X Chapter: Quardatic Equation Solved Question and Self Evaluation Question part-1

Solve $6x^2 - 5x - 25 = 0$

Solution Given $6x^2 - 5x - 25 = 0$.

First, let us find α and β such that $\alpha + \beta = -5$ and $\alpha\beta = 6 \times (-25) = -150$, where -5 is the coefficient of x. Thus, we get $\alpha = -15$ and $\beta = 10$.

Next, $6x^2 - 5x - 25 = 6x^2 - 15x + 10x - 25 = 3x(2x-5) + 5(2x-5)$
$= (2x-5)(3x+5)$.

Therefore, the solution set is obtained from $2x - 5 = 0$ and $3x + 5 = 0$

Thus, $x = \frac{5}{2}$, $x = -\frac{5}{3}$.

Hence, solution set is $\left\{-\frac{5}{3}, \frac{5}{2}\right\}$.

Solve $\frac{6}{7x-21} - \frac{1}{x^2-6x+9} + \frac{1}{x^2-9} = 0$

Solution Given equation appears to be a non-quadratic equation. But when we simplify the equation, it will reduce to a quadratic equation.

Now, $\frac{6}{7(x-3)} - \frac{1}{(x-3)^2} + \frac{1}{(x+3)(x-3)} = 0$

$\Longrightarrow \frac{6(x^2-9) - 7(x+3) + 7(x-3)}{7(x-3)^2(x+3)} = 0$

$\Longrightarrow 6x^2 - 54 - 42 = 0 \Longrightarrow x^2 - 16 = 0$

The equation $x^2 = 16$ is quadratic and hence we have two values $x = 4$ and $x = -4$.

$\therefore$ Solution set is $\{-4, 4\}$

Solve $\sqrt{24 - 10x} = 3 - 4x$, $3 - 4x > 0$

Solution Given $\sqrt{24-10x} = 3 - 4x$

Squaring on both sides, we get, $24 - 10x = (3-4x)^2$

$\Longrightarrow 16x^2 - 14x - 15 = 0 \Longrightarrow 16x^2 - 24x + 10x - 15 = 0$

$\Longrightarrow (8x+5)(2x-3) = 0$ which gives $x = \frac{3}{2}$ or $-\frac{5}{8}$

When $x = \frac{3}{2}$, $3 - 4x = 3 - 4\left(\frac{3}{2}\right) < 0$ and hence, $x = \frac{3}{2}$ is not a solution of the equation.

When $x = -\frac{5}{8}$, $3 - 4x > 0$ and hence, the solution set is $\left\{-\frac{5}{8}\right\}$.

Evaluation

Solve the following quadratic equations by factorization method.

(i) $(2x+3)^2 - 81 = 0$ (ii) $3x^2 - 5x - 12 = 0$ (iii) $\sqrt{5}x^2 + 2x - 3\sqrt{5} = 0$

(iv) $3(x^2 - 6) = x(x+7) - 3$ (v) $3x - \frac{8}{x} = 2$ (vi) $x + \frac{1}{x} = \frac{26}{5}$

(vii) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$ (viii) $a^2b^2x^2 - (a^2+b^2)x + 1 = 0$

(ix) $2(x+1)^2 - 5(x+1) = 12$ (x) $3(x-4)^2 - 5(x-4) = 12$

Solve the equation $a^2x^2 - 3abx + 2b^2 = 0$ by completing the square

Solution There is nothing to prove if $a = 0$. For $a \neq 0$, we have

$$a^2x^2 - 3abx + 2b^2 = 0$$

$$\Longrightarrow \quad x^2 - \frac{3b}{a}x + \frac{2b^2}{a^2} = 0 \qquad \Longrightarrow \quad x^2 - 2\left(\frac{3b}{2a}\right)x = \frac{-2b^2}{a^2}$$

$$\Longrightarrow \quad x^2 - 2\left(\frac{3b}{2a}\right)x + \frac{9b^2}{4a^2} = \frac{9b^2}{4a^2} - \frac{2b^2}{a^2}$$

$$\Longrightarrow \quad \left(x - \frac{3b}{2a}\right)^2 = \frac{9b^2 - 8b^2}{4a^2} \qquad \Longrightarrow \quad \left(x - \frac{3b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\Longrightarrow \quad x - \frac{3b}{2a} = \pm\frac{b}{2a} \qquad \Longrightarrow \quad x = \frac{3b \pm b}{2a}$$

Therefore, the solution set is $\left\{\frac{b}{a}, \frac{2b}{a}\right\}$.

Solve the quadratic equation $5x^2 - 6x - 2 = 0$ by completing the square.

Solution Given quadratic equation is $5x^2 - 6x - 2 = 0$

$$\Longrightarrow \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0 \qquad \text{(Divide on both sides by 5)}$$

$$\Longrightarrow \quad x^2 - 2\left(\frac{3}{5}\right)x = \frac{2}{5} \qquad \left(\frac{3}{5} \text{ is the half of the coefficient of } x\right)$$

$$\Longrightarrow \quad x^2 - 2\left(\frac{3}{5}\right)x + \frac{9}{25} = \frac{9}{25} + \frac{2}{5} \qquad \left(\text{add } \left(\frac{3}{5}\right)^2 = \frac{9}{25} \text{ on both sides}\right)$$

$$\Longrightarrow \quad \left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$\Longrightarrow \quad x - \frac{3}{5} = \pm\sqrt{\frac{19}{25}} \qquad \text{(take square root on both sides)}$$

Thus, we have $x = \frac{3}{5} \pm \frac{\sqrt{19}}{5} = \frac{3 \pm \sqrt{19}}{5}$.

Hence, the solution set is $\left\{\frac{3+\sqrt{19}}{5}, \frac{3-\sqrt{19}}{5}\right\}$.

Solution of quadratic equation by formula method

Consider a quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$.

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Longrightarrow \quad x^2 + 2\left(\frac{b}{2a}\right)x + \frac{c}{a} = 0 \qquad \Longrightarrow \quad x^2 + 2\left(\frac{b}{2a}\right)x = -\frac{c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ both sides we get, $x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

That is, $\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$$\Longrightarrow \quad x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

So, we have $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (1)

The solution set is $\left\{\frac{-b+\sqrt{b^2-4ac}}{2a}, \frac{-b-\sqrt{b^2-4ac}}{2a}\right\}$.

Self Evaluation

Solve the following quadratic equations using quadratic formula.

(i) $x^2 - 7x + 12 = 0$

(ii) $15x^2 - 11x + 2 = 0$

(iii) $x + \frac{1}{x} = 2\frac{1}{2}$

(iv) $3a^2x^2 - abx - 2b^2 = 0$

(v) $a(x^2+1) = x(a^2+1)$

(vi) $36x^2 - 12ax + (a^2 - b^2) = 0$

(vii) $\frac{x-1}{x+1} + \frac{x-3}{x-4} = \frac{10}{3}$

(viii) $a^2x^2 + (a^2-b^2)x - b^2 = 0$

The sum of a number and its reciprocal is $5\frac{1}{5}$. Find the number.

Solution Let x denote the required number. Then its reciprocal is $\frac{1}{x}$

By the given condition, $x + \frac{1}{x} = 5\frac{1}{5} \Longrightarrow \frac{x^2+1}{x} = \frac{26}{5}$

So, $5x^2 - 26x + 5 = 0$

$$\Longrightarrow \quad 5x^2 - 25x - x + 5 = 0$$

That is, $(5x-1)(x-5) = 0 \Longrightarrow x = 5$ or $\frac{1}{5}$

Thus, the required numbers are $5, \frac{1}{5}$.

The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq. cm, then find its base and altitude.

Solution Let the altitude of the triangle be x cm.

By the given condition, the base of the triangle is $(x+4)$ cm.

Now, the area of the triangle = $\frac{1}{2}(\text{base})\times\text{height}$

By the given condition $\frac{1}{2}(x+4)(x) = 48 \Longrightarrow x^2 + 4x - 96 = 0 \Longrightarrow (x+12)(x-8) = 0$ $x = -12$ or 8

But $x = -12$ is not possible (since the length should be positive)

Therefore, $x = 8$ and hence, $x + 4 = 12$.

Thus, the altitude of the triangle is 8 cm and the base of the triangle is 12 cm.

A car left 30 minutes later than the scheduled time. In order to reach its destination 150km away in time, it has to increase its speed by 25 km/hr from its usual speed. Find its usual speed.

Solution Let the usual speed of the car be x km/hr.

Thus, the increased speed of the car is $(x + 25)$ km/hr

Total distance = 150 km; Time taken = $\frac{\text{Distance}}{\text{Speed}}$.

Let T_1 and T_2 be the time taken in hours by the car to cover the given distance in scheduled time and decreased time (as the speed is increased) respectively.

By the given information $T_1 - T_2 = \frac{1}{2}$ (30 minutes = $\frac{1}{2}$hr)

$$\implies \frac{150}{x} - \frac{150}{x+25} = \frac{1}{2} \implies 150\left[\frac{x+25-x}{x(x+25)}\right] = \frac{1}{2}$$

$$\implies x^2 + 25x - 7500 = 0 \implies (x+100)(x-75) = 0$$

Thus, $x = 75$ or -100, but $x = -100$ is not an admissible value.

Therefore, the usual speed of the car is 75 km/hr.

Self Evaluation

1. The sum of a number and its reciprocal is $\frac{65}{8}$. Find the number.
2. The difference of the squares of two positive numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.
3. A farmer wishes to start a 100 sq.m rectangular vegetable garden. Since he has only 30 m barbed wire, he fences the sides of the rectangular garden letting his house compound wall act as the fourth side fence. Find the dimension of the garden.
4. A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it having an area of 111 sq. metres. Find the width of the path on the outside.
5. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.
6. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and return downstream to the original point in 4 hrs 30 minutes. Find the speed of the stream.
7. One year ago, a man was 8 times as old as his son. Now his age is equal to the square of his son's age. Find their present ages.
8. A chess board contains 64 equal squares and the area of each square is 6.25 cm^2. A border around the board is 2 cm wide. Find the length of the side of the chess board.
9. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time that B would take to finish this work by himself.
10. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after two hours, they are 50 km apart, find the average speed of each train.