10)**

Q7. The line joining the points (2, -1) and (5, -6) is bisected at P. If P lies on the line $2x + 4y + k = 0$, find the value of k. **Ans: . k = 7**

Q8. The coordinates of A and B are (1,2) and (2,3) respectively. If P lies on AB, find the coordinates of P such that $\frac{AP}{PB} = \frac{4}{3}$. **Ans : p = (11/7, 18/7)**

PRACTICE QUESTION:

MCOs

Q1. If four vertices of a parallelogram taken in order are (-3, -1),(a, b), (3,3) and (4, 3), then a:b =

- (a) 1:4 (c) 1:2
 (b) 4:1 (d) 2:1

Q2. The point on x-axis equidistant from the points P(7, 6) and Q(-3,4) is:

- (a) (-2,0) (c) (0, 4)
 (b) (3, 0) (d) (0, 3)

Q3. The x- axis divides the join of P(2, -3) and Q(5, 6) in the ratio:

- (a) 2 : 3 (c) 1 : 2
 (b) 2 : 1 (d) 3 : 5

Q4. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, -5) is the mid-point of PQ, then the coordinates of P and Q are, respectively

- (a) (0, -5), (2, 0) (c) (0, 10), (-4, 0)
 (b) (0, 4), (-10, 0) (d) (0, -10), (4, 0)

Q5. **Assertion :** The distance point P(2,3) from the x-axis is 3.

Reason: The distance from x-axis is equal to its ordinate.

- a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
 b.) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
 c.) Assertion is true but the reason is false. d.) Both assertion and reason are false.

SHORT ANSWER TYPE QUESTIONS

Q6. State whether the following statement is true or false. Justify your answer: Point P(5, -3) is one of the two points of trisection of the line segment joining the points A(7, -2) and B(1, -5).

Q7. If the joining the points A(4, -5) and B(4, 5) is divided by the point P such that $\frac{AP}{AB} = \frac{2}{5}$, find the coordinates of P.

Q8. If (1, p/3) is the midpoint of the line segment joining the points (2, 0) and (0, 2/9), then show that the line $5x + 3y + 2 = 0$ passes through the point (-1, 3p).

Q9. Three consecutive vertices of a parallelogram ABCD are A(1, 2), B(1, 0) and C(4, 0). Find the fourth vertex D.

Q10. Find the value of x for which the distance between (x, 7) and (-1, -5) is 13 units

Q11. Find a point on the y-axis which is equidistant from the point A(6, 5) and B(-4, 3).

Q12. If the distance of P(x, y) from the points A(3, 6) and B(-3, 4) are equal. Prove that $3x + y = 5$. If (-3, 2), (1, -2) and (5, 6) are the mid-points of the sides of a triangle, find the coordinates of the vertices of the triangle.

Q13. If coordinates of two adjacent vertices of a parallelogram are (3, 2), (1, 0) and diagonals bisect each other at (2, -5), find the coordinates of other two vertices.

Q14. If A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

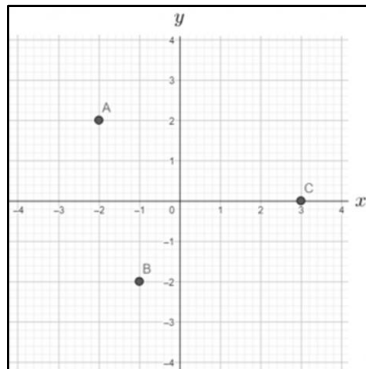
Q15. Determine the ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7). Also, find the coordinates of the point of division.

CASE BASED STUDY QUESTIONS

Q16. Three friends are standing at point A, B and C. Look at the figure below:

Questions:

- a) Write down the coordinates of A, B and C shown in the graph.
 b) Find the distance between AB, BC and CA.
 c) Are they standing at an equal distance from each other? Justify your answer.
 d) Point C lies on the line segment between points A and B such that $AC:CB = 4:3$. Find the coordinates of point C.



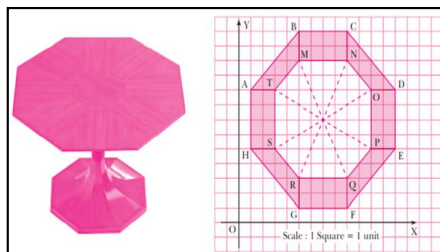
Q17. The top of a table is shown in the figure given below:

(i) The coordinates of the points H and G are respectively

- (a) (1, 5), (5, 1) (b) (0, 5), (5, 0)
 (c) (1, 5), (5, 0) (d) (5, 1), (1, 5)

(ii) The distance between the points A and B is

- (a) 4 units (b) 4 2 units
 (c) 16 units (d) 32 units



(iii) The coordinates of the mid-point of line segment joining points M and Q are

- (a) (9, 3) (b) (5, 11)
(c) (14, 14) (d) (7, 7)

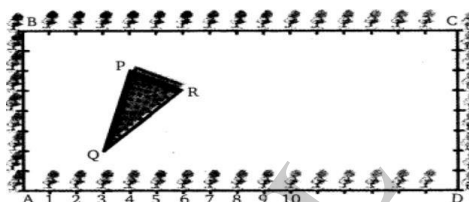
(iv) Which among the following have same ordinate?

- (a) H and A (b) T and O (c) R and M (d) N and R

v) If G is taken as the origin, and x, y axis put along GF and GB, then the point denoted by coordinate (4, 2) is

- (a) H (b) F (c) Q (d) R

Q18. The class X students school in Rishikesh have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



1. Taking A as origin, find the coordinates of P

- a) (4,6) b) (6,4) c) (0,6) d) (4,0)

2. What will be the coordinates of R, if C is the origin?

- a) (8,6) b) (3,10) c) (10,3) d) (0,6)

3. What will be the coordinates of Q, if C is the origin?

- a) (6,13) b) (-6,13) c) (-13,6) d) (13,6)

4. Calculate the area of the triangles if A is the origin

- a) 4.5 b) 6 c) 8 d) 6.25

5. Calculate the area of the triangles if C is the origin

- a) 8 b) 5 c) 6.25 d) 4.5

ANSWERS : PRACTICE QUESTIONS

Ans 1. (d) 2:1, Ans 2 (b) (3,0), Ans 3 (b) 2 : 1, Ans 4 (d) (0,-10),(4,0)

Ans 5 a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion

Ans 6 True, Ans 7. $P = (4, -15/7)$, Ans 8. Proof, Ans 9 $D(4, 2)$, Ans 10. $x = 4$ or -6

Ans 11. (0,9) Ans 12. Proof, Ans 13 (1, -12) and (3, -10),

Ans 14 $a = 1, b = 1$. Length of the sides $AB = BC = CD = DA = \sqrt{10}$ units

Ans 15 (3:4), Ans 16 a) (-2,2), (-1,-2), (3,0); b) 17, 25, 29

c) No, because distance between them is different in each case, d) (-10/7, -2/7)

Ans 17. 1. (a) 2. (b) 3. (d) 4. (b) 5. (c)

Ans 18. 1. a) (4,6) 2. c) (10,3) 3. d) (13,6) 4. a) 4.5 5. d) 4.5

TEST-1: M.M. 20

SECTION A (2 x 4=8)

Q1. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0)

Q2. Find the value of p if the distance between the points (8,p) and (4,3) is 5 units.

Q3. Find the radius of circle whose end points of diameter are (-4,1) and (2,-3).

Q4. What is the distance between the points A(c, 0) and B(0,-c) ?

SECTION B (3X 4=12)

Q5. If the distance between the points (4, p) and (1,0) is 5, find the value of p.

Q6. For what value of p are the points (2,1), (p,-1) and (-1,3) are collinear.

Q7. The line joining the points (2, -1) and (5, -6) is bisected at P. If P lies on the line $2x + 4y + k = 0$, find the value of k.

Q8. Find a point on the y-axis which is equidistant from the point A(6, 5) and B(-4, 3).

SECTION C (5 x 2=10)

Q9. a) If coordinates of two adjacent vertices of a parallelogram are (3, 2), (1, 0) and diagonals bisect each other at (2, -5), find the coordinates of other two vertices.

b) Find the value of x for which the distance between (x, 7) and (-1, -5) is 13 units.

Q10. If A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

ANSWERS : Ans1 12 units, Ans2 both (a) and (b), Ans3. 13 units, Ans4 $C\sqrt{2}$ units
Ans5. +4 and -4, Ans6 $P=5$, Ans7. $K=7$, Ans8 (0,9), Ans9 a) (1, -12) and (3, -10)
b) $x = 4$ or -6 , Ans10. $a = 1$, $b = 1$. Length of the sides $AB = BC = CD = DA = \sqrt{10}$ units.

TEST-2: M.M. 30

SECTION A (2 x 5=10)

Q1. Find the distance of the point (-3, 4) from the x-axis. (2012OD)

Q2. Find the radius of circle whose end points of diameter are (-4,1) and (2,-3)

Q3. State whether the following statement is true or false. Justify your answer: Point P(5, -3) is one of the two points of trisection of the line segment joining the points A(7,-2) and B(1, -5)

Q4. Prove that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle

Q5. Show that the points (-2, 3), (8, 3) and (6, 7) are the vertices of a right triangle.

SECTION B (3X5=15)

Q6 For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P.?

Q7. ABCD is a rectangle whose three vertices are B(4, 0), C(4, 3) and D(0, 3). Calculate the length of one of its diagonals.

Q8. Find a relation between x and y such that the point P(x, y) is equidistant from the points A (2, 5) and B (-3, 7).

Q9. Prove that the points A(2, 3), B(-2, 2), C(-1, -2) and D(3, -1) are the vertices of a square ABCD.

Q10. If A(4, 3), B(-1, y) and C(3, 4) are the vertices of a right triangle ABC, right-angled at A, then find the value of y.

SECTION C (5X3=15)

Q11. If A(5, 2), B(2, -2) and C(-2, t) are the vertices of a right angled triangle with $\angle B = 90^\circ$, then find the value of t.

Q12 Find the ratio in which y-axis divides the line segment joining the points A(5, -6), and B(-1, -4). Also find the coordinates of the point of division

Q13 Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

ANSWERS: Ans1. 4 units, Ans2. $\sqrt{13}$ units, Ans3 true, Ans4 proof, Ans5 proof
Ans6. $K=18$, Ans7. $AC=5\text{cm}$, Ans8. $10x+29=4y$ is the required relation
Ans9 proof, Ans10. $Y=-2$, Ans11. $t=1$, Ans12 (0,-13/3),
Ans13 The required ratio is 3:4 internally

INTRODUCTION TO TRIGONOMETRY

IMPORTANT CONCEPTS/RESULTS

1. Pythagoras Theorem: In right angled triangle ΔABC , right angled at B: $AC^2 = AB^2 + BC^2$
2. Trigonometric ratios:

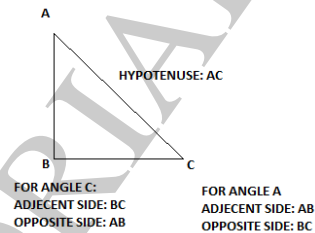
$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{opposite side}} \quad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent side}} \quad \cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin A = \frac{BC}{AC} \quad \sin C = \frac{AB}{AC}$$

$$\cos A = \frac{AB}{AC} \quad \cos C = \frac{BC}{AC}$$



Trigonometric Identities:

$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 & \rightarrow & 1 - \sin^2 \theta = \cos^2 \theta & \rightarrow & 1 - \cos^2 \theta = \sin^2 \theta \\ 1 + \tan^2 \theta &= \sec^2 \theta & \rightarrow & \sec^2 \theta - 1 = \tan^2 \theta & \rightarrow & \sec^2 \theta - \tan^2 \theta = 1 \\ 1 + \cot^2 \theta &= \operatorname{cosec}^2 \theta & \rightarrow & \operatorname{cosec}^2 \theta - 1 = \cot^2 \theta & \rightarrow & \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 \end{aligned}$$

Values of T- ratios for some standard angles:

T- ratio ↓ / angle(θ)→	0°	30°	45°	60°	90°
$\sin \theta$	0	1/2	1/√2	√3/2	1
$\cos \theta$	1	√3/2	1/√2	1/2	0
$\tan \theta$	0	1/√3	1	√3	∞
$\cot \theta$	∞	√3	1	1/√3	0
$\sec \theta$	1	2/√3	√2	2	∞
$\operatorname{cosec} \theta$	∞	2	√2	2/√3	1

Some Illustrations:

MCQs

1. If $\sin \theta = 3/5$ then $\cot \theta$ is:

- a) 4/5 b) 4/3 c) 3/4 d) 1

Sol: $\sin \theta = (\text{opposite side}) / \text{hypotenuse} = 3/5$

Opposite side = $3x$

Hypotenuse = $5x$

By Pythagoras theorem : $(\text{hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$

$$(5x)^2 = (3x)^2 + (\text{adjacent side})^2$$

$$25x^2 = 9x^2 + (\text{adjacent side})^2$$

$$25x^2 - 9x^2 = (\text{adjacent side})^2$$

$$16x^2 = (\text{adjacent side})^2$$

$$4x = (\text{adjacent side})$$

$\cot \theta = (\text{adjacent side}) / (\text{opposite side})$

$$= 4x/3x = 4/3 \quad (\text{option 'b' is the correct answer})$$

2. If $\cos\theta = 1/2$ then $\operatorname{cosec}\theta$ is :

- a) 2 b) $1/\sqrt{3}$ c) $2/\sqrt{3}$ d) $\sqrt{3}/2$

sol: $\cos\theta = 1/2 = \cos 60^\circ$ (as per table)
 $\theta = 60^\circ$

$\operatorname{cosec}\theta = \operatorname{cosec} 60^\circ = 2/\sqrt{3}$ (option 'c' is the correct answer)

3) $\sec^2 45^\circ - \tan^2 45^\circ = ?$

- a) 0 b) 1 c) -1 d) $\sqrt{2}$

Sol: $\sec^2 45^\circ - \tan^2 45^\circ = 1$ (according to identity $1 + \tan^2\theta = \sec^2\theta$)

(option 'b' is the correct answer)

4. $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 99^\circ \cdot \cos 100^\circ = ?$

- a) 1 b) -1 c) 0 d) can not be determined

sol:

$$\begin{aligned} &\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 99^\circ \cdot \cos 100^\circ \\ &= \cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \dots \cos 90^\circ \dots \cos 99^\circ \cdot \cos 100^\circ \\ &= 0 \quad (\cos 90^\circ = 0 \text{ as per table}) \end{aligned}$$

(option 'c' is the correct answer)

Short Answer type

1. If $\cot\theta = 7/8$ then evaluate $\frac{(1 - \sin\theta)(1 + \sin\theta)}{(1 - \cos\theta)(1 + \cos\theta)}$

$$\frac{(1 - \sin\theta)(1 + \sin\theta)}{(1 - \cos\theta)(1 + \cos\theta)} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$$

$$[(a + b)(a - b) = a^2 - b^2]$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta \quad (\sin^2\theta + \cos^2\theta = 1 \text{ \& } \cot\theta = \cos\theta / \sin\theta)$$

$$= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

2. If $\sec\theta = 13/12$ find all other trigonometric ratios.

Sol:

$$\sec\theta = (\text{Hypotenuse}) / (\text{adjacent side}) = 13 / 12$$

$$\text{Hypotenuse} = 13x$$

$$\text{adjacent side} = 12x$$

$$(\text{hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$$

$$(13x)^2 = (\text{opposite side})^2 + (12x)^2$$

$$169x^2 = (\text{opposite side})^2 + 144x^2$$

$$169x^2 - 144x^2 = (\text{opposite side})^2$$

$$25x^2 = (\text{opposite side})^2$$

$$5x = \text{opposite side}$$

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5x}{13x} = \frac{5}{13} \quad \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{12x}{13x} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{5x}{12x} = \frac{5}{12} \quad \operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{Opposite side}} = \frac{13x}{5x} = \frac{13}{5}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{12x}{5x} = \frac{12}{5}$$

3. Evaluate $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ \cos 60^\circ + \cot 45^\circ}$

Sol:
$$\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} - 1} = \frac{\sqrt{3} + 2\sqrt{3} - 4}{4 + \sqrt{3} - 2\sqrt{3}} = \frac{3\sqrt{3} - 4}{4 - \sqrt{3}}$$

Long Answer Type:

1. Prove that $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

Sol:
$$\begin{aligned} \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\ &= \frac{\sin \theta}{\sin^2 \theta} + \frac{\cos \theta}{\cos^2 \theta} \\ &= \frac{\sin \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos \theta}{\sin \theta (\cos \theta - \sin \theta)} \\ &= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\ &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \end{aligned}$$

$$\begin{aligned}
&= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)} \\
&= \frac{(1 + \sin \theta \cos \theta)}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \\
&= \sec \theta \operatorname{cosec} \theta + 1
\end{aligned}$$

2. Prove that $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\begin{aligned}
\sqrt{\frac{1 + \sin A}{1 - \sin A}} &= \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{(1 - \sin^2 A)}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{(\cos^2 A)}} \\
&= \frac{1 + \sin A}{\cos A} \\
&= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A
\end{aligned}$$

PRACTICE QUESTIONS:-

MCQ:-

1. If $\sec \theta - \tan \theta = 1/3$, then find the value of $(\sec \theta + \tan \theta)$.

- (a) 4 (b) 6 (c) 3 (d) 2

2. If $\sin A = \frac{1}{2}$, then find the value of $\cos A$

- (a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{3}}{5}$

3. If $\sin A + \sin^2 A = 1$, then find the value of $\cos^2 A + \cos^4 A$.

- (a) 1 (b) 2 (c) 5 (d) 2

4. If $\sec \theta = \frac{5}{4}$, then evaluate $\frac{\tan \theta}{1 + \tan^2 \theta}$. (Ans. $\frac{12}{25}$)

- (a) $\frac{12}{27}$ (b) $\frac{12}{25}$ (c) $\frac{14}{25}$ (d) $\frac{12}{35}$

5. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$ and $0^\circ < A+B < 90^\circ$, $A > B$, Find A and B.

- (a) $45^\circ, 15^\circ$ (b) $35^\circ, 15^\circ$ (c) $45^\circ, 25^\circ$ (d) $55^\circ, 35^\circ$

6. **Assertion:** The value of $\operatorname{cosec} 30^\circ + \cot 45^\circ$ is 3.

Reason: $\operatorname{cosec} 30^\circ = 2$, $\cot 45^\circ = 1$

- a) both Assertion and reason are correct and reason is correct explanation for Assertion
 b) both Assertion and reason are correct but reason is not correct explanation for Assertion
 c) Assertion is correct but reason is false
 d) both Assertion and reason are false

Short answer type question:-

1. Prove that $(\tan A - \tan B)^2 + (1 + \tan A \tan B)^2 = \sec^2 A \sec^2 B$
2. Prove that $\frac{\operatorname{Cosec} \theta}{\operatorname{Cosec} \theta - 1} + \frac{\operatorname{Cosec} \theta}{\operatorname{Cosec} \theta + 1} = 2 \sec^2 \theta$.
3. If $4 \tan \theta = 3$, then evaluate $\left[\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta} \right]$
4. Prove that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{Cosec} \theta$
5. $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

Long answer type question:-

1. Prove that $\frac{\tan \theta - \cot \theta}{\sin \theta \cdot \cos \theta} = \tan^2 \theta - \cot^2 \theta$
2. Prove that: $(\sin A + \operatorname{Cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$
3. If $x = b \cos A - a \sin A$ and $y = a \cos A + b \sin A$, then prove that $x^2 + y^2 = a^2 + b^2$.
4. Show that $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$.
5. If $\tan A = n \tan B$ and $\sin A = m \sin B$ show that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$
6. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

ANSWERS: MCQ:-

- 1.) 3 2.) $\frac{\sqrt{3}}{2}$ 3.) 1 4.) $\frac{12}{25}$ 5.) $45^\circ, 15^\circ$ 6.) (a)

Short answer type question: Q 3. $\frac{1}{2}$

APPLICATIONS OF TRIGONOMETRY

IMPORTANT CONCEPTS

1. The line drawn from the eye of an observer to a point in the object where the person is viewing is called the line of sight .
2. The angle formed by the line of sight with the horizontal when the object is above the horizontal level is called the angle of elevation.
3. The angle formed by the line of sight with the horizontal when the object is below the horizontal level is called the angle of depression.
4. The height of an object or distance between distant objects can be determined with the help of trigonometry ratios.

(I) MCQs

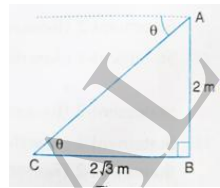
1. In the given figure point C is observed from point A. The angle of depression is

- A. 60° B. 30° C. 45° D. 75°

Sol. (B) Let θ be the angle of depression of point C from A. In rt. ΔABC

$$\tan \theta = \frac{AB}{BC}$$

$$\tan \theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$



2. The measure of the angle of elevation of the top of the tower $75\sqrt{3}$ m high from a point at a distance of 75 m from the foot of the tower in a horizontal plane is

- A. 60° B. 30° C. 45° D. 90°

Sol. (A) Let PQ be a tower of height $75\sqrt{3}$ m and let θ be the angle of elevation of its top Q from a point R at a distance of 75 from the tower.

$$\text{In right } \Delta RPQ, \text{ we obtain } \tan \theta = \frac{PQ}{PR} \Rightarrow \tan \theta = \frac{75\sqrt{3}}{75} = \sqrt{3}$$

$$\tan \theta = \tan 60^\circ$$

3. If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of sun's altitude is

- A. 45° B. 60° C. 30° D. 75°

Sol. (A) Let AB be the vertical pole of height x m and let AC be the length of its shadow when the angle of elevation of sun is θ . It is given that $AC = x$ m. In ΔACB ,

$$\tan \theta = \frac{AB}{AC} = \frac{x}{x} = 1 = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

sun's altitude is 45°

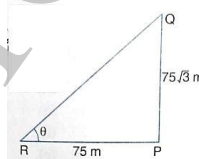
4. If the Angle of elevation of the top of the tower from a point on the ground, 100m away from the foot of the tower is 30° , then the height of the tower is

- A. 100m B. $100\sqrt{3}$ C. $\frac{100}{\sqrt{3}}$ D. $100\sqrt{3}$

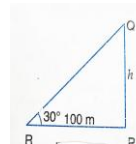
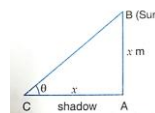
Sol. (C) Let PQ be a tower of height h m such the angle of elevation of its top Q at a point R, 100 m away from the foot P of the tower, is 30° ,

In right ΔRPQ

$$\tan 30^\circ = \frac{PQ}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100} \Rightarrow h = \frac{100}{\sqrt{3}} \text{ m}$$



height, the angle



5. **Assertion:** If shadow of pole is $\frac{1}{\sqrt{3}}$ of its height, then the altitude of the sun is 60°

Reason: If the sun's altitude is 45° , then the shadow of a vertical pole is same as height.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

- B. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
 C. Assertion (A) is true but reason (R) is false. D. Assertion (A) is false but reason (R) is true.

Soln. (B) Let PQ be height of the pole = h and RP = length of shadow = $\frac{h}{\sqrt{3}}$. sun's altitude = θ

$$\text{Then } \tan \theta = \frac{h}{h/\sqrt{3}} = \sqrt{3} \Rightarrow \tan \theta = \tan 60^\circ \Rightarrow \theta = 60^\circ$$

Let PQ be height of the pole = h and RP be length of shadow when sun's altitude is 45°

$$\text{In rt. } \triangle BAC \quad \tan 45^\circ = \frac{AB}{AC} \Rightarrow 1 = \frac{h}{AC} \Rightarrow AC = h$$

Both statements are true but statement 2 is not the correct reason of statement 1

CASE STUDY QUESTION

A group of students of class X visited India Gate on an educational trip. The teacher as well as students had interest in history behind India Gate. The teacher narrated that India Gate's official name is Delhi Memorial, originally called All India War Memorial, a monumental sandstone arch in New Delhi dedicated to the troops of British India who died in the wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath, is about 138 feet (42 m) in height.



1. What is the angle of elevation if they are standing at a distance of 42 m away from the monument?
2. What is the name of the angle formed by the line of sight with the line of horizontal when the object is viewed is below the horizontal line?
3. If the students want to see the top at an angle of 60° , then find the distance from monument where they should stand.

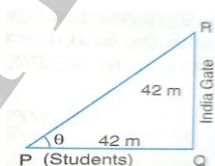
Soln. 1. Let θ be the angle of elevation of the top of India Gate. In $\triangle PQR$, we have PQ = 42m and QR = 42m. Therefore, $\tan \theta = \frac{QR}{PQ} = \frac{42}{42} = 1 \Rightarrow \theta = 45^\circ$

2. Angle of depression

3. When θ is 60° , QR = 42m,

$$\text{then in rt. } \triangle PQR \quad \tan \theta = \frac{QR}{PQ}$$

$$\tan 60^\circ = \frac{42}{PQ} \Rightarrow \sqrt{3} = \frac{42}{PQ} \Rightarrow PQ = \frac{42}{\sqrt{3}} = 14\sqrt{3} = 14 \times 1.732 = 24.24 \text{ m}$$



Short answers type of questions

1. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Soln. Let the height of the tower AC = h m

In rt. $\triangle ABP$

$$\tan 45^\circ = \frac{AB}{BP}$$

$$\Rightarrow 1 = \frac{20}{BP} \Rightarrow BP = 20 \text{ m} \dots \dots \dots (1)$$

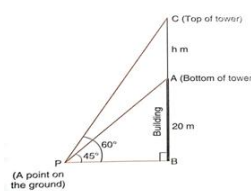
In rt. $\triangle CBP$

$$\tan 60^\circ = \frac{BC}{BP} = \frac{AB+AC}{BP}$$

$$\Rightarrow \sqrt{3} = \frac{20+h}{20}$$

$$\Rightarrow 20 + h = 20\sqrt{3}$$

$$\Rightarrow h = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1) \text{ m}$$



Height of the tower = $20(\sqrt{3} - 1)$ m

2. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

Soln. Let the height of the building AB be h m

Let PQ be the Tower.

\therefore PQ = 50m

In rt.Δ PQB ,

$$\tan 60^\circ = \frac{PQ}{BQ}$$

$$\sqrt{3} = \frac{50}{BQ}$$

$$\sqrt{3} BQ = 50 \dots\dots\dots(1)$$

In rt.Δ ABQ

$$\tan 30^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BQ}$$

$$BQ = \sqrt{3} h \dots\dots\dots(2)$$

From 1 and 2

$$\sqrt{3}\sqrt{3} h = 50 \Rightarrow 3h = 50$$

$$h = \frac{50}{3} \text{ m} = 16\frac{2}{3} \text{ m}$$

Hence the height of the building is $16\frac{2}{3}$ m

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

Soln. Let the TV of tower be AB = h m

Let BC = x m be the width of the canal

CD = 20m

In rt.Δ ABC,

$$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3} x \dots\dots\dots(1)$$

In rt.Δ ABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

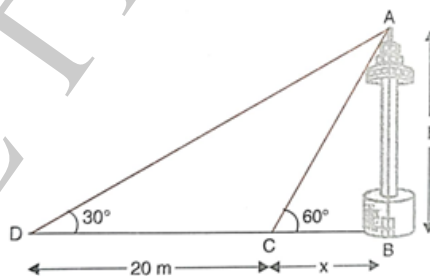
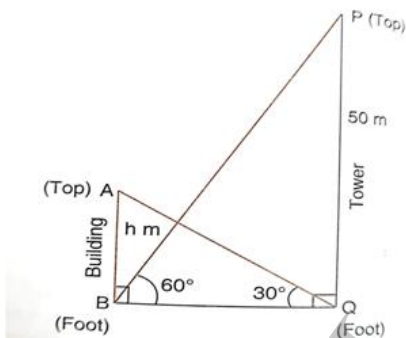
$$\Rightarrow x + 20 = \sqrt{3} h$$

Substituting $h = \sqrt{3}x$ from (1)

$$x+20 = \sqrt{3} \cdot \sqrt{3}x \Rightarrow x+20=3x$$

$$2x = 20 \Rightarrow x= 10$$

Substitute $x= 10$ in (1) $h = 10\sqrt{3}$ m



Hence height of the tower is $10\sqrt{3}$ m and the width of the canal is 10m.

PRACTICE QUESTIONS

(I) MCQs

- The length of shadow of a tower on the plane ground is $\sqrt{3}$ is height of the tower. The angle of elevation of sun is:
A. 45° B. 30° C. 60° D. 90°
- The angle of depression of a car, standing on the ground, from the top of 75 m tower, is 30° . The distance of the car from the base of the tower (in metres) is
A. $25\sqrt{3}$ B. $50\sqrt{3}$ C. $75\sqrt{3}$ D. 150
- A ladder 15m long just reaches the top of a vertical wall. If a ladder makes an angle of 60° . With the wall, then height of the wall (in metres) is
A. $15\sqrt{3}$ B. $\frac{5\sqrt{3}}{21}$ C. $\frac{15}{2}$ D. 15
- The angle of depression of car parked on the road from the top of 150 m high tower is 30° . The distance of the car from the tower (in metres) is
A. $50\sqrt{3}$ B. $150\sqrt{3}$ C. $150\sqrt{2}$ D. 75
- If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the sun at that time is
A. 30° B. 60° C. 45° D. 75°
- The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the ladder is 45° . Then the height of the tower (in metres) is
A. $50\sqrt{3}$ B. 50 C. $\frac{50}{\sqrt{2}}$ D. $\frac{50}{\sqrt{3}}$
- A ladder makes an angle of 60° with the ground when placed against the wall. If the foot of the ladder is 2m away from the wall, then the length of the ladder (in metres) is
A. $\frac{4}{\sqrt{3}}$ B. $4\sqrt{3}$ C. $2\sqrt{2}$ D. 4
- The ratio of the length of rod and its shadow is $1:\sqrt{3}$. The angle of elevation of the sun is
A. 30° B. 45° C. 60° D. 90°

9. **Assertion:** The line of sight is the line drawn from the eye of an observer to the point of the object viewed by the observer.

Reason: Trigonometric ratios are used to find height or length of an object or length of an object or distance between 2 distant objects.

- A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
B. Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
C. Assertion (A) is true but reason (R) is false. D. Assertion (A) is false but reason (R) is true.

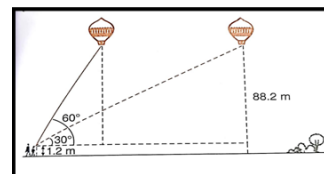
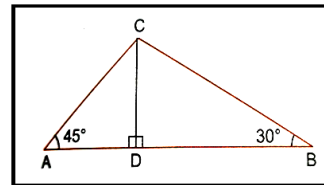
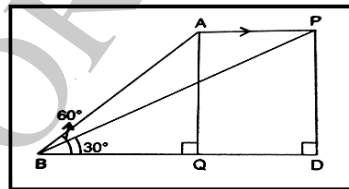
Short answers type of questions

- From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60° . If the cars are 100m apart and are on the same side of the tower. Find the height of the tower.
- Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45° . If the height of the light house is 200 m, find the distance between the two ships.
- The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, find the height of the building.

4. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles.
5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45° . Find the height of the pedestal.
6. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.
7. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30° . Find the distance of the hill from the ship and the height of the hill.
8. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the length of the flagstaff from P is 45° . Find the length of the flagstaff and distance of building from point P.
9. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant.
10. The angles of depression of the top and bottom of tower as seen from the top of 100m high cliff, are 30° and 60° respectively. Find the height of the tower.

CASE STUDY QUESTION

1. A plane is flying parallel to the ground at a constant speed. When the plane is at A its angle of elevation is noted from the point B. The plane continues to fly in the direction shown. When the plane is at P, angle of elevation is noted from B.
- (i) Find the angle of elevation of the plane from B when it is at P.
- (ii) what is the distance AP if Jet is flying with the speed 720 km per hour in 15 seconds?
- (iii) if the jet is flying at a speed of 720 km per hour, then find the constant height at which it is flying.
2. Two boats A and B are approaching a lighthouse CD in mid sea from opposite directions. The angles of elevation of the top of the lighthouse from the boats A and B are 45° and 30° respectively.
- (i) if length of AD is 90 m, then find length of CD.
- (ii) if length of CD is 60 m then find the length of DB.
- (iii) if the distance between the two boats is 100 m, then find the height of the lighthouse.
3. A 1.2 metre tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . After 30 seconds the angle of elevation reduces to 30° .
- (i) Find the distance travelled by the balloon during the interval
- (ii) find the speed of the balloon.



TEST-1 MM-30

1. A man standing on the deck of a ship which is 10 m above water level, observes angle of elevation of the top of the hill as 60° and angle of depression of the base of the hill as 30° . Calculate the distance of the hill from the ship and the height of the hill.

- Two pillars of equal heights stand on either side of the roadway 150 m wide . From a point on the roadway between the pillars, the angles of elevation of the top of the pillars are 60° and 30° . find the height of the pillars and the position of the points.
- The angle of elevation of a fighter plane from a point on the ground is 60° . After a flight of 15 seconds ,the angle of elevation changes to 30° . if the plane is flying at speed of 720 km / hr , find the constant height at which the fighter plane is flying.
- As observed from the top of the light house ,100m above sea level, the angle of depression of ship sailing towards it, changes from 30° to 60° . Determine the distance travelled by the ship during the period of observation.
- From a window 60 m above the ground of a house in a street , the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are 60° and 45° respectively . Find the height of the opposite house.
- From the foot of tower, the angle of elevation of the top of a pillar is 60° and from the top of the tower the angle of elevation of this top is 30° . Find the height of the pillar.

TEST-2 MM-20

- The angles of elevation and depression of the top and bottom of a light house from the top of a building 60m high, are 30° and 60° respectively. Find
 - Find the height of light house .
 - Distance between the light house and building.
- Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles
- From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower.
- Find the height of a mountain if the elevation of its top at an unknown distance from the base is 60° and at a distance of 10 km further off from the mountain along the same line the angle of elevation is 30° .
- From a balloon vertically above a straight road the angle of depression of two cars at an instant are found to be 30° and 45° if the cars are 80 m apart. Find the height of the balloon.

ANSWERS OF PRACTICE QUESTIONS

MCQ: 1.b 2.c 3.c 4.b 5.b 6.b 7.d 8.a 9.a

Short answers type of questions: 1). $\frac{100\sqrt{3}}{\sqrt{3}-1} = 236.6$ m, 2) $200\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) = 315.6$ (approx.)

3) 16.67 m, 4) Height of pole = $20\sqrt{3}$ m ,distances are 20m and 40m

5) $0.8(\sqrt{3} + 1)$ m, 6) $7(\sqrt{3} + 1)$ m, 7) distance = $10\sqrt{3}$,height= 40m

8) distance = $10\sqrt{3}$ m ,height = $(\sqrt{3} - 1)$ m = 7.32m, 9) 693.33m, 10) 66.67m

CASE STUDY QUESTION: 1.(i) 30° (ii) 3000m (iii) $1500\sqrt{3}$ m

2.(i) 90m (ii) $60\sqrt{3}$ m (iii) $60\sqrt{3}$ m

3.(i) $58\sqrt{3}$ m (ii) 3.35 m/ s (approx)

CIRCLES

IMPORTANT CONCEPTS / RESULTS:

A tangent to a circle is a line that intersects/touches the circle at only one point.

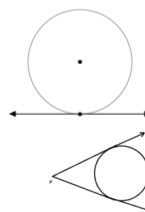
There is only one tangent at a point on a circle

There are exactly two tangents to a circle through a point lying outside the circle.

The tangent at any point of a circle is perpendicular to the radius

through the point of contact.

The length of tangents drawn from an external point to a circle is equal.



SOME ILLUSTRATIONS AND EXAMPLES:

MCQ:

1. In the below figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$.

If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

- (a) 65° (b) 60° (c) 50°
 (d) 40°

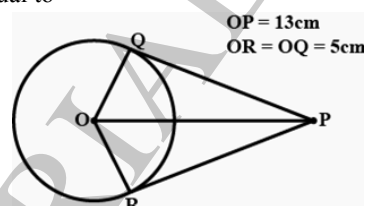
Solution: (C)

$\angle BAC = 180^\circ - (\angle ACB + \angle CBA)$ Angle sum property of triangle

$$= 180^\circ - (50^\circ + 90^\circ)$$

$$= 40^\circ$$

Now, $\angle BAC + \angle BAT = 90^\circ$ Tangent is perpendicular to radius
 $\Rightarrow \angle BAT = 50^\circ$



2. A circle has a number of tangents equal to

- (a) 0 (b) 1 (c) 2 (d) Infinite

Solution: (d)

A circle has infinitely many tangents, touching the circle at infinite points on its circumference.

3. If the angle between two radii of a circle is 110° , then the angle between the tangents at the ends of the radii is:

- (a) 90° (b) 50° (c) 70° (d) 40°

Solution: (c)

If the angle between two radii of a circle is 110° , then the angle between tangents is $180^\circ - 110^\circ = 70^\circ$. (By circles and tangents properties)

4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

- (a) 60cm^2 (b) 65cm^2 (c) 32cm^2 (d) 32.5cm^2

Solution: (a)

Step - 1: Find the length of PQ and PR (Using Pythagoras Property)

$$PQ = PR = 12\text{cm}$$

Step - 2: Find Area of quadrilateral

$$= 2 \times (\text{Area of Triangle POR or POQ})$$

$$= 2 \times \left(\frac{1}{2}\right) \times 12 \times 5$$

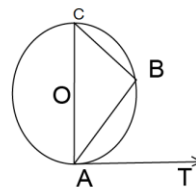
$$= 60\text{cm}^2$$

5. The length of the tangent from an external point A on a circle with centre O is

- (a) always greater than OA (b) equal to OA
 (c) always less than OA (d) cannot be estimated

Solution: (c)

- i. The tangent is perpendicular to the radius of the circle, then the angle between them is 90° .



- ii. So, OA is the hypotenuse for the right triangle OAB, which is right-angled at B.
- iii. As we know, for any right triangle, the hypotenuse is the longest side.
- iv. Therefore the length of the tangent from an external point is always less than the OA.

SHORT ANSWER TYPE QUESTIONS:

6. In Figure given, PQL and PRM are tangents to the circle with centre O at the points Q and R, respectively and S is a point on the circle such that $\angle SQL = 50^\circ$ and $\angle SRM = 60^\circ$. Find $\angle QSR$?

Solution:

Here $\angle OSQ = \angle OQS = 90^\circ - 50^\circ = 40^\circ$ and
 $\angle RSO = \angle SRO = 90^\circ - 60^\circ = 30^\circ$.

Therefore, $\angle QSR = 40^\circ + 30^\circ = 70^\circ$

7. BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 30^\circ$, then find $\angle PTA$?

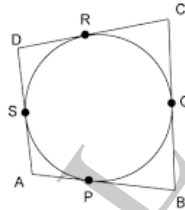
Solution:

$\angle BPA = 90^\circ$,

$\angle PAB = \angle OPA = 60^\circ$.

Also, $OP \perp PT$.

Therefore, $\angle APT = 30^\circ$ and $\angle PTA = 60^\circ - 30^\circ = 30^\circ$.



8. In the figure quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AD + BC = AB + CD$

Solution:

As we know that, length of tangents drawn from the external point are equal.
 Therefore,

$AP = AS \dots (1)$

$BP = BQ \dots (2)$

$CR = CQ \dots (3)$

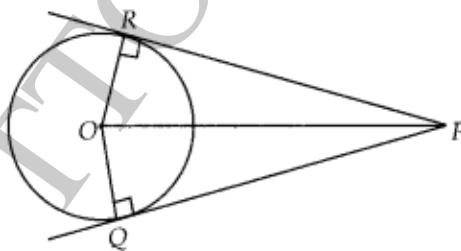
$DR = DS \dots (4)$

Adding equation (1),(2),(3) and (4), we get

$AP+BP+CR+DR=AS+BQ+CQ+DS$

$(AP+BP)+(CR+DR)=(AS+DS)+(BQ+CQ)$

$\Rightarrow AB+CD=AD+BC$

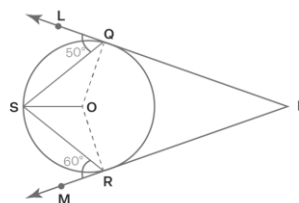


9. The lengths of tangents drawn from an external point to are equal.

Solution:

Given : A circle with centre O,

A point P lying outside the circle and two tangents PQ, PR on the circle from P



a circle

To Prove :

$$PQ = PR.$$

Join OP, OQ and OR.

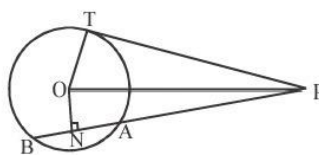
Then $\angle OQP$ and $\angle ORP$ are right angles,

$$OQ = OR \text{ (Radii of the same circle)}$$

$$OP = OP \text{ (Common)}$$

Therefore, $\triangle OQP \cong \triangle ORP$ (RHS)

This gives $PQ = PR$ (CPCT)



10. In Figure given, from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB.

- Prove that : (i) $PA \cdot PB = PN^2 - AN^2$
(ii) $PN^2 - AN^2 = OP^2 - OT^2$
(iii) $PA \cdot PB = PT^2$

Solution:

- (i) $PA \cdot PB = (PN - AN)(PN + AN)$
 $= (PN - AN)(PN + AN)$ (As $AN = BN$)
 $= PN^2 - AN^2$
- (ii) $PN^2 - AN^2 = (OP^2 - ON^2) - AN^2$ (As $ON \perp PN$)
 $= OP^2 - (ON^2 + AN^2)$
 $= OP^2 - OA^2$ (As $ON \perp AN$)
 $= OP^2 - OT^2$ (As $OA = OT$)
- (iii) From (i) and (ii)
 $PA \cdot PB = OP^2 - OT^2$
 $= PT^2$ (As $\angle OTP = 90^\circ$)

PRACTICE QUESTIONS**MCQS:**

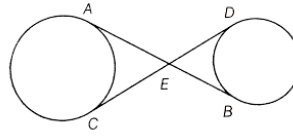
- If the circumference of a circle increases from 4π to 8π , then its area will become
A. half B. 2 times C. 4 times D. does not change
- The distance between two parallel tangents to a circle of radius 5cm is:
A. 10cm B. 11cm C. 12cm D. 14cm
- At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is
(A) 4 cm (B) 5 cm (C) 6 cm (D) 8 cm
- If angle between two radii of a circle is 130° , the angle between the tangents at the ends of the radii is :
(A) 90° (B) 50° (C) 70° (D) 40°

SHORT ANSWER TYPE QUESTIONS:

- Prove that the tangents drawn at the end-points of the diameter of a circle are parallel.
- If all the sides of a parallelogram touch a circle, then prove that the parallelogram is a rhombus.
- If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.
- Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.
- Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
- A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$
- If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by

$$r = \frac{a+b-c}{2}$$

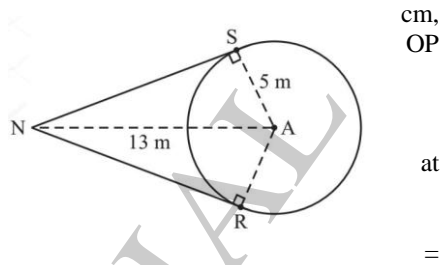
12. In Fig given, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.



13. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.
 14. Two tangents PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$

LONG ANSWER TYPE QUESTIONS:

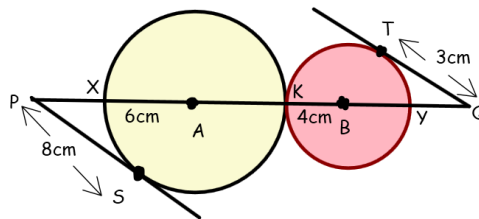
15. If a hexagon ABCDEF circumscribe a circle, prove that $AB + CD + EF = BC + DE + FA$.
 16. Two circles with centres O and O' of radii 3 cm and 4 respectively intersect at two points P and Q such that and OP are tangents to the two circles. Find the length of the common chord PQ.
 17. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
 18. In fig. circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$. If AD = 23cm, AB = 29cm, and DS 5cm, find the radius 'r' of the circle



19. Prove that opposites sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

20. Case Study I:

In a school in Chandigarh, organized a Cluster Level Football Tournament for boys. The Football team was very excited. The team captain Amar directed the team to assemble in the ground for practices. Only three boys Pavan, Salman and Arjun showed up. Amar drew a circle of radius 5 m on the ground. The center A was the position of Pavan. Amar marked a point N, 13 m away from center A as her own position. From the point N, she drew two tangential lines NS and NR and gave positions S and R to Salman and Arjun. Amar passes the ball to Pavan, Pavan passes it to Salman, Salman passes it to Amar, Amar passes it to Arjun, Arjun passes it to Pavan, Pavan passes it to Salman and so on.



- i. What is the measure of $\angle NSA$?
- ii. Find the distance between Salman and Amar.
- iii. How far does Amar have to pass the ball towards Arjun?

21. Case Study II:

A student draws two circles that touch each other externally at point K with centres A and B and radii 6 cm and 4cm, respectively as shown in the figure

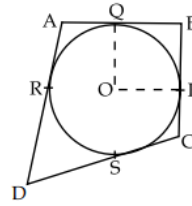
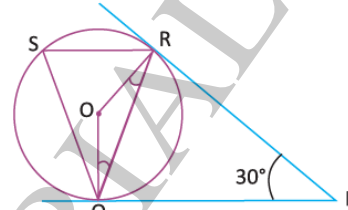
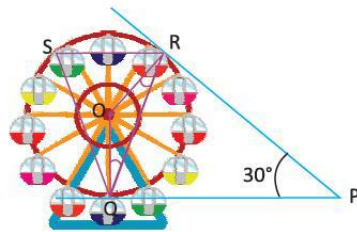
- i. Find the value of PA?
- ii. Find the value of PQ?

- iii. If two circles touch externally, then the number of common tangents can be drawn is ____

22. Case Study II:

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passengers carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.

- i. Find $\angle RQP$.
- ii. Find $\angle RSQ$.
- iii. If $PQ=40$ m and $OQ=30$ m then find PO ?



TEST PAPER – I

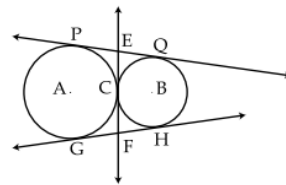
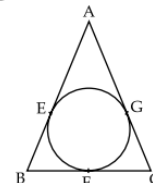
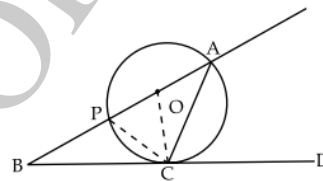
MARKS : 20

TIME : 35MIN

I. Answer the following questions

(5 × 2M = 10M)

1. O is the centre of the circle and BCD is a tangent to it at C. Prove that $\angle BAC + \angle ACD = 90^\circ$
2. Two concentric circles have centre O, $OP = 4$ cm, $OB = 5$ cm. AB is a chord of the outer circle and tangent to the inner circle at P. Find the length of AB.
3. In the isosceles triangle ABC in fig. $AB = AC$, show that $BF = FC$
4. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
5. In fig. two circles touch each other externally at C. Prove that the common tangent at C bisects the other two tangents



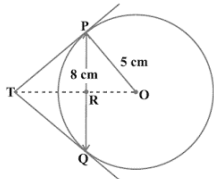
- Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
- In fig. circle touches the side BC of a triangle ABC at the point P and AB and AC produced at Q and R. Show that $AQ = \frac{1}{2}(\text{perimeter of } \triangle ABC)$
- PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

TEST PAPER – II

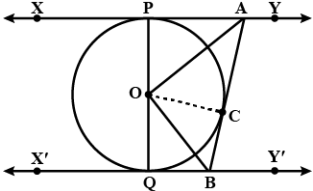
I. Answer the following questions

(5 × 2M = 10M)

- If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.



- Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.
- BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 30^\circ$, then find $\angle PTA$?
- In Figure given, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



- Prove that the line segments joining the points of contact of two parallel tangents is a diameter of the circle.

II. Answer the following questions

(4 × 3M = 12M)

- Two tangents PA and PB are drawn to a circle with centre O such that $\angle APB = 120^\circ$. Prove that $OP = 2AP$
- If all the sides of a parallelogram touch a circle, then prove that the parallelogram is a rhombus.
- Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.
- A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$

III. Answer the following questions

(2 × 4M = 8M)

- Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
- The radius of the in-circle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

SURFACE AREA AND VOLUME

In our earlier classes, we have already learnt about plane figures and some solid figures also.

Plane Figures: A plane figure has only two dimensions namely, length and breadth.

For example:

Triangles, rectangles, circles, etc

Solid Figures: A solid figure has three dimensions namely, length, breadth and height (thickness)

For examples:

Cuboids, cubes, cylinders, spheres, cones, etc.

(i) Examples of cuboids are a book, a tile, a room, etc

(ii) a cube is a cuboid whose length, breadth and height are equal.

Examples of cubes are dice, ice-cream, etc

(iii) Examples of cylinders are jars, circular pipes, circular pencils, road rollers, etc.

(iv) Examples of cones are conical tents, ice-cream cones (when it has not been filled), funnel, etc

(v) Examples of spheres are cricket balls, fully blown footballs etc.

Some useful formulae

Cuboid: Let l be the length, b be the breadth and h be the height of a cuboid

Total surface area (TSA) of the cuboid = $2(lb + bh + hl)$ unit²

Area of four walls = $2(l + b)h$ unit²

Volume of cuboid = $l \times b \times h$ unit³

Cube: Let a be the length of each edge (side) of a cube.

Total surface area (TSA) of the cube = $6a^2$ unit²

Area of four walls = $4a^2$ unit²

Volume of cube = a^3 unit³

Right circular cylinder: Let r be the radius of the circular base and h be the height of a right circular cylinder, Then

Curved surface area = $2\pi r h$

Total surface area of closed right circular cylinder = $2\pi r (h + r)$

Total surface area of right circular cylinder open at the top = $(2\pi r h + \pi r^2)$

Volume = $\pi r^2 h$

Right circular cone: Let r be the radius of the circular base, h be the height and l be the slant height of a right circular cone. $l^2 = h^2 + r^2$

Circumference of the base = $2\pi r$

Area of the circular base = πr^2

Curved surface area (CSA) = $\pi r l$

Total surface area = $\pi r (l + r)$

Volume = $\frac{1}{3}\pi r^2 h$

Sphere: Let r be the radius of a sphere, then

Curve surface area = $4\pi r^2$, Volume = $\frac{4}{3}\pi r^3$

EXAMPLES

[i] M.C.Q. (5 questions)

Q1. The surface area of a cube is 216 cm^2 , its volume is

- (a) 144 cm^3 (b) 196 cm^3 (c) 212 cm^3 (d) 216 cm^3

Q2. A solid is hemispherical at the bottom and conical above. If the surface area of the two parts equal, then the ratio of its radius and the height of its conical part is

- (a) 1 : 3 (b) $1 : \sqrt{3}$ (c) 1 : 1 (d) $\sqrt{3} : 1$

Q3. A cylinder and a cone are of same base radius and of same height. The ratio of the volume of the cylinder to that of the cone is

- (a) 2 : 1 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2

Q4. The radius (in cm) of the largest right circular cone that can be cut out of cube of edge 4.2 cm is

- (a) 4.2 cm (b) 2.1 cm (c) 8.1 cm (d) 1.05 cm

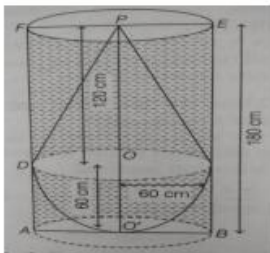
Q5. A rectangular sheet of paper 40 cm x 22 cm, is rolled to form a hollow cylinder of height 40 cm. The radius of the cylinder (in cm) is

- (a) 3.5 (b) 7 (c) 80/7 (d) 5

Answers: Q1(d), Q2(b), Q3 (b), Q4 (a), Q5 (b)

Case Study Based Question(1 question)

Q1. A flower pot is kept in one corner of the drawing room, a flower basket is kept inside the glass lies on the table. The shape of flower basket is hemisphere with radius 60 cm and upper shape is conical with height 120 cm from the bottom surface.



- (a) Find the capacity of the glass.
 (b) Find the volume of two combined figures.
 (c) Find the volume of cone.

OR

If the cost of painting the glass outside is Rs 1.20 per m², find the total cost of painting the CSA of the glass.

Solution: (a) Capacity of glass = volume of cylinder of (r=60cm ,h=180cm)
 $= \pi r^2 h = 22/7 \times 60 \times 60 \times 180 = 2036571.43 \text{ cm}^3$

(b) r=60cm ,h of cone =120cm
 volume of combine figure = volume of cone +volume hemisphere
 $= 1/3 \pi r^2 h + 2/3 \pi r^3$
 $= 452571.429 + 452571.429$
 $= 905142.858 \text{ cm}^3$

(c) r=60cm ,h of cone =120 cm
 volume of cone = $1/3 \pi r^2 h = 452571.429 \text{ cm}^3$

OR

r=60cm ,h=180cm ,

Curved surface area of glass = $2 \pi r h + \pi r^2$
 $= 2 \times 22/7 \times 0.6 \text{ m} \times 1.2 \text{ m} + 22/7 \times 0.6 \text{ m} \times 0.6 \text{ m}$
 $= 31.68 + 1.3 = 32.98 \text{ m}^2$

Cost of painting @Rs 1.20 per m²

Total cost = $32.98 \times 1.20 = \text{Rs } 39.58$

Short Answer Type Questions (3 questions)

- Q1. Two spheres have their volumes in the ratio 1:27. Find the ratio of their surface areas.
 Q2. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 21 cm.
 Q3. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π .

Answers Q1. Let the radius of two spheres are R and r respectively

Then, Ratio of volumes of two spheres = $\frac{\frac{4}{3}\pi R \times R \times R}{\frac{4}{3}\pi r \times r \times r} = \frac{1}{27}$

$$\frac{R}{r} = \frac{1}{3}$$

Ratio of surface area of two spheres = $\frac{4\pi R \times R}{4\pi r \times r} = \frac{R}{r} \times \frac{R}{r} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Q2. Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 21 = 2425 \text{ cm}^3$,

Q3. Volume of solid = volume of cone + volume of hemisphere = $\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi \times 1 \times 1 \times 1 + \frac{2}{3}\pi \times 1 \times 1 \times 1 = \text{cm}^3$

Long Answer Type Question (2 questions)

Example 1. Two cubes, each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution: Let the length of the edge of each of the two cubes be a cm.

Then, volume of each cube = $a^3 \text{ cm}^3$

Now, $a^3 = 64$

$a = 4 \text{ cm}$.

On joining both the cubes, the dimensions of the resulting cuboid

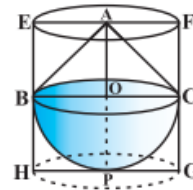
Length (l) = $4 + 4 = 8 \text{ cm}$, breadth = 4 cm , height = 4 cm .

So that, surface area of the resulting cuboid = $2(lb + bh + hl)$

$$= 2(8 \times 4 + 4 \times 4 + 4 \times 8) = 2(32 + 16 + 32)$$

$$= 2(80) = 160 \text{ cm}^2$$

Example 2. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)



Solution: Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere. The radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2} \times 4 \text{ cm} = 2 \text{ cm}$.

Volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$$= \frac{2}{3} \times 3.14 \times (2)^3 + \frac{1}{3} \times 3.14 \times (2)^2 \times 2 = 25.12 \text{ cm}^3$$

Now, let the right circular cylinder EFGH circumscribe the given solid. The radius of the base of the right circular cylinder = $HP = BO = 2 \text{ cm}$, and its height is

$$EH = AO + OP = (2 + 2) \text{ cm} = 4 \text{ cm}$$

So, the volume required = volume of the right circular cylinder – volume of the toy

$$= 3.14 (2)^2 \times 4 - 25.12 = 25.12 \text{ cm}^3$$

Hence, the required difference of the two volumes = 25.12 cm^3

Questions for Practice

M.C.Q. (5 questions)

Q1. If two solid hemispheres of same base radius are joined together along their bases, then curved surface area of this new solid is

- (a) $3\pi r^2$ (b) $4\pi r^2$ (c) $5\pi r^2$ (d) $6\pi r^2$

Q2. The ratio of surface areas of two spheres is 9:4. The ratio of their volumes

- (a) 27:8 (b) 8:27 (c) 3:2 (d) 2:3

Q3. The base radii of two right circular cones of the same height is 3:5. The ratio of their volumes is

- (a) 25:9 (b) 9:25 (c) 3:5 (d) 5:3

Q4. A cone, a hemisphere and a cylinder stand on equal bases and have equal height. The ratio of their volumes is

- (a) 1:2:3 (b) 2:3:4 (c) 1:3:4 (d) 2:3:5

Q5. What is the volume in cu cm of a cube whose surface area is 1944 sq cm?

- (a) 1728 cm³ (b) 4096 cm³ (c) 2744 cm³ (d) 5832 cm³

Answers: Q1. (b) $4\pi r^2$ Q2. (a) 27:8 Q3. (b) 9:25 Q4. (a) 1:2:3

Q5. (d) 5832 cm³

Assertion Reason type questions (1 question)

Question 1: STATEMENT - 1 (Assertion) and STATEMENT -2 (Reason) and has following four choices

(a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) Statement -1 and Statement -2 are true ; Statement -2 is a correct explanation for Statement -1.

(b) Statement -1 and Statement -2 are true ; Statement -2 is not a correct explanation for Statement -1.

(c) Statement -1 is True, Statement -2 is False.

(d) Statement -1 is False, Statement -2 is True. .

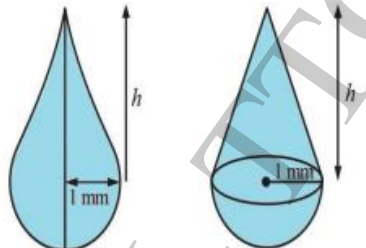
Q1. Statement -1 (Assertion) : Two cubes each of volume 125 cm³ are rejoined end to end to form a cuboid , the surface area of the resulting cuboid is 250 cm² .

Statement -2 (Reason) : If n cubes each of volume a³ cubic units are joined end to end to form a cuboid , then the surface area of the resulting cuboid is 2(2n+1) a² square units.

Answer 1. (a)

Case Study Based Questions

Q1. In the month of July in a particular year, it rained heavily throughout the day over the city of Goa. Amit observed that the rain drops as they reached him. Each rain drop was in the shape of a hemisphere surmounted by a cone of the same radius of 1 mm. Volume of one of such drops is 3.14 mm³. Anil collected the rain water in a pot having a capacity of 1099 cm³. [Use $\sqrt{2} = 1.4$]



Based on the above situation, answers the following questions.

- Find the total height of the drop.
- What is the total surface area of a hemisphere of radius r cm.
- What is the curved surface area of the drop?

OR

Find the no of drops in the vessel.

Q2. For a school Trophy :

A school decide to give a trophy of the best student in the class, which is the form of cylinder mounted on a solid hemisphere with the same radius and is made from some metal. This trophy is mounted on a wooden cuboid as shown in the figure.

Suppose the diameter of the hemisphere is 24 cm and total height of trophy is 28cm.

- Find the curve surface area of cylinder.
- Find the volume of cylinder.

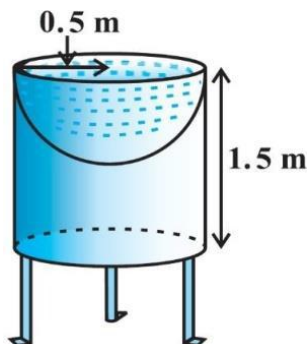


(c) Find the curved surface area of trophy.

OR

Find the weight of the metal used in making the trophy, if the weight of 1cm^3 of metal is 1.5gm.

Q3. Ashish wants to make a bird bath for his garden in the shape of cylinder of height 1.5m and radius 0.5 m with a hemispherical depression at one end, stands on three cylindrical pillars of radius 7 cm and height 2 m using POP as shown in figure.



- Find the curved surface area of the cylindrical part.
- Find the curved surface area of hemispherical depression.
- Find the volume of the three pillars.

OR

What is the curved surface area of 3 pillars?

Answer 1. (a) 2mm (b) $3\pi^2$ (c) 10.67 cm^2 OR 350000

Answer 2.

- $R=7\text{cm}$ $h= 16\text{ cm}$ CSA of cylinder = $2 \pi rh = 1206.88\text{ cm}^2$
- $R=12\text{ cm}$ $h=16\text{cm}$ volume of cylinder = $\pi r^2 h=7241.14\text{ cm}^3$
- $R=7\text{cm}$ $h= 16\text{ cm}$, CSA of trophy = CSA cylinder + CSA of hemisphere
 $=1206.88 + 3620.57= 4827.45\text{cm}^2$

OR

$R=7\text{cm}$ $h= 16\text{ cm}$

Volume of cylinder = 7241.14

Volume of hemisphere = $\frac{2}{3} \pi r^3 = 3620.57$

Total volume of trophy = 10861.71 cm^3

Weight of $1\text{ cm}^3 = 16292.565\text{ gm}$

Answer 3. (a) 4.71cm^2 (b) 1.57 cm^2 (c) 924 cm^2 OR 264 cm^2

Short Answer Type Question (10 questions)

Q1. Two cubes each of 10 cm edge are joined end to end . Find the surface area of the resulting cuboid.

Q2. The cost of painting the total outside surface of a closed cylindrical oil tank at 60 paise sq. m is Rs 237.60 and the height of the tank is 6 times the radius of the base of the tank.

(i) Find the height of the tank. [Take $\pi= 3.14$]

(ii) Also find the radius of the tank.

Q3. The radius and height of a solid right circular cone are in the ratio 5:12. If its volume is 314 cm^3 , find its total surface area. [Take $\pi= 3.14$]

Q4. The sum of radius of base and height of a right circular cylinder is 37cm. If the total surface area is 1628 cm^2 , find the volume of cylinder. [Take $\pi= \frac{22}{7}$]

Q5. The volume of a sphere is $2425\frac{1}{2}\text{ cm}^3$. Find its curved surface area.

Q6. Determine the ratio of the volume of a cube to that of a sphere which will exactly fit inside the cube.

Q7. If the height and base radius of a cone, each is increased by 50%, then what will be the ratio between the volume of the given cone and the new cone?

Q8. The radius and slant height of a right circular cone are in the ratio of 7 : 13 and its curved surface area is 286 cm^2 . Find its radius.

Q9. A toy is in the form of a cone of radius 7 cm mounted on a hemisphere of same radius. The total height of the toy is 31 cm. Find the total surface area of the toy.

Q 10. A decorative block is made of two solids-a cube and a hemisphere. The base of the block is the cube with edge of 7 cm and the hemisphere attached on the top has a diameter of 4.9 cm. If the block is to be painted, find the total area to be painted.

Answers : Q1. 1000 cm^2 Q2. (i) $h = 18\text{ m}$ (ii) $r = 3\text{ m}$, Q3. 282.6cm^2

- Q4. Volume of cylinder = 4620 cm^3 , Q5. 693 cm^2 , Q6. $6 : \pi$, Q7. $8 : 27$
 Q8. 7 cm , Q9. 858 cm^2 , Q10. 312.86 cm^2

Long Answer Type Question (5 questions)

- Q1. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.
 Q2. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use $\pi = 3.14$)
 Q3. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the volume and the total surface area of the solid. [Take $\pi = 22/7$]
 Q4. A tent is of the shape of a right circular cylinder up to a height of 3 meters and then becomes a right circular cone with a maximum height of 13.5 meters above the ground. Calculate the cost of painting the inner side of the tent at the rate of Rs 2 per square meter, if the radius of the base is 14 meters.
 Q5. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2 .

Answers : Q1. Volume of water left in the cylinder = 1.131 m^3

Q2. Total weight = 892.2624 kg

Q3. Volume of solid = 641.66 cm^3

TSA of solid = 418 cm^2

Q4. T which is to be painted = 1034 m^2

Cost of painting = Rs 2068

Q5. Total area of canvas = 44 m^2

Total cost of the canvas at the rate of Rs 500 per m^2
 = Rs 22000

Test Paper 1 MAX. MARKS: 20

Section A

- Q1. Find the edge of a cube whose volume is equal to that of a cuboid of dimensions 8 cm x 4 cm x 2 cm.
 Q2. If the perimeters of the base of two right circular cones are in the ratio 3 : 4 and their volumes are in the ratio 9 : 32. Find the ratio of their heights.
 Q3. If the volume of a solid sphere of radius r cm is equal to the volume of a solid cone of height r cm. What will be the radius of the base of the cone.

Section B

- Q4. Water flows at the rate of 10 metre per minute from a cylindrical pipe 5 mm in diameter. How long will it take to fill up a conical vessel whose diameter at the base is 40 cm and depth 247 cm?
 Q5. If four times the sum of the areas of two circular faces of a cylinder of height 8 cm is equal to twice the curve surface area. Find the diameter of the cylinder.

Section C

- Q6. A cylindrical vessel 32 cm high and 18 cm as the radius of the base, is filled with sand. This vessel is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius of its base.
 Q7. A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m. Find the total area of the canvas required in m^2 .

Test Paper 2 MAX. MARKS: 30

Section A

- Q1. If the radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio between the volume of cylinder to that of the cone.
- Q2. If a solid sphere with total surface area 48 cm² is bisected into two hemispheres Find the total surface area of any one of the hemisphere.
- Q3. Find the curved surface area of a right circular cone of height 15 cm and base diameter is 16 cm.
- Q4. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side of 3 cm to form a cone. Find the volume of the cone so formed.
- Q5. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 cm. Find the radius of the sphere.

Section B

- Q6. A solid consists of a circular cylinder surmounted by a right circular cone. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone. Find the height of the cylinder.
- Q7. Find the mass of a solid cone of silver metal having base diameter 14 cm and vertical height 51 cm. The density of silver is 10 g/cm³.
- Q8. From a solid cylinder of height 30 cm and radius 7 cm, a conical cavity of height 24 cm and same radius is hollowed out. Find the total surface area of the remaining solid.
- Q9. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel.

Section C

- Q10. An ice-cream cone is the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. Find the volume of the ice-cream cone if the height of the cone is 9 cm and the radius of its base is 2.5 cm.
- Q11. The internal and external diameters of a hollow hemispherical vessel are 12 cm and 16 cm respectively. If the cost of painting 1 cm² of the surface area is Rs 5.00, find the total cost of painting the vessel all over. ($\pi = 3.14$)

Answers

Test Paper 1

- Q1. 2 cm Q2. 1 : 2 Q3. 2r cm. Q4. 51 min 12 sec
Q5. 8 cm Q6. 36 cm Q7. 7920 m²

Test Paper 2

- Q1. 8Q2. 36 cm²Q3. 136 π cm²Q4. 12 π cm³
Q5. 6 cm Q6. 2h/3 Q7. 26.180 kg Q8. 2024 cm²
Q9. 1642.67 cm³Q10. 275/3 cm³Q11. Rs 3579.60

STATISTICS

Important Concepts & Results

- (i) Assumed Mean method or Shortcut method

$$\text{Mean} = a + \frac{\sum_{i=1}^n fidi}{\sum_{i=1}^n fi}$$

Where a = assumed mean And di = Xi - a

- (ii) Step deviation method. Mean = $a + \frac{\sum_{i=1}^n fiui}{\sum_{i=1}^n fi} \times h$

Where a = assumed mean

h = class size And ui = (Xi - a)/h

- (iii) Median of a grouped frequency distribution can be calculated by

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

Where,

l = lower limit of median class n = number of observations

cf = cumulative frequency of class preceding the median class f = frequency of median class

h = class size of the median class.

- (iv) Mode of grouped data can be calculated by the following formula.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Where

l = lower limit of modal class

h = size of class interval

f₁ = Frequency of the modal class

f₀ = frequency of class preceding the modal class

f₂ = frequency of class succeeding the modal class

- (v) Empirical relationship between the three measures of central tendency.

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$

$$\text{Or, Mode} = 3 \text{ Median} - 2 \text{ Mean.}$$

Multiple Choice Questions (5x1)

- The relationship between mean, median and mode for a moderately skewed distribution is
 - mode = median - 2 mean
 - mode = 3 median - 2 mean
 - mode = 2 median - 3 mean
 - mode = median - mean
- Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is
 - 48
 - 49
 - 50
 - 60
- Which of the following can not be determined graphically?
 - Mean
 - Median
 - Mode
 - None of these

4. Mode is the

- A. middle most frequent value
- B. least frequent value

- C. maximum frequent value
- D. none of these

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

5. Assertion (A): If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason (R): Median = (mode + 2 mean)

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

- 1. Answer: B
- 2. Answer: C
- 3. Answer: A
- 4. Answer: C
- 5. Correct Answer is Option (c)

MCQ (for practise)

1. If the arithmetic mean of x , $x + 3$, $x + 6$, $x + 9$ and $x + 12$ is 10, then $x = ?$

- A. 1
- B. 2
- C. 6
- D. 4

2. If the mean of first n natural numbers is $5n/9$, then $n = ?$

- A. 6
- B. 7
- C. 9
- D. 10

3. If 35 is removed from the data, 30, 34, 35, 36, 37, 38, 39, 40 then the median increases by:

- A. 2
- B. 1.5
- C. 1
- D. 0.5

4. The Median when it is given that mode and mean are 8 and 9 respectively, is:

- A. 8.57
- B. 8.67
- C. 8.97
- D. 9.24

5. Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

Assertion: the mode of the call received on 7 consecutive day 11,13,13,17,19,23,25 is 13.

Reason: Mode is the value that appears most frequent;

- a) Both Assertion and Reason are correct and reason is correct explanation for the.
- b) Both Assertion and Reason are false but reason is not correct explanation for assertion.
- c) Assertion is correct but reason is false.
- d) Both Assertion and reason are false.

- 1. **Answer: D**
- 2. **Answer: C**
- 3. **Answer: D**
- 4. **Answer: B**
- 5. a) both assertion and reason are correct but reason is not correct explanation for assertion.

CASE STUDY BASED QUESTIONS

Case Study 1: PARK

A decision has been made by an agency XYZ to install specially tailored playground equipment in multiple parks within different colonies. In order to determine the age group of children who frequent a specific park in a particular colony, a study was conducted. The table below illustrates the categorization of children based on their ages and their corresponding park activities.

Based on the above information, answer the following questions.

- (i) The maximum number of children are of the age-group
 - a) 12-14
 - b) 10-12
 - c) 14-16
 - d) 8-10
- (ii) The lower limit of the modal class is
 - a) 10
 - b) 12
 - c) 14
 - d) 8
- (iii) Frequency of the class succeeding the modal class is
 - a) 58
 - b) 70
 - c) 42
 - d) 27
- (iv) The mode of the ages of children playing in the park is
 - (a) 9years
 - (b) 8years
 - (c) 11.5years
 - (d) 10.6 years
- (v) If mean and mode of the ages of children playing in the park are same, then median will be equal to
 - (a) Mean
 - (b) Mode
 - (c) Both (a) and (b)
 - (d) Neither (a) nor (b)

Answer: Case study 1

- 1. (i) (b) Since, the highest frequency is 70,

therefore, the maximum number of children are of the age-group 10-12.

(ii) (a) Since, the modal class is 10-12

Lower limit of modal class = 10

(iii) (c) Here, $f_0 = 58$, $f_1 = 70$ and $f_2 = 42$

Thus, the frequency of the class succeeding the modal class is 42.

(iv) (d) **Mode** = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$= 10 + \left(\frac{70 - 58}{140 - 58 - 42} \right) \times h$$

$$= 10.6$$

(v) (c) Given that, Mean = Mode

By Empirical relation, we have

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mode}$$

$$3 \text{ Mode} = 3 \text{ Median}$$

$$\text{Median} = \text{Mode} = \text{Mean}$$

CASE STUDY BASED QUESTIONS (for practice)

CASE STUDY 1: COVID-19

COVID-19 Pandemic The COVID-19 pandemic, also known as coronavirus pandemic, is an ongoing pandemic of coronavirus disease caused by the transmission of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) among humans.

The following tables show the age distribution of case admitted during a day in two different hospitals.

Table 1

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	6	11	21	23	14	5

Table 2

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	8	16	10	42	24	12

Refer to table 1

1. The average age for which maximum cases occurred is

a) 32.24

c) 36.82

b) 34.36

d) 42.24

2. The upper limit of modal class is

a) 15

c) 35

b) 25

d) 45

3. The mean of the given data is

a) 26.2

b) 32.4

c) 33.5

d) 35.4

Refer to table 2

4. The mode of the given data is

a) 41.4

c) 55.3

b) 48.2

d) 64.6

5. The median of the given data is

JSUNIL TUTORIAL

- a) 32.7
- b) 40.2
- c) 42.3
- d) 48.6

CASE STUDY 2:

Electricity consumption pertains to the utilization of electric energy. The worldwide consumption of electricity rises at a faster pace compared to the population growth, which results in an augmented average consumption of electricity per person, otherwise known as per capita electricity consumption.

A survey is conducted for 56 families of a Colony A. The following table gives the weekly consumption of electricity of these families.

Weekly consumption (in units)	0-10	10-20	20-30	30-40	40-50	50-60
No. of families	16	12	18	6	4	0

The similar survey is conducted for 80 families of Colony B and the data is recorded as below:

Weekly consumption (in units)	0-10	10-20	20-30	30-40	40-50	50-60
No. of families	0	5	10	20	40	5

Refer to data received from Colony A

1. The median weekly consumption is
 - a) 12 units
 - b) 16 units
 - c) 20 units
 - d) None of these

2. The mean weekly consumption is
 - a) 19.64 units
 - b) 22.5 units
 - c) 26 units
 - d) None of these

3. The modal class of the above data is I
 - a) 0-10
 - b) 10-20
 - c) 20-30
 - d) 30-40

Refer to data received from Colony B

4. The modal weekly consumption is
- a) 38.2 units
 - b) 43.6 units
 - c) 26 units
 - d) 32 units
5. The mean weekly consumption is
- a) 15.65 units
 - b) 32.8 units
 - c) 38.75 units
 - d) 48 units

Case Study 3 -Toll Tax Collection

On a particular day, National Highway Authority of India (NHAI) checked the toll tax collection of a particular toll plaza in Rajasthan. The following table shows the toll tax paid by drivers and the number of vehicles on that particular day.

Toll tax (in Rs)	30-40	40-50	50-60	60-70	70-80
No. of vehicles	80	110	120	70	40

Based on the above information, answer the following questions.

- (i) If A is taken as assumed mean, then the possible value of A is
- a) 32
 - b) 42
 - c) 85
 - d) 55
- (ii) The mean of toll tax received by NHAI by assumed mean method is
- (a) 52
 - (b) 52.14
 - (c) 52.50
 - (d) 53.50
- (iii) 'The mean of toll tax received by NHAI by direct method is
- (a) equal to the mean of toll tax received by NHAI by assumed mean method
 - (b) greater than the mean of toll tax received by NHAI by assumed mean method
 - (c) less than the mean of toll tax received by NHAI by assumed mean method
 - (d) none of these
- (iv) The average toll tax received by NHAI in a day, from that particular toll plaza, is
- (a) 21000
 - (b) 21900
 - (c) 30000
 - (d) none of these

Solutions:

Case study 1

Answer: c) 36.82

Answer: d) 45

Answer: d) 35.4

Answer: a) 41.4

Answer: b) 40.2

Case Study 2

Answer: c) 20 units

Answer: a) 19.64 units

Answer: c) 20-30

Answer: b) 43.6 units

Answer: c) 38.75 units

Case Study 3 :

Answer:

Class	Class marks (x_i)	$d_i = x_i - A$	Frequency (f_i)	$f_i d_i$
30-40	35	-20	80	-1600
40-50	45	-10	110	-1100
50-60	$55 = A$	0	120	0
60-70	65	10	70	700
70-80	75	20	40	800
Total			$\Sigma f_i = 420$	$\Sigma f_i d_i = -1200$

(i) (d): Clearly, the possible values of assumed mean (A) are 35, 45, 55, 65, 75.

(ii) (b): Required Mean = $a + \frac{\Sigma_{i=1}^n f_i d_i}{\Sigma_{i=1}^n f_i}$

$$= 55 - \frac{1200}{420}$$

$$= 52.14$$

(iii) (a): Mean by direct and assumed mean method are always equal.

(iv) (d): Average toll tax received by a vehicle = Rs 52.14

Total number of vehicles = 420

Average toll tax received in a day = 52.14×420

Rs 21898.80

Long answer type question (solved)

1 Find the mode of the following distribution:

Class Interval	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	30	45	75	35	25	15

Answer:

$$f_1 = 75, f_0 = 45, f_2 = 35$$

$$\text{Model class} = 20-25 \quad h=5 \quad l=20$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 20 + \frac{75 - 45}{2 \times 75 - 45 - 35} \times 5$$

$$= 20 + \frac{30}{70} \times 5 = 22.14$$

2 Find the median marks for the following distribution:

Classes	0-10	10-20	20-30	30-40	40-50
Number of students	2	12	22	8	6

Answer:

Class	Frequency	c.f.
0-10	2	2
10-20	12	14
20-30	22	36
30-40	8	44
40-50	6	50

$$N=50 \quad \frac{N}{2} = 25 \quad \text{median class} = 20-30$$

$$l = 20 \quad f = 22 \quad cf = 14 \quad h = 10$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h = 20 + \frac{25 - 14}{22} \times 10 = 20 + 5 = 25$$

Long answer type question (Practise)

- 1 The given distribution shows the number of runs scored by some top batsman of the world in one day international cricket matches.

RUNS SCORED	NUMBER OF BATSMAN
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

Find the mode of the data.

- 2 If the median of the distribution given below is 28.5. Find the value of 'x' and 'y'.

C.I	FREQUENCY
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5

- 3 Find the median of the following frequency distribution

MARKS	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
FREQUENCY	2	5	9	12	17	20	15	9	7	4

- 4 The median of the following data is 20.75. Find the missing frequencies 'x' and 'y'.
If the total frequency is 100.

CI	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
FREQUENCY	7	10	x	13	y	10	14	9

- 5 If the mean of the following distribution is 6, find the value of 'p'.

X	2	4	6	10	P+6
Y	3	2	3	1	2

6 The following distribution gives the weight of 60 students of a class. Find the mean and mode weights of the students.

Weight (in kg)	40-44	44-48	48-52	52-56	56-60	60-64	64-68	68-72
No. of Students	4	6	10	14	10	8	6	2

7 The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2 :

Class	0-20	20-40	40-60	60-80	80-100	100-120	Total
Frequency	5	f_1	10	f_2	7	8	50

Answers :

(1) 4608.7 runs

(2) $\begin{pmatrix} x = 8 \\ y = 8 \end{pmatrix}$

(3) 525

(4) $y = 20$, $x = 17$

(5) $P = 7$

(6) By using formula for mean and mode. Correct mean = 55.2 mode = 54

(7) By using direct method to find mean. We get $f_1 + f_2 = 20$ and $3f_1 + 7f_2 = 108$

After solve $f_1 = 8$ and $f_2 = 12$

TEST-1 (20 Marks)

1 The mean of the first 10 multiples of 6 is (1)

A. 3.3

B. 33

C. 34

D. none of these

2 While computing mean of the grouped data, we assume that the frequencies are: (2)

A. evenly distributed over all the classes

B. centered at the class marks of the classes

C. centered at the upper limits of the classes

D. centered at the lower limits of the classes

3 CASE STUDY

(5)

Analysis of Electricity Consumption in a Locality

x	30-40	40-50	50-60	60-70	70-80	80-90	90-100
f	6	10	16	X	10	5	2

An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summaries in the following table.

Monthly Consumption (in kwh)	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Number of families	2	5	x	12	17	20	y	9	7	4

Based on the above information, answer the following questions.

- (i) The value of $x + y$ is
 a) 100 (b) 42 (c) 24 (d) 200
- (ii) If the median of the above data is 525, then x is equal to
 (a) 10 (b) 8 (c) 9 (d) none of these
- (iii) What will be the upper limit of the modal class?
 (a) 400 (b) 600
 (c) 650 (d) 700
- (iv) The average monthly consumption of a family of this locality is approximately
 (a) 520kwh (b) 522kwh (c) 540 kwh (d) none of these
- (v) If A be the assumed mean, then A is always
 (a) $>$ (Actual mean) (b) $<$ (Actual Mean)
 (c) $=$ (Actual Mean) (d) can't say

4 If the mode of the following distribution is 57.5. Find the value of x . (4)

5 A life insurance agent found the following data for distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to persons having age 18 yrs onwards but less than 60 years. (4 mark)

Age (in yrs)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No. of policy holders	2	6	24	45	78	89	92	98	100

Age (in yrs)	Frequency (f)	cf
Below 20	2	2
20-25	6-2=4	6
25-30	24-6=18	24
30-35	45-24=21	45(cf)

35-40 (Median class)	33(f)	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100(N)

6 Following distribution gives the marks scored by a class of 20 students. If median of given data is 14.4, find x and y. (4)

Marks	0-6	6-12	12-18	18-24	24-30
No. of students	4	X	5	Y	1

Answer Key TEST (20):

1 Answer: B

2 **Answer: B**

3 Answer:(i) (c) : Here, it is given that total frequency = 100

$$76+x+y=100 = x+y=24$$

ii) c) x=9

(iii) (b) upper limit of modal class is 600.

(iv) (b): 522

(v) d

4 Mode=57.5 modal class=50-60

$$l=50 \quad f_0=10 \quad f_1=16 \quad f_2=x \quad h=10$$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 50 + \frac{16 - 10}{32 - 10 - x} \times 10$$

$$57.5 = 50 + \frac{60}{22 - x} \quad \text{Or } 7.5 = \frac{60}{22 - x}$$

$$X=14$$

5 Median number = $\frac{N}{2} = 50$

$$Median = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

l=35, h=5 put value in the formula and solve

6 Median=14.4 median class=12-18

$$l=12 \quad f=5 \quad cf=4+x \quad h=6$$

Put above values in median formula get x=4

$$x+y=10 \quad 4+y=10 \quad \text{or } y=6$$

TEST-2 (30)

1 Find mode, using an empirical relation, where it is given that mean and median are 10.5 and 9.6 respectively.

- 2 In a frequency distribution, if a = assumed mean=55, $\sum_{i=1}^n fi=100$, $h=10$ and $\sum_{i=1}^n fix_i=-30$, then find mean of the distribution. (2)
- 3 Write the median class of the following distribution. (2)

Classes	Frequency
0-10	4
10-20	4
20-30	8
30-40	10
40-50	12
50-60	8
60-70	4

- 4 Find the mean of the given data is 15.45 (3)

Class	Frequency
0-6	6
6-12	8
12-18	p
18-24	9
24-30	7

- 5 The median of the distribution given below is 14.4. Find the values of x and y, if the sum of frequency is 20. (3)

C.I	Frequency
0-6	4
6-12	X
12-18	5
18-24	Y
24-30	1

- 6 Find the median of the following data (3)

C.I	5-15	15-25	25-35	35-45	45-55	55-65	65-75
Freq.	6	10	16	15	24	8	7

7 Determine the missing frequency x , from the following data when Mode is 67 (3)

Class	40-50	50-60	60-70	70-80	80-90
Frequency	5	x	15	12	7

8 Find the mean, median and mode of the following data (4)

C.I	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Freq.	6	8	10	12	6	5	3

9 Find the missing frequency (f_1, f_2 and f_3) in the following frequency distribution when it is given that $f_1/f_3=4:3$ and mean =50. (4)

C.I	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	f_2	f_3	19	120

10 If the mean of the following frequency distribution is 65.6, find the missing frequencies. (5)

Class	Frequency
10-30	5
30-50	8
50-70	f_1
70-90	20
90-110	f_2
110-130	2
Total	50

PROBABILITY

IMPORTANT NOTES:

Experimental or empirical probability P (E) of an event E is

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

The theoretical probability (also called classical probability) of an event A, written as P(A), is defined as

$$P(A) = \frac{\text{number of favourable outcomes of A}}{\text{total number of favourable outcomes}}$$

Two or more events of an experiment, where occurrence of an event prevents occurrences of all other events, are called **Mutually Exclusive Events**.

COMPLIMENTARYEVENTSANDPROBABILITY

We denote the event 'notE' by \overline{E} . This is called the **complement** event of event E.

So, $P(E) + P(\text{not}E) = 1$

i.e., $P(E) + P(\text{not } E) = 1$, which gives us $P(\text{not } E) = 1 - P(E)$. In general, it is true that

for an event E, $P(E) = 1 - P(\text{not } E)$





















































- ☞ The probability of an event which is impossible to occur is 0. Such an event is called an **Impossible event**.
- ☞ The probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.
- ☞ The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$
- ☞ An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

DECKOF CARDSANDPROBABILITY

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠), red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards.

Example set of 52 poker playing cards

Suit	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

Equally likely events: Two or more events are said to be equally likely if each one

of them has an equal chance of occurrence.

Mutually Exclusive events: Two or more events are mutually exclusive if the occurrence of each event prevents the every other event.

Complementary events: Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favorable event is called Complementary event.

Exhaustive events: All the events are exhaustive events if their union is the sample space.

Sure events: The sample space of a random experiment is called sure or certain event as any one of its elements will surely occur in any trail of the experiment.

Impossible event: An event which will occur on any account is called an impossible event.

ILLUSTRATIONS:

(A) MCQ TYPE QUESTIONS:

1. Cards are marked with numbers between 1 to 50 are placed in the box and mixed thoroughly. One card is drawn at random. What is the probability of getting a multiple of 5?

- (a) $\frac{1}{5}$ (b) $\frac{9}{48}$ (c) $\frac{4}{5}$ (d) $\frac{7}{48}$

Ans. (b)

2. A coin is tossed 1000 times and 560 times the Head occurs. The empirical probability of occurrence of Head in this case is

- (a) 0.5 (b) 0.56 (c) 0.44 (d) 0.056

Ans. (b)

3. What is the probability of getting an ace from a well shuffled deck of cards?

- (a) $\frac{1}{2}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{1}{13}$

Ans. (d)

4. A die is thrown once what is the probability of getting a number less than or equal to 4?

- (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) 0

Ans. (a)

5. Assertion: The probability of an event that cannot happen or which is impossible is equal to zero.

Reason: The probability lies between 0 and 1. Hence, it cannot be negative.

- a) Both Assertion and Reason are correct and reason is correct explanation for the.
b) Both Assertion and Reason are false but reason is not correct explanation for assertion.
c) Assertion is correct but reason is false.
d) Both Assertion and reason are false.

Ans. (a)

(B) CASE BASED STUDY QUESTIONS:

Building Block kit

Prateek goes to a toy shop to purchase a building block kit for his son. He found that the kit contains 120 blocks, of which 40 are red, 25 are blue, 30 are green and the rest are yellow.

His son picks up a block at random.

1. Find the probability that the block is

i) of red color (ii) not blue color

Ans. i) Probability of getting red colour is $= \frac{40}{120} = \frac{1}{3}$

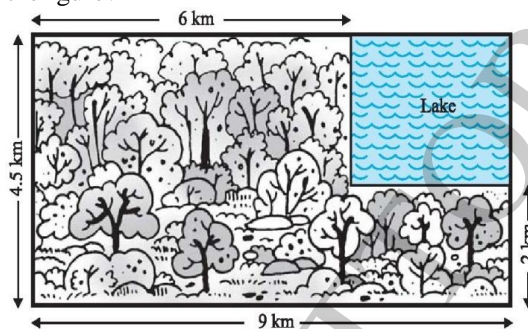
ii) Probability of not blue colour $= \frac{95}{120} = \frac{19}{24}$

2. A blue colour block taken out is not placed back in the kit. What is the probability of getting a blue or green block taken out at random?

Ans. Probability of getting blue or green block $= \frac{24+30}{119} = \frac{54}{119}$

(C) SHORT ANSWER TYPE QUESTIONS:

1. A missing helicopter is reported to have crashed somewhere in the rectangular region shown in Figure. What is the probability that it crashed inside the lake shown in the figure?



Ans. Probability $= \frac{\text{area of lake}}{\text{area of rectangular region}} = \frac{3 \times 2.5}{9 \times 4.5} = \frac{7.5}{40.5} = \frac{5}{81}$

2. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Ans. Probability of getting black ball $= \frac{x}{12}$

Probability of after adding 6 more black balls is $= \frac{x+6}{18}$

Given that $\frac{x+6}{18} = 2 \times \left\{ \frac{x}{12} \right\}$

$$\frac{6(x+6)}{18} = x$$

$$\frac{x+6}{3} = x$$

$$x+6 = 3x$$

$$3x - x = 6$$

$$2x = 6$$

$$x = 3$$

3. A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone if it is good but the trader will only buy a mobile if it has no major defect. One phone is selected at random from the lot. What is the probability that it is

- (i) A good phone (ii) a bad phone

$$\text{Ans. Probability of getting a good phone} = \frac{42}{48} = \frac{7}{8}$$

$$\text{Probability of getting a bad phone} = \frac{6}{48} = \frac{1}{8}$$

QUESTIONS FOR PRACTICE:

MCQ TYPE QUESTIONS:

- Two dice were thrown simultaneously. What is the probability of getting numbers on both the dice such that their sum is a prime number?
(a) $\frac{13}{36}$ (b) $\frac{12}{36}$ (c) 0 (d) 1
- A box contains marbles numbered from 12 to 78. What is the probability of getting a multiple of 12?
(a) $\frac{7}{66}$ (b) $\frac{6}{67}$ (c) $\frac{5}{66}$ (d) $\frac{9}{67}$
- What is probability of getting 53 Sundays in 2024?
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$
- From a well shuffled deck of cards what is the probability of getting a red non-face card?
(a) $\frac{2}{13}$ (b) $\frac{3}{13}$ (c) $\frac{4}{13}$ (d) $\frac{5}{13}$

5. **Assertion: If $P(E) = 0.07$ then $P(\text{not } E) = 0.93$.**

Reason: $P(E) + P(\text{not } E) = 1$

- a) Both Assertion and Reason are correct and reason is correct explanation for the.
b) Both Assertion and Reason are false but reason is not correct explanation for assertion.
c) Assertion is correct but reason is false.
d) Both Assertion and reason are false.

SHORT ANSWER TYPE QUESTIONS:

- Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12
- One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting neither a face card nor an ace?
- A coin is tossed two times. Find the probability of getting (i) at most one head (ii) no head.
- A box contains 90 discs numbered 1 to 90. What is the probability of getting
(i) A perfect cube
(ii) A multiple of 3 and 5.
- A number is selected from the numbers 1, 2, 3 and then a second number is randomly selected from the number 1, 4, 9. What is the probability that the

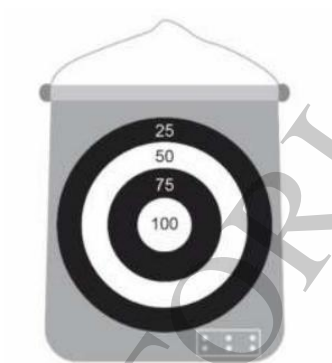
product xy of the two numbers will be less than 9?

CASE STUDY BASED QUESTIONS:

1. **DART AND ARROW GAME:**

Nishant and Kapil are playing a game of darts. They use this dart board. If your dart strikes outside the circles, you get zero points. Here are the rules of the game: When your dart is placed in any circular region, you get the points mentioned in that region. During the game all the darts of both Nishant and Kapil fall in the circular region. The radius of the innermost circle is 7 cm and the width of all other circular regions is 7 cm.

Answer the following questions.



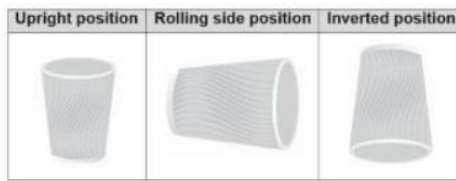
- (a) Nishant throws the first dart. What is the probability of Nishant getting a score of 100 in first throw?
- (b) Kapil threw a dart which hits the board. What is the probability of dart hitting the outermost circular region on the board?

2. **TOSSING A PAPER CUP:**

This is a paper cup.



Jaya tossed the cup. When the cup lands on the table, it can land in three possible positions as shown in the figure below.

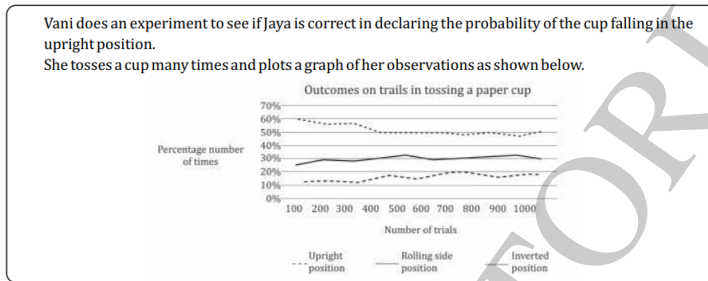


To calculate the probability of falling in each position, Jaya tosses the cup her observations in the table below.

Cup after toss	Frequency
Upright	20
Rolling side	5
Inverted	35

60 times. She records

- (a) Jaya tosses the cup one more time. What would be the probability of the cup falling in the upright position.
- (b)



Based on the graph and Vani's observations what should be the probability of the cup falling in the upright position if Jaya were to toss the cup again?

3. FREE TICKETS FOR WORLD CUP:

Geeta wanted to watch football world cup final match. She saw an advertisement that a radio station has 25 free tickets to football world cup final match to give away. Radio announced that one participant can send only one SMS for free ticket. SMS's are received from 20000 listeners out of which 12000 are female. SMS's are then selected at random one at a time until all free tickets are given away.



- (a) The first 24 tickets have been given away to the participants and Gita's SMS's has yet not been selected. What is Geeta's chance of winning the last ticket, based on above said information.

- (b) Out of first 24 tickets 14 males have already won the ticket and remaining are won by females.
Chances that last ticket is won by Geeta.

ANSWERS:

MCQ TYPE QUESTIONS: CASE STUDY BASED QUESTIONS:

1. A 2. B 3. B 4. D 5. A 1. (a) $\frac{1}{16}$ (b) $\frac{7}{16}$

SHORT ANSWER TYPE QUESTIONS: 2. (a) $\frac{1}{12}$ (b) 0.2


1. $\frac{5}{36}, \frac{1}{36}$ 5. $\frac{5}{9}$ 3. (a) $\frac{1}{19976}$ (b) $\frac{11990}{19976}$
 2. $\frac{9}{13}$
 3. $\frac{3}{4}, \frac{1}{4}$
 4. $\frac{4}{90}, \frac{2}{30}$

CLASS TEST-1

(CHAPTER: PROBABILITY)

CLASS: X MARKS: 20

SN O	QUESTION	MAR KS
	SECTION A	1X7= 7
1.	Two dice are thrown simultaneously. The probability of getting a sum of 9 is: (A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{1}{9}$ (D) $\frac{4}{9}$	1
2.	What is the probability of getting 53 Mondays in a leap year? (A) $\frac{1}{7}$ (B) $\frac{53}{366}$ (C) $\frac{2}{7}$ (D) $\frac{7}{366}$	1
3.	A jar contains 24 marbles. Some are red and others are white. If a marble is drawn at random from the jar, the probability that it is red is $\frac{2}{3}$, then the number of white marbles in the jar is: (A) 10 (B) 6 (C) 8 (D) 7	1
4.	If $P(E) = 0.07$, then what is the probability of 'not E'? (A) 0.93 (B) 0.95 (C) 0.89 (D) 0.90	1
5.	A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is: (A) 4 (B) 13 (C) 48 (D) 51	1
6.	A number x is chosen at random from the numbers -2, -1, 0, 1, 2. Then the probability that $x^2 < 2$ is? (A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$	1
7.	. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, then the number of blue balls in a bag is: (A) 5 (B) 10 (C) 15 (D) 20	1

SECTION-B		2X4 =8
8.	A carton of 24 bulbs contains 6 defective bulbs. One bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that these cond bulb is defective?	2
9.	A letter of English alphabets is chosen at random. Determine the probability that the letter is a consonant.	2
10.	Savita and Hamida are friends. What is the probability that both will have (i)different birthdays? (ii)the same birthday?(Ignoring a leap year)	2
11.	A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the number 1, 4, 9. What is the probability that the product xy of the two numbers will be less than 9?	2
SECTION-C		
CASE BASED QUESTION		5
12.	During the admission procedure in a school, the number of students seeking admission is more than that of the seats available in the class so that school administration decides to organize a draw so that each student has equal possibility of getting admission in the school. The following category of students applied for admission. 	2 2 1

- (A) If all the admission forms are shuffled and one form is drawn randomly, what is the probability that an OBC student belonging to either of the categories 1,2,3 or 4 will get admission?
- (B) If SC and ST category's admission forms are shuffled and one form is drawn randomly, what is the probability that student from service category 1 either SC or ST will get admission?
- (C) What is the probability of a general student of any category will get admission?

Service/ Social Category.	GEN	SC	OBC	ST
1	60	40	25	12
2	45	15	18	10
3	20	17	12	8
4	18	13	10	15
5	72	50	45	25
TOTAL	215	135	110	70

ANSWERS:

1. C5. D7. B

2. C6. C 8. $\frac{3}{4}, \frac{5}{23}$

11. $\frac{5}{9}$
12. (a) $\frac{65}{570}$ (b) $\frac{52}{205}$ (c) $\frac{43}{114}$

3. C9. $\frac{21}{26}$

4. A 10. (a) $\frac{364}{365}$ (b) $\frac{1}{365}$

CLASS TEST-1

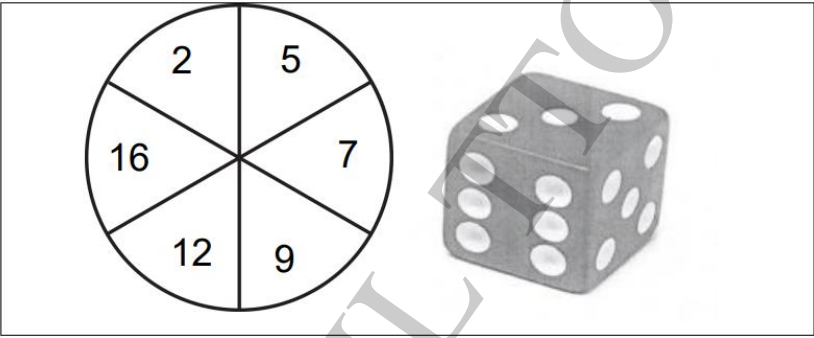
(CHAPTER: PROBABILITY)

CLASS: X

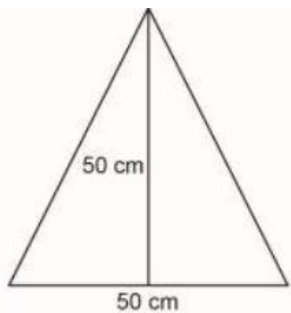
MARKS: 30

SN O	QUESTION	MA RK S
	SECTION-A	1X 10= 10
1.	If R is the event that it will rain tomorrow, such that $P(R)=0.03$, then $P(\bar{R}) =$ A) 0.07 B) 0.09 C) 0.79 D) 0.97	1
2.	Cards numbered 7 to 40 were put in a box. Anish selects a card at random. What is the probability that the selected card is a multiple of 7? A) $\frac{7}{34}$ B) $\frac{5}{34}$ C) $\frac{6}{35}$ D) $\frac{7}{35}$	1
3.	Which of the following cannot be the probability of an event? A) $\frac{-5}{7}$ B) 19% C) 0 D) 1	1

4.	If all cards of diamond are removed from the deck, find the probability that a card drawn at random from the deck is a red jack? A) $\frac{1}{52}$ B) $\frac{3}{52}$ C) $\frac{1}{39}$ D) $\frac{2}{39}$	1
5	A card is drawn at random from a pack of well-shuffled 52 cards. What is the probability that the card drawn is not an ace? A) $\frac{1}{13}$ B) $\frac{4}{13}$ C) $\frac{9}{13}$ D) $\frac{12}{13}$	1
6	What is the probability of choosing a black card or a ten from a deck of playing cards? A) $\frac{1}{2}$ B) $\frac{7}{13}$ C) $\frac{1}{13}$ D) $\frac{2}{13}$	1
7	T-shirts marked with numbers 2 to 101 are placed in a box. Sarita is fond of numbers which are perfect squares. When her turn comes, she randomly takes out a T-shirt from this box; what is the probability of getting her favourite T-shirt? A) $\frac{9}{100}$ B) $\frac{3}{10}$ C) $\frac{1}{10}$ D) $\frac{19}{100}$	1
8	A number x is chosen at random from the numbers -2, -1, 0, 1, 2. Then the probability that $x^2 < 2$ is? (A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$	1
9	A box contains marbles numbered from 12 to 78. What is the probability of getting a multiple of 12? A) $\frac{7}{66}$ B) $\frac{6}{67}$ C) $\frac{5}{66}$ D) $\frac{9}{67}$	1
10.	What is the probability of getting 53 Mondays in a non-leap year? (A) $\frac{1}{7}$ (B) $\frac{53}{366}$ (C) $\frac{2}{7}$ (D) $\frac{7}{366}$	1
SECTION -B		2×6 =12
11.	(i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective? (ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?	2
12.	A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded. (i) How many different scores are possible? (ii) What is the probability of getting a total of 7?	2
13.	There are 1000 sealed envelopes in a box, 100 of them contain a cash prize of Rs 100 each, 100 of them contain a cash prize of Rs 50 each and 200 of them contain a cash prize of Rs 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?	2

14.	A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.	2
15.	Box A contains 25 slips of which 19 are marked Re 1 and other are marked Rs 5 each. Box B contains 50 slips of which 45 are marked Re 1 each and others are marked Rs 13 each. Slips of both boxes are poured into a third box and reshuffled. A slip is drawn at random. What is the probability that it is marked other than Re 1?	2
16.	<p>1. A child has a die whose six faces show the letters as given below:</p> <p style="text-align: center;"> A B C D E A </p> <p>The die is thrown once. What is the probability of getting (i) A? (ii) D?</p>	2
SECTION-C		
CASE STUDY QUESTIONS		
4 X 2 = 8		
17.	<p>Diwali Fest is an annual South Asian arts & culture festival produced by the Diwali Celebration Society. In the Diwali fest, a game is played with a fair spinner, shown above. The numbers on the spinner are 2, 5, 7, 9, 12, 16. Sometimes the owner will invite a player who does not win with the spinner to throw the dice as a free bonus.</p> <div style="text-align: center; border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;">  </div> <p>(a) What is the probability that a player will get a special prize because the spinner stops on a perfect square?</p> <p>(b) If the player gets a chance to throw a dice, what is the probability of getting a multiple of 2 on the dice?</p>	2 2

1 In a classroom a rectangular board is fitted on a wall. The length of the board is 8
2m and width is 1m. Anuj draws this triangle on the board.



(a) Anuj

all

expected to fall in the triangular region?

(b) What is the probability of chunk of chalks no hitting the board?

throws a bunch of chalk
randomly at the board with
the chalks striking the board.
What proportion of chalks is

2
2

ANSWERS:

1. D 2. B 3. A 4. C. 5. D 6. B 7. C. 8. C 9. B 10. C

11. $\frac{1}{5}$, $\frac{15}{19}$ 12. $18, \frac{1}{2}$ 13. 0.69 14. $\frac{3}{4}$ 15. $\frac{64}{75}$ 16. $\frac{1}{3}, \frac{1}{6}$

17. $\frac{1}{3}, \frac{1}{2}$ 18. $\frac{1}{16}, 0$